FCS Midterm Question

- 1. Without using the truth table show the following statements are true. Explain your reasoning.
 - (a) $(p o q) \lor (p o r) \equiv \neg r o (\neg p \lor q)$ [3 marks]
 - (b) $(p
 ightarrow \neg q) \land \neg p \equiv p
 ightarrow (\neg q \land \neg p)$ [3 marks]
 - (c) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \lor r)$ [3 marks]
- 2. Prove the following statement by induction.

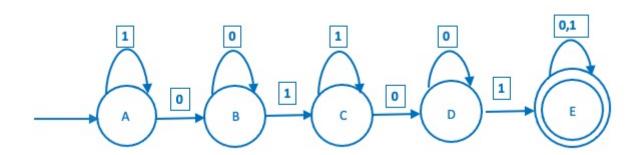
For all nonnegative integers n, 3 divides $n^3 + 2n + 3$. State the mathematical induction and show your work clearly. [9 marks]

3. Students are required to create **6-character** long passwords to access the library. The letters must be from lowercase letters or digits. Each password must contain **at least two** lowercase-letters, **at least one** digit and contains **no repeated digits**. How many valid passwords are there? You are required to show your work step-by-step. [9 marks]

Note:

1hfgt1 is invalid because 1 appears more than once. 134ggg is valid because there are 3 lower-case letters and digits are not repeated.

4. Consider the following automaton.



- a) Give an example of a string containing 11 that is **accepted** by the following automaton. [2 marks]
- b) Give an example of a string of length 8 that is **rejected** by the following automaton. [2 marks]
- c) Describe the language of this automaton in plain English. [4 marks]
- d) Describe the language of this automaton using Regular expression. [3 marks]
- 5. Given $R = (0*10^+)^+1^*$ and $S = (1*01^+)^*$
 - a) Give an example of a string that is neither in the language of R nor in S. [2marks]
 - b) Give an example of a string that is in the language of S but not R. [2 marks]
 - c) Give an example of a string that is in the language of R but not S. [2 marks]
 - d) Give an example of a string that is in the language of *R* and *S*. [2 marks]
 - e) Design a regular expression that accepts the language of all binary strings with no occurrences of bab [4 marks]