UNIVERSITY OF LONDON

CM2035

BSc EXAMINATION

COMPUTER SCIENCE

Algorithms and Data Structures II

Release date: Monday 2 October 2023 at 12:00 midday British Summer Time

Submission date: Tuesday 3 October 2023 by 12:00 midday British Summer Time

Time allowed: 4 hours to submit

INSTRUCTIONS TO CANDIDATES:
Part A of this assessment consists of a set of **TEN** Multiple Choice Questions (MCQs). You should attempt to answer **ALL** the questions in **Part A.** The maximum mark for Part A is **40**.

Candidates must answer **TWO** out of the **THREE** questions in **Part B**. The maximum mark for Part B is **60**.

Part A and Part B will be completed online together on the Inspera exam platform. You may choose to access either part first upon entering the test area but must complete both parts within **4 hours** of doing so.

Calculators are **not** permitted in this examination. Credit will only be given if all workings are shown.

Do not write your name anywhere in your answers.

PART A

Candidates should answer the **TEN** Multiple Choice Questions (MCQs) in Part A of the test area.

PART B

Candidates should answer any **TWO** questions in Part B.

Question 1

This question concerns max-heaps.

In this question, the indices of any array, `A`, range from `1` to `A.length` i.e. the first element of `A` is `A[1]` and the last is `A[A.length]`.

a. What properties must a max-heap satisfy?

[4 marks]

b. Suppose a max-heap is implemented by an array. Write three pseudocode functions `PARENT(i)`, `LEFT(i)` and `RIGHT(i)`, that return the indices of the parent, left and right nodes of node i.

[6 marks]

c. The inputs to function `X` below are an array (`heap`), and a value (`k`). What is the purpose of `X`? Explain how `X` accomplishes its objective.

```
function X(heap, k)
     pos = heap_size + 1
     heap[pos] = k
     heap_size++
     while pos > 1 AND heap[PARENT(pos)] < heap[pos]
          swap(heap[PARENT(pos)], heap[pos])
          pos = PARENT(pos)
```

[6 marks]

d. Write a function `BUILD(A)` that uses `X` to build a heap from an unsorted array `A` of values. [4 marks]

e. How does heap sort embody the good properties of insertion sort and merge sort? [4 marks]

f. Provide a real-world application of the heap data structure. Explain why a heap is particularly suited to the application.

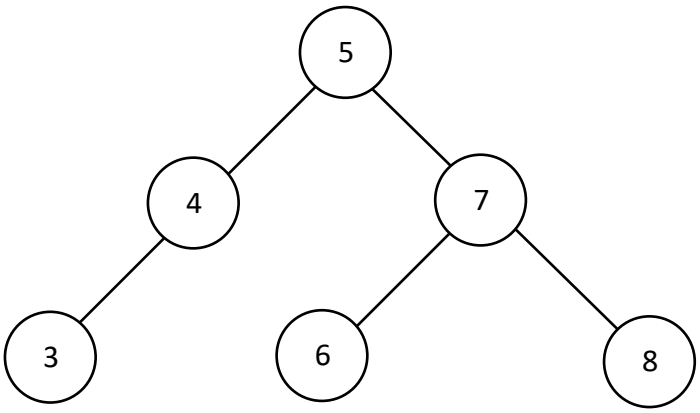
[6 marks]

Question 2

This question concerns Binary Search Trees (BSTs).

a. What property distinguishes a binary search tree (BST) from any other binary tree? [4 marks]

b. Consider the binary search tree in the diagram below:



Suppose the tree is traversed in post-order and the content of each node is printed when visited. What is printed to the screen?

[4 marks]

c. Write pseudocode for a function `HIGH-TO-LOW` that takes the root of a BST as a single argument and prints the values of the tree in descending order.

[4 marks]

d. Write a pseudocode function `TREE-MAX` that returns the maximum-value node of a BST. (The maximum-value node of the BST in part (a) is the node with value 8.)

[4 marks]

e. What is the time complexity of `TREE-MAX` as a function of the number of nodes, n, of the tree. Justify your answer.

[4 marks]

f. i. What is the average and worst-case time complexity for searching a BST?

[2 marks]

ii. Draw a 4-node tree which has worst-case search time complexity.

[2 marks]

iii. What circumstance would lead to the construction of this worst-case tree?

[2 marks]

iv. What steps could be taken to mitigate effects of the worst-case circumstance?

[4 marks]

Question 3

This question concerns topics from part 1 of the syllabus.

a. Explain why $5n = O(n^2)$ but $5n \neq \Theta(n^2)$.

[4 marks]

b. What is the running time of `SUM`? Show your working.

```
function SUM(n)
     if (n == 0) return 1;
     else return n * SUM(n - 1)
end function
```

[4 marks]

c. When is it better to use counting sort rather than mergesort to sort an array in ascending order?

[4 marks]

d. When is it better to use mergesort rather than counting sort to sort an array in ascending order.

[4 marks]

e. The array [23, 59, 14, 93, 1] is sorted by radix sort. Write down the array after a single pass of the algorithm.

[4 marks]

f. Briefly explain why hash tables are useful for storing many integers.

[4 marks]

g. Explain how hash functions are used to build a hash table. Ignore collisions and illustrate your answer by considering a function h(k) = k mod 5 hashing into a table with 5 slots.

[6 marks]

END OF PAPER