Question 1 This question is about number systems

a)

i. Convert the decimal number 103_{10} to binary. **Show all your working**

ii. Write the binary number 101100.101_2 in expanded form and hence find its decimal equivalent

iii. Convert the hexadecimal number $A5.7_{16}$ into binary. **Show all your working**

iv. Hence or otherwise convert the hexadecimal number $A5.7_{16}$ number into octal.

[6]

b) Working in binary perform the following calculation $100010_2 - 1101_2$

[2]

c) Convert the recurring decimal number $0.429429\ldots$ to a fraction in its simplest form.

[2]

Question 2 This question is about set theory

1.

a.

i. Describe the following set by the listing method

$$\{2^n : n \in \mathbb{Z} \; and \; 3 \leq n \leq 6\}$$

ii. Describe the following set by giving a suitable universal set and rules of inclusion $\{3, 5, 7, 9\}$

[3]

b. Let $X = \{a, e, i\}$ and $Y = \{a, b, c, d, e\}$ be subsets of a universal set $U = \{a, b, c, d, e, f, g, h, i, j\}$. Find the following

i. $\bar{X} \cap \bar{Y}$

ii. $\overline{X \cup Y}$

[3]

c. Let A, B and C be subsets of a universal set U

i. Construct a membership table for the set $A \cap \overline{B \cup C}$

ii. By using membership tables or otherwise show that $A \cap \overline{B \cup C} = A \cap \bar{B} \cap \bar{C}$.

[4]

Question 3 This question is about trigonometric and exponential functions

Give your answers to 1 decimal place or the nearest degree.

a. ABC is a right angled triangle with $A = 90°, AB = 10cm, BC = 23cm$
   i. Find length AC
   ii. Find angle B

[2]

b. FGH is a triangle with $F = 122°, GH = 23cm, G = 43°$. Find length FH

[2]

c. XYZ is a triangle with $XY = 15cm, XZ = 21cm, YZ = 18cm$. Find angle Y

[2]

d.

   i. By completing a table of values or otherwise, sketch the function $f(x) = \cos x$ for $x \in [-360°, 360°]$

   ii. Find all the values of $x \in [-360°, 360°]$ for which $\cos x = 0.3$

[4]

Question 4 This question is about graph sketching

a. Given the function $f(x) = 2x + 3$
   i. Find the gradient
   ii. Find the y-intercept
   iii. Draw the graph of the function
   iv. Draw the graph of the inverse function $f^{-1}(x)$

[5]

b. Find the minimum value of the function $g(x) = (x - 4)(x + 1)$

[2]

c. Sketch the function $h(x) = x(x + 2)(x + 3)$

[3]

Question 5 This question is about the binomial theorem, calculus and velocity and acceleration

d.

i. Write out the first four terms of following binomial expansion. (You may use the formula)

$$(1+x)^8$$

[2]

ii. Find the value of the 1.003^8 to 2 decimal places by using the binomial expansion of $(1+x)^8$ with an appropriate value of x. You may use your answer to part a. above

[2]

e. Find $\frac{dy}{dx}$ the gradient function of $y = x^5 + 3x + 2$

[2]

f. The acceleration of a particle is given by $a = 3t^2 + 2$, where a is the acceleration in $ms^{-2}$and t is the time in seconds

i. Use your expression for the acceleration to find the acceleration of the particle when
   1. $t = 0$ seconds
   2. $t = 2$ seconds

[2]

ii. Which of the following could give the velocity of the particle $v = 6t + 2,\ v = 6t, v = t^3 + 2t, v = t^3 + 2t + 2$

[2]

Question 6 This question is about vectors and matrices

a. Given the vectors $\mathbf{u} = \begin{pmatrix} 1 \\ \sqrt{5} \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ \sqrt{5} \\ 2 \end{pmatrix}$

i. Write **u** in terms of the unit vectors **i, j** and **k**
ii. Find the magnitudes of **u** and **v**
iii. Compute **u** . **v,** the dot product of **u** and **v**
iv. Find the angle between **u** and **v**

[7]

b.

i. Find the determinant of the matrix $M = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$
ii. State the determinant of M^{-1}

[3]

Question 7 This question is about matrix transformations

a. Given matrices

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

i. Describe the transformations represented by A and B

[2]

ii. Calculate BA the product of the matrices A and B

[2]

iii. A triangle has vertices (1, 1) (1, 2) and (2, 1). Find the vertices of the triangle after it has been transformed by matrix A followed by matrix B

[2]

b.

i. Find the 3 by 3 matrix R which represents a rotation of 90° anticlockwise about the origin

[2]

ii. Given that R represents a rotation of 90° anticlockwise about the origin, describe the single transformations represented by

1. R^2
2. R^{-1}

[2]

Question 8 This question is about complex numbers

a. Given complex numbers $z_1 = -1 + i$ and $z_2 = 1 + 2i$

i. Find $z_1 \times z_2$, give your answer in the form $a + ib$

[1]

ii. Find $z_1 \div z_2$, give your answer in the form $a + ib$. You may use the complex conjugate

[1]

iii. Convert z_1 to polar form

[2]

iv. Hence convert z_1 to exponential form

[1]

b. Given complex numbers $z_1 = 2e^{\frac{\pi}{4}i}$ and $z_2 = \frac{1}{2}e^{-\frac{\pi}{3}i}$

v. Find $z_1 \times z_2$, give your answer in exponential form

[1]

vi. Find ${z_1}^3$, give your answer in exponential form

[2]

vii. One possible value of ${z_1}^{\frac{1}{3}}$ is $\sqrt[3]{2}e^{\frac{\pi}{12}i}$. Find the remaining values of ${z_1}^{\frac{1}{3}}$. Give your answers in exponential form

[2]

END OF EXAMINATION