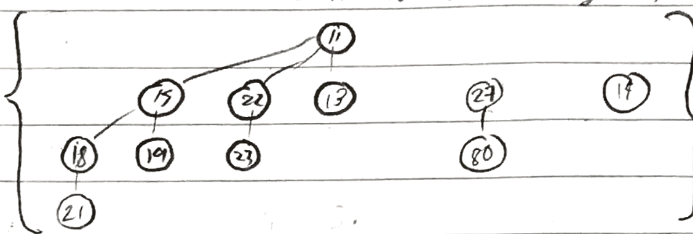
CSC263 Jan 17, 2018

	ADT	DS.	Opprations
Recall:	Mergeable PQ	Binomial heap	· min(S) · Insert(S,x) · Extractmin(S) · merge(P,Q) → in $O(\log n)$

Ex: $m = 11 = \langle 1011 \rangle_2$ ∴
∴ $F_{11} = \langle S_3, S_1, S_0 \rangle$



MERGE: $|T| = 3 = \langle 11 \rangle_2 = \langle S_1, S_0 \rangle$ ∴ T+Q =
(T, Q) $|Q| = 7 = \langle 111 \rangle_2 = \langle S_2, S_1, S_0 \rangle$

$$\begin{array}{rrrr} & S_2 & S_1 & \\ & & S_1 & S_0 \\ + & S_2 & S_1 & S_0 \\ \hline S_3 & X & S_1 & X \end{array} \quad \Leftrightarrow \quad \begin{array}{rrrr} & 1 & 1 & \\ & 0 & 1 & 1 \\ + & 1 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 \end{array} = 10 = |T+Q|$$

· $|T| \le m$, $|Q| \le m$ ∴ Merge(T,Q) $\in O(\log m)$ # of carries
↳ there are at max log m colums ∴ max log m comparisons
* ↳ note: unclear to me why this is max log n

INSERT(Q,7): equiv to 'merge' with an S_0 tree ∴ $|Q| \le m \to O(\log n)$

MIN: will be root nodes of one of the trees; compare roots
* ↳ note: unclear to me why this is max log n

Extract Min: removing root of S_k tree results in $\{S_i\}$ for $i \in \mathbb{N}$, $0 \le i \le k-1$
becomes a merge of these subtrees & trees in F whose roots ≠ min

· when you do a bunch of merges
$|T| = 27 = 16 + 8 + 2 + 1 = \langle 11011 \rangle_2$

T + {x_1}: 11011 + 1 = 11100 ∴ 2 comps
T + {x_2}: + 1 = 11101 ∴ 0 comp
T + {x_3}: + 1 = 11110 ∴ 1 comp
T + {x_4}: + 1 = 11111 ∴ 0 comp
T + {x_5}: + 1 = 100000 ∴ 5 comp

↳ $|T + \{x_i \mid i \in \mathbb{N}, 1 \le i \le 5\}| = 32$

- there is a new edge iff comparison
- average cost of insert = average comparisons = average edges
- to find the average cost of insert, must evaluate average edges on S_k trees
↳ related homework problem

- DICTIONARY (ADT): set S of elements with keys
  ↳ OPS: INSERT(X); DELETE(X); SEARCH(X)
    ↳ goal to have these occur in $O(\log n)$

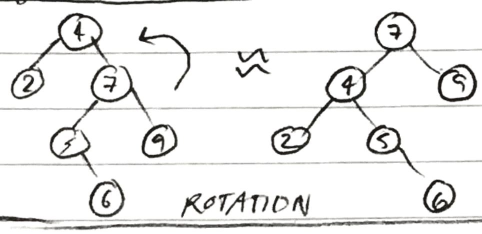
  How could we implement this?

- BINARY SEARCH TREE:
  Recall property given root x: L-child keys $\leq x$, R keys $\geq x$

SEARCH = In order transversal: (recursively) check left side, root, right side
↳ Results in keys listed in increasing order

EX: $S = \{2, 4, 5, 6, 7, 9\}$

TREES ARE NOT UNIQUE



DELETE: leaf nodes are trivial to remove
what if we remove node x?
↳ get smallest node from tree, new x
↳ recurse to replace smallest node deletion