CSC263 Monday Lecture: Depth-First Search

Mar 12, 2018

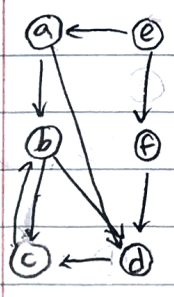
- Breadth-first: first discovered, first explored // Queue
- Depth-first: last discovered, first explored // stack

- As in breadth-first search, the DFS algo tracks at node v:
  - colour[v]: WHITE (undiscovered), GREY (discovered), BLACK (explored).
  - P[v] = u: "u discovered v", "u is parent of v in discovery tree"
- Differences new to DFS:
  - d[v]: "time" at which node v was discovered
  - f[v]: "time" at which node v's exploration completes
  - "time" tracked by counter that starts at 0, increments as single events take place in algo.

```
DFS(G)
    for each v ∈ V:    //initialize
        colour[v] ← W; d[v] ← ∞; f[v] ← ∞; P[v] ← NIL;
    time ← 0
    for each v ∈ V:
        if colour[v] == W: then DFS-EXPLORE(G,V)

DFS-EXPLORE(G,u)
    colour[u] ← GREY, TIME++, d[u] ← TIME    // discover u
    for each edge (u,v) ∈ E    //explore edges
        if colour[v] == W:
            P[v] ← u; DFS-EXPLORE(G,v)    // discover & explore v
    colour[u] ← BLACK, TIME++, f[u] ← TIME
```

Ex:

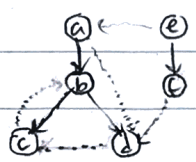


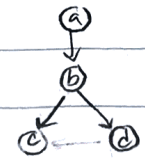
- a → b → d
- b → c → d
- c → b
- d → c
- e → a → f
- f → d

B	1	8
B	2	7
B	3	4
B	5	6
B	9	12
B	10	11

c	d	f

Reconstructed tree of discovery paths:





without attempted lines, called DFS forest

- time boundaries $(d[v], f[v])$ of an ancestor node v form the boundaries of time intervals of its children
  - ie: children discovered & searched within function call searching ancestor nodes

- A DFS of a directed $G=(V,E)$ classifies edge (u,v) as follows:
- ① Tree edge: iff v discovers v

  Non-tree edges:
  - ② "forward edge": from ancestor to descendant in discovery tree
  - ③ "backwards edge": from descendant to ancestor
  - ④ "cross edge": no decendancy relation

- worst case running time of DFS:
  - constant time for every element in adjacency list
  - $\therefore$ worst case $\in O(|V|+|E|) = O(n+m)$

CL1 • u ancestor of v iff $d[u] < d[v] < f[v] < f(u)$

CL2 • for any u, v can never have $d[u] < d[v] < f[u] < f[v]$

CL3 • If v discovered v, $d[v] < f[u]$

- Next Class: Applications of DFS

  Will prove that a backwards edge in the DFS discovery tree indicates the presence of a cycle in the graph explored.