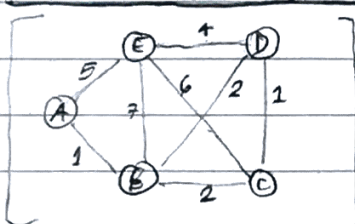


- Overview: using material presented in 03-19-18, introduce two algos for building Minimum Spanning Trees
 - One in detail today, one in Friday tutorial.

Recall: MST construction theorem (Monday & proof online)

KRUSKAL'S ALGORITHM

OVERVIEW



• Store ordered list of edges:

AB 1	BD 2	ED 4	EC 6
CB 1	BC 2	EA 5	EB 7

* Steps to the algorithm is as follows:

- build min heap of edges

CHECK MIN EDGE	EDGE IN MST?	FOREST
		$\{A\} \{B\} \{C\} \{D\} \{E\}$
(A,B) 1	YES	$\{A, B\} \{C\} \{D\} \{E\}$
(C,D) 1	YES	$\{A, B\} \{C, D\} \{E\}$
(B,D) 2	YES	$\{A, B, C, D\} \{E\}$
(B,C) 2	NO	"
(ED) 4	YES	$\{A, B, C, D, E\}$

- check min weight edge
- use disjoint sets & union/find to check if edge connects diff set.
 - connect if different sets
- continue until one set left

ALGORITHM CODE:

[G: connected undirected weighted graph $G=(V,E)$, $|V|=n$, $|E|=m$
 $V = \{1, 2, 3, \dots, n\}$ • $E = \text{array of edges } [(1,4)4], [(2,3)7], \dots]$

• BUILD-MINHEAP(E), FOREST $\leftarrow \{\{1\} \{2\} \dots \{n\}\}$, MST-EDGES $\leftarrow \emptyset$

• WHILE ($|MST-EDGES| \leq n-1$)

$(u,v) \leftarrow \text{EXTRACT_MIN}(E)$

$T_u \leftarrow \text{FIND}(u)$; $T_v \leftarrow \text{FIND}(v)$

IF ($T_u \neq T_v$)

$\text{UNION}(T_u, T_v)$

$MST-EDGES \leftarrow MST-EDGES \cup \{(u,v)\}$

• Some notes about implementation:

- use tree-based UNION/FIND DS with weighted union & path compr.

→ Upper bound on running time of KRUSKAL's:

- Building min heap $\in O(m)$
- initialize node forest $\in O(n)$
- INSIDE WHILE LOOP: (occurs $< n-1$ times)
 - extract.min(E) $\in O(m \log m)$ \otimes
 - $n-1$ unions, $2m$ finds $\in O(m \log^* n)$

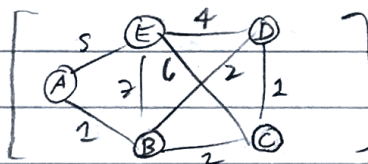
$$[m \leq n^2]$$

$$\therefore O(m) + O(n) + O(m \log m) + O(m \log^* n) \in O(m \log m)$$

$$= O(m \log m) = O(m \log n^2) \quad // \quad m \leq n^2$$

$$= O(m \log n) \equiv O(|E| \log |V|)$$

• Preview of Prim's ALGO:

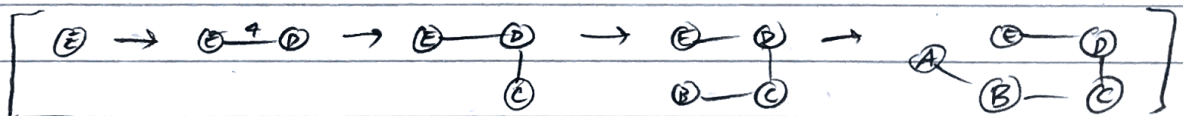


• start w/ single node

• take min edge out of node, add

• take min edge from component, add

• continue until all nodes connected



• Next week: NP / Problems