CSC263 Wednesday lecture: cont. DFS Mar 14, 2018

Recall:

CL1 • (u ancestor of v) $\longleftrightarrow$ ($d[u] < d[v] < f[v] < f[u]$)

CL2 • for any $u, v \in V$, can never have $d[u] < d[v] < f[u] < f[d]$

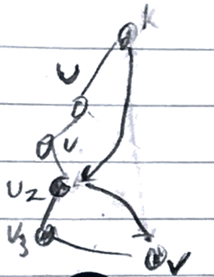
CL2 • ($P[v] = u$) $\longrightarrow$ $d[v] < f[u]$

• note that a single graph G can have many distinct DFS discovery forests, depending on initial node searched

Today:

• <u>WHITE PATH THEOREM</u> - for all G & DFS of G:

(v becomes descendant of u) $\longleftrightarrow$ (at $d[u]$, $\exists$ path from $u \to v$ with only white nodes)



• <u>Consider any G and any DFS of G:</u>

(A) suppose v becomes a descendant of u in this DFS:

- ie $\exists$ discovery path from u to v
- if at $d[u]$ nodes on discovery path are already discovered, v would not be made descendant of u (other node would reach v first) $\therefore$ they must be white

$\boxed{\therefore \text{(v descendant of u)} \longrightarrow (\exists \text{ white discovery path})}$

I anticipated the second half of this proof but wrote it informally

(B) suppose at $d[u]$, there is a path $u \to v$ of only white nodes

<u>Claim:</u> all nodes in the path (including v) become descendant of u.

- Suppose the claim is false. Let z be closest node to u on path that does not become descendant of u.
- Then u has edge back to w, which is white (becuz z first undisc

① z white when u discovered $\longrightarrow$ $d[u] < d[z]$

② by CL3 applied to (w, z) $\longrightarrow$ $d[z] < f[w]$

③ $w = u$ or descendant of u $\longrightarrow$ $f[w] \le f[u]$

$\to$ ($d[u] < d[z] < f[u]$) $\longrightarrow$ $d[u] < d[z] < f[z] < f[u]$ $\longrightarrow$ z descendant of u!

Then $\therefore$ contradiction, so claim must not be false!

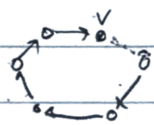
$\boxed{\therefore \text{(disc. path of white nodes)} \longrightarrow \text{(all on path descendants of u)}}$

Thm 22.11 · $\forall$ directed graphs G, $\forall$ DFS of G:

(G has a cycle) $\longleftrightarrow$ (G has backedge)

· Suppose DFS of G has a back edge $v \cdots\to u$.
We know $\exists$ solid discovery path from u to v.
The back edge then completes cycle containing u, v.
$\therefore$ (G has back edge) $\longrightarrow$ (G has cycle)

· Suppose G has a cycle C.



u: first node of c discovered by DFS
v becomes descendant of u by WPT
then when link from v to u searched, back edge.

$\therefore$ (G has cycle) $\longrightarrow$ (G has backedge)