CSC263 Wednesday Lecture Feb 14, 2018

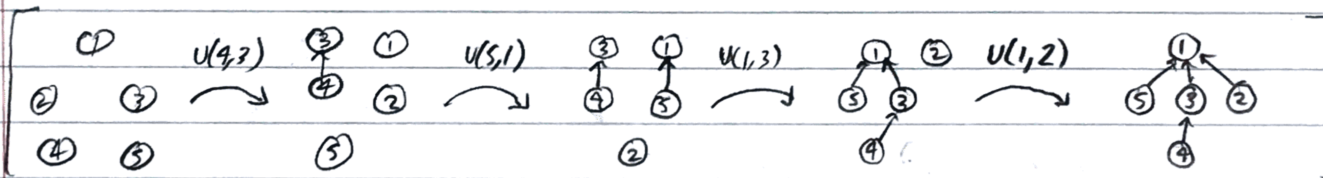
... continued

Starting from $\{1\}, \{2\}, \ldots, \{n\}$: want to execute sequence of operations R: $n-1$ unions, $m \geq n$ finds.

FOREST STRUCTURE

- cost per union: $O(1)$
- cost per find: $O(1+$ length of find path
- weighted union rule: make smaller tree child of larger
- root of tree is representative element

EX



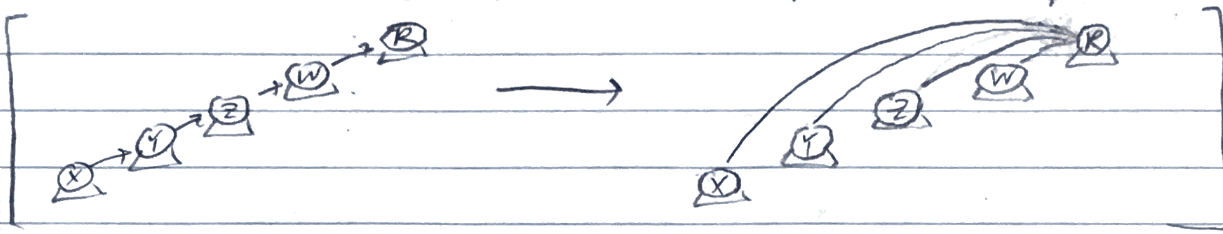
- weighted union by:
  - size - fewer nodes tree child of more nodes tree
  - rank - smaller height tree child of taller height tree

➤ Claim: using weighted union rule produces trees of height $\leq \log n$

- Claim: any tree T of height h created during the execution of R has at least $|T| \geq 2^h$ nodes. Prove via induction on h.
  - $h=0$, $|T| = 1 = 2^0$ ∴ holds for $h=0$
  - Assume holds for some arbitrary $h \geq 0$. Must show holds for $h+1$. Tree T of height $h+1$ is created by unioning 2 trees A, B. One child of T, A, has height h (by I.H. has $\geq 2^h$ nodes). We know (WU rule used) $B_h \geq A_h$.
    ∴ $|T| = |A| + |B| \geq 2^h + 2^h = 2^{h+1}$ nodes ∴ holds for $h+1$.

$2^h \leq |T| \leq m \longmapsto h \leq \log_2 n$

- **Path compression**; PC rule
  - after once traversing Find path and arriving at representative node, store direct pointer to representative



  - decreases height of trees, saves work during FIND operations for all future children of path-compressed nodes.

→ What is the cost of doing R with weighted union and path compression rules??

- **Define** $x^{*}y$: $x^{*0} = 1$; $x^{*(n+1)} = x^{x*(n)}$

Ex $2^{*0} = 1$; $2^{*1} = 2^{1} = 2$; $2^{*2} = 2^{2} = 4$; $2^{*3} = 2^{2^{2}} = 16$ } Grows very quickly!
$2^{*4} = 2^{2^{2^{2}}} = 2^{16} = 65536$; $2^{*5} = 2^{65536} = 10^{19729}$

- **Define** $\log_{*} n = m\{k : 2^{*k} \geq m\}$

Ex

n	2	3 4	5 6 ... 16	17 ... 65536	65537 ...... 2^{65536}
$\log^{*}n$	1	2 2	3 ... 3	4 ... 4	5 5

} Grows <u>very</u> slowly!

1973 - Somebody proved that cost of executing R with WU & PC is $O(m \cdot \log^{*} n)$
  - mathematically $\log^{*}n$ increases forever, realistically near constant

1975 - Somebody proved $R \in O(m \cdot \alpha(m,n))$
  - $\alpha(m,n)$: inverse ackerman's function, grows slower than $\log^{*}n$!
  - what if ackermans is artifact of analysis? Find Θ?

1979 - Somebody proved $R \in \Omega(m \cdot \alpha(m,n))$ relying on assumptions

1989 - Somebody proved $R \in \Omega(m \cdot \alpha(m,n))$ with no assumptions

1964 - Algo for finds & unions first created