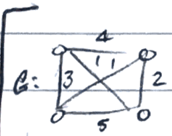
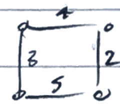
CSC263 Monday Lecture: Travelling Salesman Prob   Mar 26, 2018

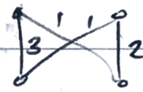
- G: completely connected, undirected graph with non-negative weights (ie. each edge (u,v) has cost $c(u,v) \geq 0$)

- Tour of G: cycle C that includes every node of G exactly once

* Q: what is the cheapest tour of a graph G?
↳ called the "optimal tour" or "TSP tour"

EX [G: 

| T_1: 

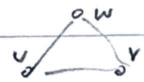
$c(T_1) = 14$ | T_2: 

$c(T_2) = 7$]

- There is as of yet no "efficient" (polynomial time) algo to solve!

- The TSP problem is one of the class of NP-complete problems; finding polynomial algo to solve it proves all NP complete probs can be solved via an efficient algorithm!!!

➤ SIMPLIFIED TSP: Δ-TSP



- Δ-INEQUALITY: $\forall\ u,v,w \in V$: $c(u,v) \leq c(u,w) + c(w,v)$
- Assume Δ-inequality to simplify the TSP...
- Δ-TSP is still an NP complete problem
- We can find an approximate solution to the Δ-TSP efficiently, and the resulting tour of G, T has $c(T) \leq 2 \cdot c(T_*)$ where T_* is the optimal tour of G.

Lemma 1
- $c(MST) \leq c(OT)$: MST = Min span tree of G, OT: optimal tour of G
- Proof: Consider any OT. Take $(u,v) \in T$ and remove it. The result is a spanning tree ST whose cost is less than OT. Then we have

$$c(MST) \leq c(ST) \leq c(OT)$$

- What is the efficient approximate solution of the Δ-TSP?

Δ-TSP Algorithm

- find an MST of G using Kruskal's algo $O(m \log n)$
- do a full walk of the MST:
  - result is a cycle C_{MST}, with $2|E_{MST}|$ edges, some repeat nodes.
  - Note that $c(C) = 2 \cdot c(MST)$
- whenever there is repeated nodes in C, replace w. direct edge
  - complete graph, $\therefore$ such edge exists
  - Δ-inequality, $\therefore$ direct edges cheaper than indirect
- Result C_* is a tour (no repeat nodes)
  - $c(C_*) \leq c(C) = 2 \cdot c(MST$
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- With Lemma 1: $c(C^*) \leq 2 \cdot c(MST) \leq c(OT)$, $\therefore$ approximate soln.

- Someone has figured out how to get soln. within factor of 1.5
  - ↳ at some point problem becomes NP complete w/ better approximations