CSC263 Friday Tutorial: Midterm Solutions — Mar 16, 2018

```
RANDOM SEARCH(A, x)
while true:
    pick i uniformly from {1, ..., n}
    if A[i] == [x]
        return i
    else: with probability p, return "not found"
```

a) • Given $x \notin A$, $P(k \text{ iterations of loop}) = P(k-1 \text{ no returns; } k^{th} \text{ returns})$
$= (1-p)^{k-1} p$

• Expected # iterations of the while loop?
• let random variable k = # of iterations of while loop.
• $E[k] = \sum_{k=1}^{\infty} kP(k=k) = \sum_{k=1}^{\infty} (1-p)^{k-1} p = p \sum_{k=1}^{\infty} (1-p)^{k-1}$

Notice that: $\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$ $\therefore (d/dr) \sum r^k = (d/dr) \frac{1}{1-r}$ $\therefore \sum k r^{k-1} = \frac{1}{(1-r)^2}$
$\therefore E[k] = p \sum_{k=1}^{\infty} k(1-p)^{k-1} = p(\frac{1}{p^2}) = \frac{1}{p}$

b) Exactly one $j \in A$. s.t. $A[j] = x$. Probability of exactly k iter of while?
• $P(\text{return in an iteration}) = P(\text{return in if statement}) + P(\text{return in else})$

$$q = \frac{1}{n} + (1 - \frac{1}{n})p$$

• $(1-q)^{k-1} q = (1 - \frac{1}{n} - (1-\frac{1}{n})p)^{k-1} (\frac{1}{n} + (1-\frac{1}{n})p)$

7 Give a linear time algo to determine if BST is AVL tree.

```
Height(v)
  if v == NIL : return -1
  else :
    h_L = height(v.left)
    h_R = height(v.right)
    if (h_L == unbalanced ||
        h_L == unbalanced ||
        |h_R - h_L| > 1 ) return unbalanced
    else return max(h_L, h_R) + 1;
```

• Height called once per node exactly
• all non-recursive calls & computation is constant
∴ significant factor of running time is # of nodes
∴ linear time

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- max # rotations for insertion from AVL $\in O(1)$
- max # rotations to delete from AVL $\in O(\log n)$
- inserting n unsorted elements into binomial heap $\in O(n)$

➢ how to determine if undirected graph connected with BFS? Run BFS, if any nodes white after finishing first node, graph is not connected.

➢ how to label connected components of undirected graph?
- A: start counter at 1, do BFS for one node, all coloured during search of this node get labelled w/ counter.
- when node searched, increment counter, go to next white node.