CSC263 Wednesday Lecture (Guest Lecturer) Feb 7, 2018

- What is the difference between average & expected running time?

Running time of a single execution of an algorithm depends on:

1) the particular input data
2) the "random choices" made within the algorithm

- worst case: worst cases of both ① and ②
- average case: only discussed re: deterministic algorithms
  - average running time of all inputs
  - should be averaged over distribution of inputs (not equally likely)
  - cannot be computed without knowing input probability distribution
- expected (worst case): assume worst case input ①, then compute expected runtime given distribution of ②

```
RAND-PARTITION(A, p, r)
    i = RAND(p, r)
    exchange A[r] with A[i]
    return PARTITION*(A, p, r)
```

* PARTITION from regular quicksort

```
RAND-QUICKSORT(A, p, r)
    if p<r
        q = RAND_PARTITION(A, p, r)
        RAND_QUICKSORT(A, p, q-1)
        RAND_QUICKSORT(A, q+1, r)
```

To Prove: → **Theorem**: the expected # of comparisons in RAND_QUICKSORT is at most $O(n \log n)$ (ie. worst-case expected running time $O(n \log n)$).

- Variability in work done in quicksort all comes from comparisons in Partition
- elements are only compared to pivot
- i^{th} & j^{th} smallest elements are "split" by Partition exactly once
- write total # of comparisons (X) as sum of simpler random variables
  - → $X = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij}$ where $X_{ij} = 1$ if i,j compared, 0 if not compared

$\therefore E(X) = E\left[\sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij}\right] = \sum_{i=1}^{n} \sum_{j=1}^{n} E(X_{ij}) = \sum_{i=1}^{n} \sum_{j=1}^{n} P(X_{ij} = 1)$

* Recall expected value of indicator value of Bernoulli trials is the probability of the event occurring.

→ Now find P(i and j are compared):

- What is probability that i or j are pivot when split?
  → only in this circustance will they be compared
- Consider point of execution where they are split.

  $\boxed{i-1 \, [i, i+1, \ldots, \ldots, j-1, j] \, j+1}$

  split only occurs iff pivot chosen between i^{th} & j^{th} element in A
  so we know that pivot x : $i \leq x \leq j$
- Comparison between i & j only occurs if one is pivot
  ∴ $P(i, j \text{ compared}) = P(i \text{ or } j \text{ chosen as pivot from } [i, \ldots, j]) = 2/(j-i+1)$

- $E(X) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left[\frac{2}{j-i+1}\right]$; Let $k = j-i$;

  $= \sum_{i=1} \sum_{k=1} \left[2/(k+1)\right] < \sum_{i=1}^{n} \sum_{k=1}^{n} 2/k = O(n \log n)$

- Recall $\int_1^n 1/x \, dx = \log(n)$