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MANAGING INVENTORIES: THE NEWSVENDOR MODEL

Managing inventories is important in every company and under many circumstances, but it is especially critical for firms facing uncertain demand in only one selling season. This problem is commonly referred to as the *newsvendor problem* or *newsvendor model*,[1] derived from the problem faced by a newsvendor who must decide on the number of newspapers to purchase at the beginning of the day before demand can be observed. This model, however, can be applied to any situation where the selling season is short enough that ordering decisions must be made before the season starts, such as for fashion goods, influenza vaccines and other health care products, short-life-cycle technology products (consumer electronics and telecommunications equipment), and perishable goods.

Determining the Optimal Inventory Level

When determining how much inventory to hold before the selling season starts, a firm needs to balance two possible outcomes. If the firm orders too much, inventory remaining at the end of the selling season will have to be sold at a discount or thrown away. But if the firm doesn't order enough, it may lose potential sales and possibly also cause customers to obtain the product from a competitor. To take those two possible outcomes into account, we define two types of costs: the *overstocking cost*, C_o, which is the loss for the firm for each unit of leftover, and the *understocking cost*, C_u, which is the loss of margin for the firm for each unit of lost sales.

By the newsvendor model, the firm's optimal order quantity (inventory level) Q^*, can be determined by equating the expected cost of overstocking with the expected cost of understocking where (**Equation 1**):

$$\begin{aligned}\text{Expected cost of overstocking} &= \text{Prob (Overstocking)} \times C_o \\ &= \text{Prob (Demand} \leq Q^*) \times C_o = F(Q^*) \times C_o \qquad (1)\end{aligned}$$

[1] First mentioned by Philip M. Morse and George E. Kimball, *Methods of Operations Research* (New York and Cambridge: Technology Press of MIT and John Wiley, 1951).

Expected cost of understocking = Prob (Understocking) × C_u
= Prob (Demand > Q^*) × C_u
= [1 – Prob (Demand ≤ Q^*)] × C_u = [1 – $F(Q^*)$] × C_u (2)

and $F(x)$ = Prob (Demand $\leq x$) is the cumulative probability function that is equal to the probability of demand being less than x.

Equating the two costs in **Equation 1** and **Equation 2** and rearranging, we get the well-known newsvendor critical ratio where:

$$F(Q^*) \equiv \frac{C_u}{C_u \pm C_o} \quad (3)$$

As can be seen in **Equation 3**, when the understocking cost is lower than the overstocking cost, $F(Q^*) < 0.5$, and thus the order quantity the firm will choose will be lower than the mean demand (assuming a symmetric distribution). Intuitively, because it is more expensive to order too many, the firm will want to order a quantity lower than the mean demand. On the other hand, when the understocking cost is higher than the overstocking cost, it is more expensive to not order enough, so the firm will order more than the mean demand.

Consider a specific example: TAL Corporation is a web-based costume retailer. In preparation for Halloween, the company needs to decide how many witch costumes to order in advance from its supplier, which is based in China. On the basis of past experience, TAL estimates that demand for the costumes will be between 2,600 and 3,000; probabilities are shown in **Table 1** below.

Table 1. Probabilistic forecast for costumes.

Demand, *D*	2,600	2,700	2,800	2,900	3,000
Probability	15%	25%	20%	25%	15%
F(*D*)	15%	40%	60%	85%	100%

Source: Created by case writer.

Each costume, which can be sold at a price of \$15.00, costs TAL \$6.00 to make, plus an additional variable overhead of about \$3.00. TAL also incurs a cost of \$2.00 per costume to transport it from the supplier in China. Any remaining costumes unsold after Halloween are scrapped, and TAL receives \$5.00 for each.

To find the optimal ordering quantity for the costumes, we use the newsvendor model and the critical ratio. When deciding on the optimal ordering quantity, we have two types of costs to take into account:

The overstocking cost, C_o = (\$6 + \$3 + \$2) – \$5 = \$6.00.
The understocking cost, C_u = \$15 – (\$6 + \$3 + \$2) = \$4.00.

By the newsvendor model, we should choose an inventory level that makes the probability that demand is less than the order quantity equal to the critical ratio $\frac{C_u}{C_u + C_o}$

Thus, Q^*, the optimal order quantity, can be found by using **Equation 4**:

$$F(Q^*) = \frac{C_u}{C_u + C_o} = \frac{\$4}{\$4 + \$6} = 0.40\ . \tag{4}$$

Using the probabilistic forecast from **Table 1**, the optimal ordering quantity therefore is Q^* = 2,700 costumes.[2] Because the understocking cost is lower than the overstocking cost, it is more expensive in this case to order too many, so the firm orders 2,700 units, which is less than the mean demand of 2,800 units.

Next, we would like to calculate the expected profit for TAL. The expected profit has to take into account TAL's revenues from the sale of the costumes and the salvage revenues, as well as the cost of producing the costumes. The expected profit is therefore (**Equation 5**):

$$\begin{aligned}\text{Expected Profit} &= \text{Expected Revenues} + \text{Expected Salvage Revenues} - \text{Total Cost}\\ &= \text{Price} \times \text{E [Sales]} + \text{Salvage} \times \text{E [Overstock]} - \text{Cost} \times Q^* \end{aligned} \tag{5}$$

Starting with the cost: for each of the units ordered (2,700 units), TAL will have to pay \$11 per unit (\$6 cost per unit + \$3 variable overhead + \$2 transportation cost). The total cost is therefore:

$$\text{Total Cost} = \$11 \times 2{,}700 = \$29{,}700.$$

In order to calculate the revenue from sales, recall that TAL receives \$15 for every unit sold, and by **Equation 5**, the expected revenue is the price per unit (\$15) multiplied by the expected sales. To calculate the expected sales, we need to take into account TAL's optimal order quantity (2,700 units) as well as the different possible demands and appropriate probabilities. In each case, the sales will be equal to the minimum of the units ordered and the demand, and thus:

- If demand is 2,600 (with probability 0.15), TAL will sell 2,600 units.
- If demand is 2,700 units or more (with probability 0.85), since TAL ordered 2,700 units, it will sell the minimum between the two, which is 2,700 units.

[2] Note that when demand is discrete, the critical ratio might not be exactly equal to $F(Q^*)$. For example, if the critical ratio is equal to 0.35, it is still optimal to order 2,700 units. In fact, we will order 2,700 units so long as the critical ratio is in the range $0.25 < C_u / (C_u + C_o) \leq 0.40$.

The expected revenues are therefore:

$$\text{Expected Revenues} = \$15 \times (0.15 \times 2{,}600 + 0.85 \times 2{,}700) = \$40{,}275.$$

Finally, the firm will receive $5.00 for any unit leftover at the end of the season. If demand is 2,700 units or more (with probability 0.85), there would be no leftovers, but when demand is 2,600 (with probability 0.15), the firm will have 100 units leftover at the end of the season. Thus, the expected salvage revenues are:

$$\text{Expected Salvage Revenues} = \$5 \times (0.15 \times 100 + 0.85 \times 0) = \$75.$$

The total profits for TAL are therefore equal to:

$$\text{TAL Expected Profits} = \$40{,}275 + \$75 - \$29{,}700 = \$10{,}650.$$

When demand is normal

While some firms have a forecast of demand similar to the one presented in **Table 1**, other firms might expect demand to be continuous and follow a normal distribution. Suppose that, instead of having the demand distribution described in **Table 1**, the demand for the costumes for TAL follows a normal distribution; mean demand $\mu = 2{,}800$ and standard deviation $\sigma = 200$.

We can use the critical ratio in **Equation 4** (where $F(Q^*) = 0.4$) to calculate the optimal order quantity for TAL in this case. To calculate Q^*, we have to use the NORMINV function in Excel. Knowing that the mean demand $\mu = 2{,}800$ and the standard deviation $\sigma = 200$, we derive the following (**Equation 6**):

$$Q^* = \text{NORMINV}(0.40, 2{,}800, 200) = 2{,}749. \tag{6}$$

Thus, considering this normal demand distribution, TAL should order 2,749 costumes.

Expected profits when demand is normal

By **Equation 5**, the expected profit is given by:

$$\text{Expected Profit} = \text{Price} \times \text{E [Sales]} + \text{Salvage} \times \text{E [Overstock]} - \text{Cost} \times Q^*$$

Note that since every unit ordered is either sold or salvaged, the order quantity is equal to the sum of the expected sales and the expected overstock. Using this, the expected profit can be simplified to:

$$\text{Expected Profit} = (\text{Price} - \text{Cost}) \times Q^* - (\text{Price} - \text{Salvage}) \times \text{E [Overstock]}$$

$$= C_u \times Q^* - (C_u + C_o) \times \text{E [Overstock]}) \tag{7}$$

For the case when demand is normal, the expected overstock is given by **Equation 8**:

$$\text{E[Overstock]} = \left(Q^{*} - \mu\right) \times \text{F}\left(Q^{*}\right) + \sigma \times f_{s}\left(\frac{Q^{*} - \mu}{\sigma}\right) \tag{8}$$

where $f_s(x)$ is the density function of the standard normal distribution.

In the case of TAL, we can use **Equations 7** and **8** and the function NORMDIST() in Excel to calculate expected profit. By **Equation 8**, the expected overstock for TAL is equal to:

$$\begin{aligned}\text{E[Overstock]} &= \left(Q^{*} - \mu\right) \times \text{F}\left(Q^{*}\right) + \sigma \times \text{NORMDIST}\left(\frac{Q^{*} - \mu}{\sigma}, 0,1,0\right) \\ &= (2{,}749 - 2{,}800) \times 0.4 + 200 \times \text{NORMDIST}\left(\frac{2{,}749 - 2{,}800}{200}, 0,1,0\right) = 57.\end{aligned}$$

Thus by **Equation 7**, the expected profit is equal to:

$$\text{Expected Profit} = 4 \times 2{,}749 - (4 + 6\ ) \times 57 = \$10{,}427.$$