

CSE6140 Final Project

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A clear and well-documented L^AT_EX document is presented as an article formatted for publication by ACM in a conference proceedings or journal publication. Based on the “acmart” document class, this article presents and explains many of the common variations, as well as many of the formatting elements an author may use in the preparation of the documentation of their work.

CCS Concepts: • **Theory of computation** → **Graph algorithms analysis**; **Branch-and-bound**; **Approximation algorithms analysis**.

Additional Key Words and Phrases: minimum vertex cover, spanning, topology, heuristics

ACM Reference Format:

Avery Bodenstein, Adrian Thinnyun, Jai Jacob, and Zheyi Zhang. 2022. CSE6140 Final Project. *Proc. ACM Meas. Anal. Comput. Syst.* 37, 4, Article 111 (August 2022), 11 pages. <https://doi.org/XXXXXXX.XXXXXXX>

1 INTRODUCTION

short summary of the problem, the approach and the results you have obtained

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2 PROBLEM DEFINITION

formal definition of the problem

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3 RELATED WORK

a short survey of existing work on the same problem, and important results in theory and practice

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4 ALGORITHMS

Detailed description of each algorithm you have implemented, with pseudo-code, approximation guarantee (if any), time and space complexities, etc. What are the potential strengths and weaknesses of each type of approach? Did you use any kind of automated tuning or configuration for your local search? Why and how you chose your local search approaches and their components? Please cite any sources of information that you used to inform your algorithm design

4.1 Branch and Bound

4.1.1 Description. The branch and bound approach is a non-polynomial time algorithm which is guaranteed to return an optimal solution. The BnB algorithm at worst does not perform any better than a brute force search, but can have significant performance gains where portions of the search space can be pruned. This pruning is performed by iteratively defining sub-problems and determining lower bounds on the cost to go for these sub-problems. If the best possible cost for a sub-problem exceeds the current best cost that subproblem (and therefore all its derivatives) can be pruned from the search space. The sub-problems presented in this algorithm are defined based on inclusion or exclusion of nodes from the cover set.

4.1.2 Pseudo Code. -

The overall branch and bound algorithm has the following structure:

```

Data: P
1  $F \leftarrow (\emptyset, P)$ ;
2  $B \leftarrow (+\infty, (\emptyset, P))$ ;
3 while  $F$  not empty do
4   choose  $(X, Y)$  in  $F$ ;
5   expand  $(X, Y)$ ;
6   let  $(x_1, y_1), (x_2, y_2)$  be new configurations;
7   foreach  $(x_i, y_i)$  do
8     if solution found then
9       if  $\text{cost}(x_i) < B$  then
10         $B \leftarrow (\text{cost}(x_i), (x_i, y_i))$ ;
11      end
12    end
13    if not dead end then
14      if  $\text{lowerBound}(x_i) < B$  then
15         $F \leftarrow F \cup (x_i, y_i)$ ;
16      end
17    end
18  end
19 end
20 return  $(B)$ 

```

Algorithm 1: Branch and Bound

where P is the graph, x_i are all the vertices in the cover set, and y_i are the vertices remaining for selection. There are four sub functions, "choose", "expand", "checkSolution", "checkDeadEnd", and "lowerBound". In the implementation presented here, choose selects the subproblem in F with the lowest lowerBound on cost to go. Expand takes that subproblem and returns two subproblems, one with the node with fewest unique edges selected and one without that node as a possible selection. checkSolution checks to see if all edges are covered by x_i . checkDeadEnd checks if $x_i \cup y_i$ covers all edges. Finally lowerBound runs the heuristic algorithm described in this report on y_i then divides by the approximation ratio (2) to get a lower bound on the possible additional nodes required to cover all edges. This is then added to the number of nodes already in x_i .

4.1.3 Algorithm Analysis. Additional description

Gives exact solution

Time and space complexity

4.2 Construction Heuristic

4.2.1 Description. The construction heuristic approach implemented here is the *Greedy Independent Cover (GIC)* algorithm, presented in Halldórsson and Radhakrishnan [1994] for the independent set problem. This is an approximation approach which runs in polynomial time and yields an answer within a bound of the optimal solution. Runtime and Approximation Ratio analysis is provided in 4.2.3. This algorithm lends itself well to use with priority queues. All vertices in the graph are stored in a priority queue, ordered by the number of adjacent edges remaining in G : $q[\text{nodeLabel}] = [\text{nEdges}, \text{destinationNodes}]$. At each iteration the vertex with the fewest remaining edges is popped from the queue. Each of its neighbors is then removed from the queue and added to the cover set. For each remaining edge in each neighbor, the source node is removed from the list of edges in the destination node and the number of edges in the destination node is lowered by one. This process continues until all edges are covered.

4.2.2 Pseudo Code. -

```

Data:  $G = (V, E)$ 
1  $C \leftarrow \emptyset$ ;
2 while  $E \neq \emptyset$  do
3   select a vertex  $u$  of minimum degree;
4    $C \leftarrow C \cup N(u)$ ;
5    $V \leftarrow V - (N(u) \cup u)$ ;
6 end
7 return  $C$ ;

```

Algorithm 2: Greedy Independent Cover (GIC)

Where $N(u)$ is the neighborhood (all adjacent vertices to) u .

```

Data:  $q, \text{nEdges}$ 
1  $C \leftarrow 0$ ;
2 while  $C < \text{nEdges}$  do
3    $u \leftarrow q.\text{peek}()$ ;
4   foreach  $\text{neighbor } i \text{ in } N(u)$  do
5      $C \leftarrow C + q[i][0]$ ;
6     foreach  $\text{edge } (i, j) \text{ in neighbor}$  do
7        $q[j][0] \leftarrow q[j][0] - 1$ ;
8        $q[j][1] \leftarrow q[j][1] - \{i\}$ ;
9     end
10  end
11   $q = q - u$ ;
12 end
13 return  $C$ ;

```

Algorithm 3: Detailed Implementation

4.2.3 Algorithm Analysis. -

Additional description

Approximation ratio discussion

Time and space complexity

4.3 Local Search 1 (specific descriptor)

4.3.1 Description. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Data: B,C

```

1   $n_a = 0$ ;
2   $i = \text{len}(B)$ ;
3   $j = \text{len}(C)$ ;
4   $A = [0]^*(i+j-2)$ ;
5  for  $l = i+j-2$ ;  $l \geq 0$ ;  $l=1$  do
6      if  $i == 0$  then
7           $A[0:l] = C[0:j]$ ;
8          return  $(A, n_a)$ 
9      else if  $j == 0$  then
10          $A[0:l] = B[0:i]$ ;
11         return  $(A, n_a)$ 
12     if  $C[j-1] \geq B[i-1]$  then
13          $A[l] = C[j-1]$ ;
14          $j = j-1$ ;
15     else
16          $A[l] = B[i-1]$ ;
17          $n_a = n_a + j$ ;
18          $i = i-1$ ;
19 end
20 return  $(A, n_a)$ 

```

4.3.2 Pseudo Code.

Algorithm 4: merge

4.3.3 Algorithm Analysis. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices

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4.4 Local Search 2 (specific descriptor)

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Data: B,C

```

1   $n_a = 0$ ;
2   $i = \text{len}(B)$ ;
3   $j = \text{len}(C)$ ;
4   $A = [0]^{*(i+j-2)}$ ;
5  for  $l = i+j-2$ ;  $l \geq 0$ ;  $l=1$  do
6      if  $i == 0$  then
7           $A[0:l] = C[0:j]$ ;
8          return  $(A, n_a)$ 
9      else if  $j == 0$  then
10          $A[0:l] = B[0:i]$ ;
11         return  $(A, n_a)$ 
12     if  $C[j-1] \geq B[i-1]$  then
13          $A[l] = C[j-1]$ ;
14          $j = j-1$ ;
15     else
16          $A[l] = B[i-1]$ ;
17          $n_a = n_a + j$ ;
18          $i = i-1$ ;
19 end
20 return  $(A, n_a)$ 

```

4.4.2 Pseudo Code.

Algorithm 5: merge

4.4.3 Algorithm Analysis. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur

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5 EMPIRICAL EVALUATION

a detailed description of your platform (CPU, RAM, language, compiler, etc.), experimental procedure, evaluation criteria and obtained results (plots, tables, etc.). What is the lower bound on the optimal solution quality that you can drive from the results of your approximation algorithm and how far is it from the true optimum? How about from your branch-and-bound?

Table 1. Algorithm Performance

	Branch and Bound			Construction Heuristic			Local Search 1			Local Search 2	
Dataset	Time(s)	VC Value	RelErr	Time(s)	VC Value	RelErr	Time(s)	VC Value	RelErr	Time(s)	VC Value
jazz				0.0037	159	0.063					
karate				0.0005	14	0.00					
football				0.0014	95	0.011					
as-22july06				0.21	3303	0.00					
hep-th				0.098	3943	0.0043					
star				0.24	7069	0.024					
star2				0.21	4674	0.029					
netscience				0.017	901	0.0022					
email				0.015	604	0.017					
delaunay n10				0.012	733	0.043					
power				0.051	2226	0.010					

6 DISCUSSION

a comparative analysis of how different algorithms perform with respect to your evaluation criteria, or expected time complexity, etc

7 CONCLUSION

8 MATH EQUATIONS

You may want to display math equations in three distinct styles: inline, numbered or non-numbered display. Each of the three are discussed in the next sections.

8.1 Inline (In-text) Equations

A formula that appears in the running text is called an inline or in-text formula. It is produced by the **math** environment, which can be invoked with the usual `\begin ... \end` construction or with the short form `$... $`. You can use any of the symbols and structures, from α to ω , available in L^AT_EX [?]; this section will simply show a few examples of in-text equations in context. Notice how this equation: $\lim_{n \rightarrow \infty} x = 0$, set here in in-line math style, looks slightly different when set in display style. (See next section).

8.2 Display Equations

A numbered display equation—one set off by vertical space from the text and centered horizontally—is produced by the **equation** environment. An unnumbered display equation is produced by the **displaymath** environment.

Again, in either environment, you can use any of the symbols and structures available in L^AT_EX; this section will just give a couple of examples of display equations in context. First, consider the equation, shown as an inline equation above:

$$\lim_{n \rightarrow \infty} x = 0 \tag{1}$$

Notice how it is formatted somewhat differently in the **displaymath** environment. Now, we'll enter an unnumbered equation:

$$\sum_{i=0}^{\infty} x + 1$$

and follow it with another numbered equation:

$$\sum_{i=0}^{\infty} x_i = \int_0^{\pi+2} f \tag{2}$$

just to demonstrate L^AT_EX's able handling of numbering.

9 FIGURES

The “**figure**” environment should be used for figures. One or more images can be placed within a figure. If your figure contains third-party material, you must clearly identify it as such, as shown in the example below.

Your figures should contain a caption which describes the figure to the reader.

Figure captions are placed *below* the figure.

Every figure should also have a figure description unless it is purely decorative. These descriptions convey what's in the image to someone who cannot see it. They are also used by search engine crawlers for indexing images, and when images cannot be loaded.

A figure description must be unformatted plain text less than 2000 characters long (including spaces). **Figure descriptions should not repeat the figure caption – their purpose is to capture important information that is not already provided in the caption or the main text of the paper.** For figures that convey important and complex new information, a short text description may not be adequate. More complex alternative descriptions can be placed in an appendix and referenced in a short figure description. For example, provide a data table capturing the information in a bar chart, or a structured list representing a graph. For additional information regarding how best to write figure descriptions and why doing this is so important, please see <https://www.acm.org/publications/taps/describing-figures/>.

10 CITATIONS AND BIBLIOGRAPHIES

The use of BibT_EX for the preparation and formatting of one's references is strongly recommended. Authors' names should be complete — use full first names (“Donald E. Knuth”) not initials (“D. E. Knuth”) — and the salient identifying features of a reference should be included: title, year, volume, number, pages, article DOI, etc.

The bibliography is included in your source document with these two commands, placed just before the `\end{document}` command:



Fig. 1. 1907 Franklin Model D roadster. Photograph by Harris & Ewing, Inc. [Public domain], via Wikimedia Commons. (<https://goo.gl/VLCRBB>).

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Citations and references are numbered by default. A small number of ACM publications have citations and references formatted in the “author year” style; for these exceptions, please include this command in the **preamble** (before the command “\begin{document}”) of your L^AT_EX source:

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be present since it has no vol. no.), a chapter in a divisible book [?], a chapter in a divisible book in a series [?], a multi-volume work as book [?], a couple of articles in a proceedings (of a conference, symposium, workshop for example) (paginated proceedings article) [? ?], a proceedings article with all possible elements [?], an example of an enumerated proceedings article [?], an informally published work [?], a couple of preprints [? ?], a doctoral dissertation [?], a master's thesis: [?], an online document / world wide web resource [? ? ?], a video game (Case 1) [?] and (Case 2) [?] and [?] and (Case 3) a patent [?], work accepted for publication [?], 'YYYYb'-test for prolific author [?] and [?]. Other cites might contain 'duplicate' DOI and URLs (some SIAM articles) [?]. Boris / Barbara Beeton: multi-volume works as books [?] and [?]. A couple of citations with DOIs: [? ?]. Online citations: [? ? ?]. Artifacts: [?] and [?].

11 ACKNOWLEDGMENTS

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This section has a special environment:

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ACKNOWLEDGMENTS

To Robert, for the bagels and explaining CMYK and color spaces.

REFERENCES

Received 4 December 2022