Homework #1

1.) Evaluate the following sums:

a. $\sum_{i=1}^{\infty} \frac{1}{2^i} = 1$ b. $\sum_{i=1}^{\infty} \frac{i}{2^i} = 2$

2.) Order the following functions by growth rate

$37 < \sqrt{N} < \frac{n}{2} \leq n < n \log\log n < n \log n \leq n \log n^2 < n \log^2 n$
$< n \log^2 n < n^{1.5} < n^2 < n^2 \log n < n^3 < 2^{n/2} < 2^n$

3.) a.) $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$

$\{ f_3(n) = f_1(n) + f_2(n);\ f_3 \in O(\max(g_1, g_2))$

b.) $f_1(n) * f_2(n) = O(g_1(n) * g_2(n))$

$\{ f_1 \in O(g_1);\ f_2 \in O(g_2);\ f_1 \in O(n);\ f_2 \in O(n^2)$
$f_3 = 15n^3;\ f_3 = 3n^2 * 5n;\ f_3\ O(n^2 * n);\ n^3$

4.) Proof by O

b.) $5n^3 + 2n + 8$

$C = 6;\ 5n^3 + 2n + 8 \leq 6n^3$
$n_0 = 3;\ 2n + 8 \leq n^3$
$2(3) + 8 \leq (3)^3$
$14 \leq 27$

Proof by Ω

b.) $5n^3 + 2n + 8$

$C = 5;\ 5n^3 + 2n + 8 \geq 5n^3$
$n_0 = 1;\ 2n + 8 \geq n^3$
$2(1) + 8 \geq (1)^3$
$10 \geq 1$

c.) $(n^2+1)(3n+5)$

$C = 4;\ 3n^3 + 5n^2 + 3n + 5$
$n_0 = 6;\ 3n^3 + 5n^2 + 3n + 5 \leq 4n^3$
$5n^2 + 3n + 5 \leq n^3$
$5(6)^2 + 3(6) + 5 \leq (6)^3$
$203 \leq 216$

c.) $(n^2+1)(3n+5)$

$C = 3;\ 3n^3 + 5n^2 + 3n + 5$
$n_0 = 1;\ 3n^3 + 5n^2 + 3n + 5 \geq 3n^3$
$5n^2 + 3n + 5 \geq n^3$
$5(1)^2 + 3(1) + 5 \geq (1)^3$
$13 \geq 1$

5.) Find two funtions that neither $f(n) = O(g(n))$ nor $g(n) = O(f(n))$

$f(n) = \sin(n)$ $g(n) = \cos(n)$

6 & 7 are included in the folder in seperate files.