1. (1 point) Using Sarrus' Rule or Triangle Rule, evaluate the following determinant:

$$\begin{vmatrix} 2 & 1 & 2 \\ -4 & 0 & -2 \\ 1 & -1 & 1 \end{vmatrix}.$$

$$\left|\begin{matrix} 2 & 1 & 2 \\ -4 & 0 & -2 \\ 1 & -1 & 1 \end{matrix}\right|\begin{matrix} 2 & 1 \\ -4 & 0 \\ 1 & -1 \end{matrix}\right| = (0)+(-2)+(8)-(0)-(4)-(-4) = 6$$

2. (1 point) Choose the values of i and j so that the product $a_{47}a_{63}a_{1i}a_{55}a_{7j}a_{24}a_{31}$ enters into a 7-th order determinant (see Definition 1) with the plus sign.

$$\begin{array}{c|ccccccc|} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & a & a & a & a & a & a_{16} & a \\ 2 & a & a & a & a_{24} & a & a & a \\ 3 & a_{31} & a & a & a & a & a & a \\ 4 & a & a & a & a & a & a & a_{47} \\ 5 & a & a & a & a & a_{55} & a & a \\ 6 & a & a & a_{63} & a & a & a & a \\ 7 & a & a_{72} & a & a & a & a & a \end{array}$$

$(3163)(247)$

$j = 2$

$i = 6$

3. (2 point) Find the value of the determinant of a matrix whose sum of rows is equal to a zero row. [**hint:** use $l_{r,s,\lambda}(A)$.]

Because $l_{r,s,\lambda}(A)$ doesn't affect $|A|$ and sum of rows gives 0, we can use l to get a zero row, then $|A| = 0$

4. (2 points) Transforming the matrix into Row Echelon Form, evaluate the following determinant

$$\begin{vmatrix} 2 & -1 & 3 & 4 \\ -2 & 0 & -5 & -1 \\ 4 & -1 & 13 & 6 \\ 6 & -3 & 14 & 12 \end{vmatrix}.$$

$$\xrightarrow{L_{2,1,1},\ L_{3,1,-2},\ L_{4,1,-3}} \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & -1 & -2 & 3 \\ 0 & 1 & 7 & -2 \\ 0 & 0 & 5 & 0 \end{vmatrix} \xrightarrow{L_{3,2,1}} \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & -1 & -2 & 3 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 5 & 0 \end{vmatrix} \xrightarrow{L_{4,3,-1}}$$

$$\rightarrow \begin{vmatrix} 2 & -1 & 3 & 4 \\ 0 & -1 & -2 & 3 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & -1 \end{vmatrix} = 2 \cdot (-1) \cdot 5 \cdot (-1) = 10$$

5. (1 points) Find the value of the following determinant

$$\begin{vmatrix} 0 & 0 & \cdots & 0 & 0 & a_{1,n} \\ 0 & 0 & \cdots & 0 & a_{2,n-1} & a_{2,n} \\ 0 & 0 & \cdots & a_{3,n-2} & a_{3,n-1} & a_{3,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & a_{n-1,2} & \cdots & a_{n-1,n-2} & a_{n-1,n-1} & a_{n-1,n} \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n-2} & a_{n,n-1} & a_{n,n} \end{vmatrix}.$$

[**hint:** use $t_{r,s}(A)$.]

Consider 2 cases:

$n \% 2 = 0$

then we use $t_{r,s}(A)$ until we get an upper triangular matrix, so $|A| = a_n \cdot a_{n-1} \dots a_2 \cdot a_1$

$n \% 2 \neq 0$

The same approach, but there will be an extra $t_{r,s}$ we have to account for by multiplying by (-1).

$|A| = (-1) \cdot a_n \cdot a_{n-1} \dots a_2 \cdot a_1$

6. (2 points) For any $n \in \mathbb{N}$, evaluate $\det(A)$, where

$$A = \begin{bmatrix} 1 & 2 & 3 & \dots & n \\ n+1 & n+2 & n+3 & \dots & 2n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n^2-n+1 & n^2-n+2 & n^2-n+3 & \dots & n^2 \end{bmatrix}.$$

[**hint:** take a look at the solution of Problem 6 from Seminar 6, almost the same approach works in this case; do not forget about "small" values of n.]

for $n \geq 3$:

$$\xrightarrow{L_{2,1,-1}} \begin{vmatrix} 1 & 2 & 3 \dots & n \\ n & n & n \dots & n \\ 2n+1 & 2n+2 & 2n+3 \dots & \vdots \\ \dots & \dots & \dots & \dots \end{vmatrix} \xrightarrow{L_{3,1,-1}} \begin{vmatrix} 1 & 2 & 3 \dots & n \\ n & n & n & \vdots \\ 2n & 2n & 2n & \vdots \\ \dots & \dots & \dots & \dots \end{vmatrix} \rightarrow$$

$$\xrightarrow{L_{3,2,-2}} \begin{vmatrix} 1 & 2 & 3 \dots & n \\ n & n & n \dots & \\ 0 & 0 & 0 \dots & 0 \\ \dots & \dots & \dots & \dots \end{vmatrix} = 0$$

$n = 2$: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$

$n = 1$: $|1| = 1$