

$$4) a) \lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \rightarrow 0} \frac{1 + x + \frac{x^2}{2} + o(x^2) - 1 - x}{x^2} = \lim_{x \rightarrow 0} \frac{x^2}{2} \cdot \frac{1}{x^2} = \frac{1}{2}$$

$$b) \lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x \rightarrow 0} \frac{1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^4) - 1 + \frac{x^2}{2}}{x^4} =$$

$$= \lim_{x \rightarrow 0} \frac{x^4}{24} \cdot \frac{1}{x^4} + o(x^4) = \frac{1}{24}$$

$$5) a) \lim_{x \rightarrow 0} \frac{\cosh 3x + \cos 3x - 2}{x^4} = \lim_{x \rightarrow 0} \frac{\left(1 + \frac{9x^2}{2} + \frac{81x^4}{24} + o(x^4)\right) +$$

$$+ \left(1 - \frac{9x^2}{2} + \frac{81x^4}{24} + o(x^4)\right) - 2}{x^4} = \lim_{x \rightarrow 0} \frac{162x^4}{24} \cdot \frac{1}{x^4} = \frac{162}{24} = \frac{27}{4}$$

$$b) \lim_{x \rightarrow 0} \frac{\sinh 2x - 2 \sinh x}{x^3} = \lim_{x \rightarrow 0} \frac{\left(2x + \frac{8x^3}{6}\right) - 2x - \frac{2x^3}{6}}{x^3} =$$

$$= \lim_{x \rightarrow 0} \frac{6x^3}{6} \cdot \frac{1}{x^3} = 1$$

$$6) a) \lim_{x \rightarrow 0} \frac{e^x - \sqrt{1+2x}}{\ln \cos x} =$$

$$= \lim_{x \rightarrow 0} \frac{\left(1 + x + \frac{x^2}{2} + o(x^2)\right) - \left(1 + x - \frac{x^2}{2} + o(x^2)\right)}{(-1 + \cos x) - \frac{(1 - 2\cos x + \cos^2 x)}{2} + o(x^4)} \quad \textcircled{=}$$

$$t = -1 + \cos x$$

$$\ln \cos x = \ln(1+t) = x - \frac{x^2}{2} + o(x^2) = (-1 + \cos x) - \frac{(1 - 2\cos x + \cos^2 x)}{2} + o(x^2)$$

$$\textcircled{=} \lim_{x \rightarrow 0} \frac{\frac{2x^2}{2} \cdot 2}{-3 + 4\cos x - \cos^2 x} = \lim_{x \rightarrow 0} \frac{2x^2}{-3 + 4\cos x - \cos^2 x} =$$

$$= \lim_{x \rightarrow 0} \frac{2x^2}{-3 + 4\left(1 - \frac{x^2}{2} + o(x^2)\right) - \left(1 - \frac{x^2}{2} + o(x^2)\right)^2} =$$

$$= \lim_{x \rightarrow 0} \frac{2x^2}{-3 + 4 - \frac{4x^2}{2} - 1 + x^2 + \frac{x^4}{4}} = \lim_{x \rightarrow 0} \frac{2x^2}{\frac{-4x^2 + x^4}{4}} = \frac{8}{-4} = -2$$

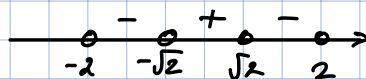
$$b) \lim_{x \rightarrow 0} \frac{3 \cos x + \arcsin x - 3\sqrt[3]{1+x}}{\ln(1-x^2)} =$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{3} - \frac{3x^2}{2} + o(x^2) + \cancel{x} + o(x^2) - \cancel{3} - \cancel{x} + \frac{x^2}{3} + o(x^2)}{x^2 + o(x^2)} =$$

$$= \lim_{x \rightarrow 0} \frac{-\frac{4}{6}x^2 + o(x^2)}{-x^2 + o(x^2)} = \frac{4}{6}$$

$$9) a) y = x \sqrt{4-x^2} \quad D(y) = [-2, 2]$$

$$f'(x) = \frac{4-2x^2}{\sqrt{4-x^2}}$$



From  $(-2, -\sqrt{2}] \cup [\sqrt{2}, 2)$  - decreasing

$[-\sqrt{2}; \sqrt{2}]$  - increasing

$x = \sqrt{2}$  is local maximum

$x = -\sqrt{2}$  is local minimum

$x = \pm 2$  - critical point

$x = \pm \sqrt{2}$  - stationary points

$$b) y = \frac{2x^2-1}{x^4} \quad x \neq 0 \quad D(y) = (-\infty, 0) \cup (0, +\infty)$$

$$f'(x) = \frac{-4x^2+4}{x^5}$$

$x = 0$  - critical point

$x = \pm 1$  - stationary point

$(-\infty, -1] \cup (0, 1]$  - increasing

$[-1, 0) \cup [1, +\infty)$  - decreasing

$x = \pm 1$  - local maximum