1. Consider the schematic map of another city's downtown.

$B(x,y)$
x = avenue
y = street

Avenues ↓ \ streets →	1	2	3	4	5
1	H			S	
2		$H\ R$		R	
3	S	$B\ S$	S	S	$S\ H$
4	B				B
5	B		R		$S\ R$

Translate the following statements into/from logical symbolism and check if they are true:

a) There is a street with at least two different Bank offices.

b) Only Restaurants and Supermarkets can share a crossing with a Hotel.

c) Every avenue with a Supermarket has a Restaurant as well.

d) If you mistake streets for avenues and vice versa, the map is still accurate.

e) $\exists m \forall n\, S(n,m) \to \forall n \exists m\, R(n,m)$.

f) $\exists i \exists j \exists n \exists m\, (B(i,j) \wedge S(i,m) \wedge S(n,j) \wedge B(n,m))$.

a) $\exists y \exists x_1 \exists x_2 (B(x_1,y) \wedge B(x_2,y))$ Such street does exist: street 4

b) $\exists x \exists y ((H(x,y) \to R(x,y)) \vee (H(x,y) \to S(x,y)))$ That's true. A bank is never located on the same crossing as a hotel.

c) $\forall x \exists y_1 \exists y_2 (S(x,y_1) \wedge R(x,y_2))$ Not true. Avenue 1, for example, doesn't have a R.

d) Let R, S, H, B = I (infrastructure) $\forall x \forall y (I(x,y) = I(y,x))$
Not true, the map is completely different.

e) If there exists a street all crossings of which have an S, then there is an avenue all crossings of which have an R. It's true, because the assumption is false.

f)

	i	n
j	B	S
m	S	B

There are 4 crossings, 2 of which have an S and the other 2 have a B.

2. Put the following arguments in symbols and check their validity:

a) Only birds have feathers. No mammal is a bird. Therefore each mammal is featherless.

b) Everyone loves himself. Therefore someone is loved by somebody.

c) Any mathematician can solve this problem if anyone can. Paul is a mathematician and cannot solve the problem. Therefore the problem cannot be solved.

d) Anyone who can solve this problem is a mathematician. Paul cannot solve this problem. Therefore Paul is not a mathematician.

e) Anyone who can solve this problem is a mathematician. No mathematician can solve this problem. Therefore the problem cannot be solved.

a) Domain: living-beings

1. $\forall x(\underbrace{F(x)}_{\text{has feathers}} \Rightarrow \underbrace{B(x)}_{\text{is a bird}})$

2. $\forall x(\underbrace{M(x)}_{\text{is a mammal}} \Rightarrow \neg B(x))$

3. $\forall x(M(x) \Rightarrow \neg F(x))$

Assume:

$$[(\forall x(F(x) \Rightarrow B(x)) \wedge \forall x(M(x) \Rightarrow \neg B(x))) \Rightarrow \forall x(M(x) \Rightarrow \neg F(x))] = 0$$

then:

$$[(\forall x(F(x) \Rightarrow B(x)) \wedge \forall x(M(x) \Rightarrow \neg B(x)))] = 1$$

$$[\forall x(M(x) \Rightarrow \neg F(x))] = 0$$

$$\Downarrow$$

$[\forall x M(x)] = 1$ $\qquad 1 \Rightarrow B(x)$

$[\neg F(x)] = 0$ $\qquad 1 \Rightarrow \neg B(x)$ $\qquad \Rightarrow$ contradiction, the assumption is wrong.

$$\Downarrow$$

$[F(x)] = 1$

The statement is valid.

b) 1. $\forall x (L(x,x)$

loves who by whom

2. $\exists x \exists y (L(x,y))$

Assume:

$[\forall x (L(x,x) \rightarrow \exists x \exists y (L(x,y))] = 0$

Then:

$[\forall x (L(x,x)] = 1$

$[\exists x \exists y (L(x,y))] = 0$

If $x=y$, there's a contradiction. $\Rightarrow$ arguement is valid.

c) 1. $\forall x (S(x) \rightarrow \forall y (M(y) \wedge S(y))$

solve mathematician

2. $\forall x (P(x) \wedge \neg S(x))$

Paul

3. $\forall x (\neg S(x)$

Assume:

$[(\forall x (S(x) \rightarrow \forall y (M(y) \wedge S(y)) \wedge \forall x (P(x) \wedge \neg S(x))) \rightarrow \forall x (\neg S(x)] = 0$

then:

$[(\forall x (S(x) \rightarrow \forall y (M(y) \wedge S(y)) \wedge \forall x (P(x) \wedge \neg S(x)))] = 1$

$[\forall x (\neg S(x)] = 0$

$[\forall y (M(y)] = 1$

$[\forall x (S(x)] = 1$ $\Rightarrow$ No contradiction, the arguement is invalid.

$[S(y)] = 1$

$[P(x)] = 1$

d) 1. $\forall x\,(S(x) \Rightarrow M(x))$

2. $\forall x\,(P(x) \wedge \neg S(x))$

3. $\forall x\,(P(x) \Rightarrow \neg M(x)$

Assume:

$$(\forall x\,(S(x) \Rightarrow M(x)) \wedge \forall x\,(P(x) \wedge \neg S(x))) \Rightarrow (\forall x\,(P(x) \Rightarrow \neg M(x))$$

then:

$[(\forall x\,(S(x) \Rightarrow M(x)) \wedge \forall x\,(P(x) \wedge \neg S(x)))] = 1$

$\underline{[\forall x\,(P(x) \Rightarrow \neg M(x)] = 0}$

$[P(x)] = 1$

$[M(x)] = 1$

$S(x) \Rightarrow 1$

$1 \Rightarrow \neg S(x)$

If x is Paul, there are no contradictions, thus, the statement is false.

e) 1. $\forall x(S(x) \Rightarrow M(x)$

2. $\forall x(M(x) \wedge \neg S(x)$

3. $\forall x(\neg S(x)$

Assume:

$$(\forall x(S(x) \Rightarrow M(x) \wedge \forall x(M(x) \wedge \neg S(x)) \Rightarrow \forall x(\neg S(x)$$

then:

$[(\forall x(S(x) \Rightarrow M(x) \wedge \forall x(M(x) \wedge \neg S(x))] = 1$

$\underline{[\forall x(\neg S(x)] = 0}$

$S(x) = 1$

$\forall x\,(1 \Rightarrow M(x)) \wedge \forall x\,(M(x) \wedge 0) = 1$

$\forall x\,(1 \Rightarrow 0) \wedge \forall x\,(0 \wedge 0) = 1$

No contradiction.
The statement is false.