2. (HW) Find the following limits:

(a) $\lim\limits_{x\to 0}\dfrac{\tan 4x}{7^{3x}-1}$; (b) $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{x\cdot\arctan 8x}$; (c) $\lim\limits_{x\to 0}\dfrac{\ln(1-5x)}{\sqrt[3]{7x+8}-2}$;

(d) $\lim\limits_{x\to 0}\dfrac{\ln^3(1+2x)}{x\cdot(e^{x^2}-1)}$; (e) $\lim\limits_{x\to 0}\dfrac{\arcsin(\ln(1+x))}{2x+x^2}$; (f) $\lim\limits_{x\to 0}\dfrac{8^x-6^x}{x}$.

a) $$\lim_{x\to 0}\frac{\tan 4x}{7^{3x}-1}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{4\cancel{x}}{3\cancel{x}\ln 7}=\frac{4}{3\ln 7}$$

b) $$\lim_{x\to 0}\frac{9\cancel{x^2}}{2}\cdot\frac{1}{\cancel{x}\cdot 8\cancel{x}}=\frac{9}{16}$$

c) $$\lim_{x\to 0}\frac{-5x}{\sqrt[3]{(\frac{7x}{8}+1)8}-2}=\lim_{x\to 0}\frac{-5x}{2(\frac{7x}{8}+1)^{\frac{1}{3}}-2}=\frac{1}{2}\lim_{x\to 0}\frac{-5x}{(\frac{7x}{8}+1)^{\frac{1}{3}}-1}=$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{-5}{\frac{1}{3}\cdot\frac{7}{8}}=\frac{1}{2}\cdot\left(-\frac{5}{1}\right)\cdot\frac{24}{7}=-\frac{60}{7}$$

d) $$\lim_{x\to 0}\frac{8x^3}{x\cdot x^2}=8$$

e) $$\lim_{x\to 0}\frac{\arcsin(x)}{2x+x^2}=\lim_{x\to 0}\frac{x}{x(2+x)}=\lim_{x\to 0}\frac{1}{2+x}=\frac{1}{2}$$

f) $$\lim_{x\to 0}\frac{(8^x-1)-(6^x-1)}{x}=\lim_{x\to 0}\frac{x\ln 8-x\ln 6}{x}=\lim_{x\to 0}\frac{x(\ln 8-\ln 6)}{x}=$$

$$=\ln 8-\ln 6=\ln\left(\frac{8}{6}\right)=\ln\left(\frac{4}{3}\right)$$

5. (HW) Use L'Hospital's rule, if applicable, to evaluate the following limits:

$$\text{(a)} \lim_{x\to 3} \frac{x^3-7x-6}{\ln(x^2-8)}; \qquad \text{(b)} \lim_{x\to 1} \frac{\sqrt[6]{x}-5/6-x/6}{\sqrt[8]{x}-7/8-x/8}; \qquad \text{(c)} \lim_{x\to 0} \frac{\sin x - x}{3x^3};$$

$$\text{(d)} \lim_{x\to 0+} x\ln x; \qquad \text{(e)} \lim_{x\to 0} \frac{1-\cos 2x}{\ln(1+x)-x}.$$

a) $\frac{27-21-6}{\ln(9-8)} = \left[\frac{0}{0}\right] \overset{L}{=} \lim_{x\to 3} \frac{3x^2-7}{\left(\frac{1}{x^2-8}\right)\cdot(2x)} = \lim_{x\to 3} \frac{(3x^2-7)\cdot(x^2-8)}{2x} =$

$= (27-7)\left(\frac{1}{6}\right) = \frac{20}{6} = \frac{10}{3}$

b) $\frac{1-\frac{5}{6}-\frac{1}{6}}{1-\frac{7}{8}-\frac{1}{8}} = \left[\frac{0}{0}\right] \overset{L}{=} \lim_{x\to 1} \frac{\frac{1}{6\sqrt[6]{x^5}}-\frac{1}{6}}{\frac{1}{8\sqrt[8]{x^7}}-\frac{1}{8}} \overset{L}{=} \lim_{x\to 1} \frac{-\frac{5}{36x^{\frac{11}{6}}}}{-\frac{7}{64x^{\frac{15}{8}}}} = \frac{5}{36}\cdot\frac{64}{7} =$

$= \frac{5\cdot 16}{9\cdot 7} = \frac{80}{63}$

c) $\frac{0-0}{3\cdot 0} = \left[\frac{0}{0}\right] \overset{L}{=} \lim_{x\to 0} \frac{\cos x - 1}{9x^2} \overset{L}{=} \lim_{x\to 0} \frac{-\sin x}{18x} \overset{L}{=} \lim_{x\to 0} \frac{-\cos x}{18} = -\frac{1}{18}$

d) $[0\cdot\infty] = \lim_{x\to 0+} \frac{\ln x}{\frac{1}{x}} = \left[\frac{\infty}{\infty}\right] \overset{L}{=} \lim_{x\to 0+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0} \frac{1}{\cancel{x}}\cdot\left(-\frac{x^{\cancel{2}}}{1}\right) = 0$

e) $\frac{1-1}{0-0} = \left[\frac{0}{0}\right] \overset{L}{=} \lim_{x\to 0} \frac{2\sin(2x)}{\frac{1}{x+1}-1} = \lim_{x\to 0} \frac{2\sin(2x)\cdot(1+x)}{-x} =$

$= \lim_{x\to 0} \frac{2\cdot 2x}{-x}\cdot(1+x) = -4$

7. (HW) Use L'Hospital's rule, if applicable, to evaluate the following limits:

$$\text{(a)}\ \lim_{x\to 1} x^{\frac{x+1}{x-1}}; \qquad \text{(b)}\ \lim_{x\to \pi/2} (\tan x)^{\cos x}; \qquad \text{(c)}\ \lim_{x\to 0} \left(2\sqrt[4]{x}+x\right)^{1/\ln x}.$$

a) $1^{\frac{2}{0}} = [1^{\infty}] = \lim\limits_{x\to 1} e^{\frac{(x+1)(\ln x)}{x-1}} \overset{L}{=} \lim\limits_{x\to 1} e^{x\ln x + x + \frac{1}{x}} = e^2$

b) $[\infty^0]$ $\ln L = \lim\limits_{x\to\frac{\pi}{2}} \ln(\tan x)^{\cos x} = \lim\limits_{x\to\frac{\pi}{2}} \cos x \cdot \ln\tan x =$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\ln\tan x}{\frac{1}{\cos x}} \overset{L}{=} \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\frac{1}{\sin x\cos x}}{\frac{\sin x}{\cos^2 x}} = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{1}{\sin x\cos x} \cdot \dfrac{\cos^2 x}{\sin x} =$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos x}{\sin^2 x} = \dfrac{0}{1^2} = 0 \Rightarrow \ln L = 0 \Rightarrow \lim\limits_{x\to\frac{\pi}{2}} (\tan x)^{\cos x} = 1$

c) $\left[\frac{0}{0}\right]$ $\ln L = \lim\limits_{x\to 0} \frac{1}{\ln x} \cdot \ln(2\sqrt[4]{x}+x) \overset{L}{=}$

$\overset{L}{=} \lim\limits_{x\to 0} \dfrac{\frac{1+2\sqrt[4]{x^3}}{4x+2x\sqrt[4]{x^3}}}{\frac{1}{x}} = \lim\limits_{x\to 0} \dfrac{(1+2\sqrt[4]{x^3})x}{x(4+2\sqrt[4]{x^3})} = \lim\limits_{x\to 0} \dfrac{1+2\sqrt[4]{x^3}}{4+2\sqrt[4]{x^3}} =$

$= \frac{1}{4} \Rightarrow \ln L = \frac{1}{4} \Rightarrow L = e^{\frac{1}{4}} = \sqrt[4]{e}$