

$$(b) \text{ (HW) } \lim_{n \rightarrow \infty} \frac{(-1)^n}{5n-2} = 0;$$

Solution by definition:

$$\forall \varepsilon > 0 \exists N \in \mathbb{N} \quad \forall n \geq N \quad |x_n - L| < \varepsilon$$

$$\left| \frac{(-1)^n}{5n-2} \right| < \varepsilon$$

$$\frac{1}{5n-2} < \varepsilon$$

$$5n-2 > \frac{1}{\varepsilon}$$

$$5n > \frac{1+2\varepsilon}{\varepsilon}$$

$$n > \frac{1+2\varepsilon}{5\varepsilon}$$

$$N = \left\lceil \frac{1+2\varepsilon}{5\varepsilon} \right\rceil + 1$$

$$(c) \text{ (HW) } \lim_{n \rightarrow \infty} \frac{2n-1}{3n+5} = \frac{2}{3}.$$

$$\left| \frac{2n-1}{3n+5} - \frac{2}{3} \right| < \varepsilon \quad \forall n \in \mathbb{N}:$$

$$\frac{2}{3} - \frac{2n-1}{3n+5} < \varepsilon$$

$$\frac{2(3n+5) - 3(2n-1)}{3(3n+5)} < \varepsilon$$

$$\frac{6n+10-6n+3}{9n+15} < \varepsilon$$

$$\frac{13}{9n+15} < \varepsilon$$

$$9n+15 > \frac{13}{\varepsilon}$$

$$9n > \frac{13-15\varepsilon}{\varepsilon}$$

$$n > \frac{13-15\varepsilon}{9\varepsilon}$$

$$N = \left\lceil \frac{13-15\varepsilon}{9\varepsilon} \right\rceil + 1$$

3. (HW) Prove that the sequence $\{(-1)^n n - 7\}$ is divergent.

Let's prove that X is unbounded, where X - the given sequence.

$$\forall A > 0 \exists n \in \mathbb{N}: |(-1)^n n - 7| > A$$

Consider $n > 7$, then:

$$|(-1)^n n - 7| > A$$

$$n - 7 > A$$

$$n > A + 7$$

Thus, $|X|$ - unbounded \Rightarrow divergent, because every unbounded sequence is divergent by definition.

6. (HW) Suppose $\lim_{n \rightarrow \infty} x_n = L < b$. Prove that $\exists N \in \mathbb{N} \forall n \geq N: x_n < b$.

Solution:

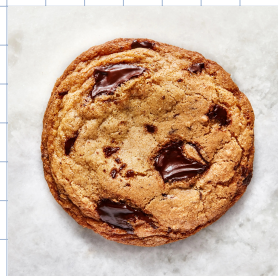
Definition of a limit: $\forall \varepsilon > 0 \exists N \in \mathbb{N} \forall n \geq N: |x_n - L| < \varepsilon$

If $L < b$, then $b - L > 0 \Rightarrow \exists N_1 \in \mathbb{N} \forall n \geq N_1: |x_n - L| < b - L$

$$|x_n| < b$$

$$x_n < b$$

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Here's a cookie for your checking!