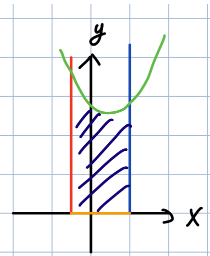
2. (HW) Find the area of the region bounded by the given curves:

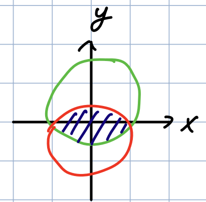
(a) $y = 3x^2 - 4x + 8$, $y = 0$, $x = -1$, $x = 2$; (b) $x^2 + y^2 = 4$, $x^2 + (y-2)^2 = 8$.

a)



$$\int_{-1}^{2} (3x^2 - 4x + 8)\,dx = \left[x^3 - 2x^2 + 8x \right]_{-1}^{2} = 8 - 8 + 16 + 1 + 2 + 8 = 27$$

b)



1. $x^2 + y^2 = 4$
2. $x^2 + (y-2)^2 = 8$

$x^2 + y^2 - 4y + 4 = 8$

$$\begin{cases} x^2 + y^2 = 4 \\ x^2 + y^2 - 4y + 4 = 8 \end{cases} \Leftrightarrow -4y + 4 = 4 \Rightarrow y = 0 \Rightarrow x = \pm 2$$

$$y^2 = 4 - x^2 \Rightarrow y = \sqrt{4 - x^2}$$

$$(y-2)^2 = 8 - x^2$$

$$y - 2 = \pm \sqrt{8 - x^2}$$

$$y = 2 - \sqrt{8 - x^2}$$

abs because the area is under the graph

$$\int_{-2}^{2} \sqrt{4 - x^2}\,dx + \left| \int_{-2}^{2} 2 - \sqrt{8 - x^2}\,dx \right| = \text{Area}$$

$$\int_{-2}^{2} \sqrt{4 - x^2}\,dx \left| \begin{matrix} x = 2\sin t \\ dx = 2\cos t\,dt \end{matrix} \right| = 4 \int_{x=-2}^{x=2} \sqrt{1 - \sin^2 t}\,\cos t\,dt =$$

$$= 4\int_{x=-2}^{x=2} \cos^2 t\,dt = 2\int_{x=-2}^{x=2} (1+\cos 2t)\,dt = 2t + \sin 2t \Big|_{x=-2}^{x=2} =$$

$$= 2\arcsin\frac{x}{2} + \frac{x\sqrt{4-x^2}}{2}\Big|_{-2}^{2} = \pi + 0 + \pi - 0 = \boxed{2\pi}$$

$$\int_{-2}^{2} 2-\sqrt{8-x^2}\,dx = 8 - \int_{-2}^{2}\sqrt{8-x^2}\,dx \left| \begin{array}{l} x=\sqrt{8}\sin t \\ dx = \sqrt{8}\cos t\,dt \end{array} \right| =$$

$$= 2x - \int_{-2}^{2}\sqrt{8-8\sin^2 t}\,\sqrt{8}\cos t\,dt = 2x - 8\int_{-2}^{2}\cos^2 t\,dt =$$

$$= 2x - 4\int_{-2}^{2} 1+\cos 2t\,dt = 2x - 4t - 2\sin 2t\Big|_{x=-2}^{x=2} =$$

$$= 2x - 8\arcsin\frac{x}{\sqrt{8}} - 2\sin\left(2\arcsin\frac{x}{\sqrt{8}}\right)\Big|_{-2}^{2} =$$

$$= \left[2x - 4\arcsin\frac{x}{\sqrt{8}} - x\frac{\sqrt{8-x^2}}{2}\right]_{-2}^{2} = 8 - \pi - 2 - \pi - 2 =$$

$$= \boxed{4 - 2\pi}$$

$$\int_{-2}^{2}\sqrt{4-x^2}\,dx + \left|\int_{-2}^{2} 2-\sqrt{8-x^2}\,dx\right| = 2\pi + |4-2\pi| = 4\pi - 4$$

Final

5. (HW) Find the volume of the solid obtained by revolving the region bounded by

$$\text{(a) } y = x^2,\ y^2 = x, \qquad \text{(b) } x^2 + \frac{y^2}{9} = 1$$

about the x-axis.

a) $\pi\int_0^1 (x - x^4)\,dx = \pi\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$

b) Due to symmetry:

$2\pi\int_0^1 9(1-x^2)\,dx = 18\pi\left(x - \frac{x^3}{3}\right)\Big|_0^1 = 18\pi - \frac{18\pi}{3} = 12\pi$

6. (HW) Find the volume of the solid obtained by revolving the region bounded by $x^2 - y^2 = 4$, $y = 2$, $y = -2$, about the y-axis.

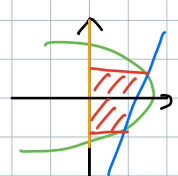
$x^2 = 4 + y^2$

$x = \pm\sqrt{4+y^2}$

$\pi\int_{-2}^{2} 4+y^2\,dy = 2\pi\int_0^2 4+y^2\,dy = 2\pi\left[4y + \frac{y^3}{3}\right]_0^2 = 2\pi\left[8 + \frac{8}{3}\right] =$

$= \frac{64\pi}{3}$

7. (HW) Find the volume of the solid obtained by revolving the region within the parabola $x = 9 - y^2$ and between $y = x - 7$ and the y-axis, about the y-axis.



$9 - y^2 = 7 + y$

$y = 1, -2$

$\pi\int_{-2}^{1} (7+y)^2\,dy = 129\pi$