a) $\int \cos^5(7x+3)\,dx \;\left|\begin{array}{l} u=7x+3 \\ du=7dx \\ dx=\frac{du}{7} \end{array}\right. = \frac{1}{7}\int \cos^5 u\,du =$

$= \frac{1}{7}\int \cos^4 u \cdot \underbrace{\cos u\,du}_{d(\sin u)} = \frac{1}{7}\int (1-\sin^2 u)^2 d(\sin u) \,|\, t=\sin u =$

$= \frac{1}{7}\int (1-t^2)\,dt = \frac{1}{7}\int (1-2t^2+t^4)\,dt = \frac{t}{7} - \frac{2}{21}t^3 + \frac{t^5}{35} + C =$

$\frac{\sin(7x+3)}{7} - \frac{2}{21}\sin^3(7x+3) + \frac{\sin^5(7x+3)}{35} + C$

b) $\int \cosh^4 x\,dx = \int \left(\frac{\cosh 2x+1}{2}\right)^2 dx = \frac{1}{4}\int (\cosh 2x+1)^2 dx =$

$= \frac{1}{4}\int (\underset{t=2x}{\cosh^2 2x} + 2\cosh 2x + 1)\,dx = \frac{1}{4}\int \cosh^2 2x + \frac{1}{2}\int \cosh 2x + \int dx =$

$= \frac{1}{4}\int \frac{\cosh^2 t}{2}\,dt + \frac{1}{4}\sinh 2x + x + C = \frac{1}{8}\int \frac{\cosh 2t+1}{2}\,dt + \frac{1}{4}\sinh 2x + \frac{x}{4} + C =$

$= \frac{1}{16}\int (\cosh 2t+1)\,dt + \frac{1}{4}\sinh 2x + \frac{x}{4} + C =$

$= \frac{1}{32}\sinh 4x + \frac{1}{8}x + \frac{1}{4}\sinh 2x + \frac{x}{4} + C$

c) $\int \sin^4(2x)\cos^2(2x)\,dx \;\left|\begin{array}{l} u=2x \\ du=2dx \\ dx=\frac{du}{2} \end{array}\right| = \frac{1}{2}\int \sin^4 u \cos^2 u\,du =$

$= \frac{1}{2}\int \left(\frac{1-\cos 2u}{2}\right)^2 \left(\frac{1+\cos 2u}{2}\right) du = \frac{1}{16}\int (1-2\cos 2u+\cos^2 2u)\cdot$

$\cdot(1+\cos 2u)\,du = \frac{1}{16}\int (1+\cos 2u - 2\cos 2u - 2\cos^2 2u +$

$+\cos^2 2u + \cos^3 2u)\,du = \frac{1}{16}\int (1-\cos 2u - \cos^2 2u + \cos^3 2u)\,du =$

$= \frac{u}{16} - \frac{\sin 2u}{32} - \frac{1}{16}\int \cos^2 2u\,du + \frac{1}{16}\int \cos^3 2u\,du =$

$$\int \cos^2 2u\,du = \frac{1}{2}\int (1+\cos 4u)\,du = \frac{u}{2} + \frac{\sin 4u}{8}$$

$$\int \cos^3 2u\,du = \int (1-\sin^2 2u)\cos 2u\,du \left| \begin{array}{l} t=\sin 2u \\ dt = 2\cos 2u\,du \\ du = \frac{dt}{2\cos 2u} \end{array} \right. =$$

$$= \frac{1}{2}\int (1-t^2)\,dt = \frac{t}{2} - \frac{t^3}{6} = \frac{\sin 2u}{2} - \frac{\sin^3 2u}{6}$$

$$= \frac{u}{16} - \frac{\sin 2u}{32} - \frac{u}{32} - \frac{\sin 4u}{128} + \frac{\sin 2u}{32} - \frac{\sin^3 2u}{96} + C =$$

$$= \frac{x}{16} - \frac{\sin 8x}{128} - \frac{\sin^3 4x}{96} + C$$

4) a) $$\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx = \int \frac{\cos x(1-\sin^2 x)}{\sqrt{\sin x}}\,dx \left| \begin{array}{l} u=\sin x \\ du = \cos x\,dx \end{array} \right. =$$

$$= \int \frac{1-u^2}{\sqrt{u}}\,du = \int \frac{du}{\sqrt{u}} - \int u^{\frac{3}{2}}\,du = 2\sqrt{u} - \frac{2}{5}u^{\frac{5}{2}} + C =$$

$$= 2\sqrt{\sin x} - \frac{2}{5}\sin x^{\frac{5}{2}} + C$$

b) $$\int \coth^3(2x+3)\,dx \left| \begin{array}{l} u=2x+3 \\ du = 2dx \end{array} \right. = \frac{1}{2}\int \coth^3 u\,du =$$

$$= \frac{1}{2}\int \coth u \cdot \left(\frac{1}{\sinh^2 u} + 1\right) du = \frac{1}{2}\left[\int \frac{\coth u}{\sinh^2 u}\,du + \int \coth u\,du\right] =$$

$$\int \frac{\coth u}{\sinh^2 u}\,du = \int \frac{\cosh u}{\sinh^3 u}\,du \left| \begin{array}{l} v=\sinh u \\ dv = \cosh u\,du \end{array} \right. = \int \frac{dv}{v^3} = -\frac{1}{2v^2} =$$

$$= -\frac{1}{2\sinh^2 u} + C$$

$$\int \coth u\,du = \int \frac{\cosh u}{\sinh u}\,du \left| \begin{array}{l} v=\sinh u \\ dv=\cosh u\,du \end{array} \right. = \int \frac{dv}{v} = \ln|\sinh u| + C$$

$$= \frac{1}{2}\ln|\sinh(2x+3)| - \frac{1}{4\sinh^2(2x+3)} + C$$

c) $$\int \cot^4(2-x)\,dx \left| \begin{array}{l} u=2-x \\ du=-dx \end{array} \right. = -\int \cot^4 u\,du =$$

$$= -\left[\int\left(\frac{1}{\sin^2 u}-1\right)\cot^2 u\,du\right] = -\int \frac{\cot^2 u}{\sin^2 u}\,du + \int \cot^2 u\,du =$$

$$\int \frac{\cot^2 u}{\sin^2 u}\,du \left| \begin{array}{l} v=\cot u \\ dv=-\frac{dx}{\sin^2 x} \end{array} \right. = -\int v^2\,dv = -\frac{v^3}{3} + C$$

$$\int \cot^2 u\,du = -\cot u - u + C$$

$$= \frac{\cot^3(2-x)}{3} - \cot(2-x) - (2-x) + C$$

d) $$\int \sin 5x \sin 7x\,dx = \int \frac{\cos(-2x) - \cos(12x)}{2}\,dx =$$

$$= \int \frac{\cos 2x}{2}\,dx - \int \frac{\cos 12x}{2}\,dx = \frac{\sin 2x}{4} - \frac{\sin 12x}{24} + C$$

e) $$\int \frac{\sin x}{1+5\cos x}\,dx \left| \begin{array}{l} u=1+5\cos x \\ du=-5\sin x\,dx \end{array} \right. = -\frac{1}{5}\int \frac{du}{u} =$$

$$= -\frac{1}{5}\ln|1+5\cos x|$$

6. a) $$\int \frac{dx}{2\sin x + \cos x + 3} \left| \begin{array}{l} x = 2\arctan t \\ dx = \frac{2\,dt}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ \sin x = \frac{2t}{1+t^2} \end{array} \right. =$$

$$=\int \frac{\frac{2dt}{1+t^2}}{\frac{4t}{1+t^2}+\frac{1-t^2}{1+t^2}+3} = \int \frac{2dt}{1+t^2}\cdot\frac{1+t^2}{2t^2+4t+4} = \int\frac{dt}{t^2+2t+2} =$$

$$=\int\frac{dt}{(t+1)^2+1^2} = \arctan(t+1)+C = \arctan(\tan\tfrac{x}{2}+1)+C$$

b) $\int \frac{dx}{3\sin x-4\cos x}$ | same stuff $=\int\frac{\frac{2dt}{1+t^2}}{\frac{6t}{1+t^2}-\frac{4-4t^2}{1+t^2}} =$

$$=\int\frac{2dt}{1+t^2}\cdot\frac{1+t^2}{4t^2+6t-4} = \int\frac{dt}{2t^2+3t-2} = -\frac{1}{5}\int\frac{dt}{t+2}+\frac{2}{5}\int\frac{dt}{2t-1} =$$

$$=-\frac{\ln|t+2|}{5}+\frac{\ln|2t-1|}{5}+C = -\frac{\ln|\tan\frac{x}{2}+2|}{5}+\frac{\ln|2\tan\frac{x}{2}-1|}{5}$$

$+C$

7) a) $\int\frac{dx}{\sqrt{1-x^2}\cdot\sqrt[7]{\arcsin x}}$ $\left|\begin{array}{l} u=\arcsin x \\ du=\frac{dx}{\sqrt{1-x^2}}\end{array}\right.$ $=\int\frac{du}{\sqrt[7]{u}} =$

$$=\frac{7}{6}(\arcsin x)^{6/7}$$

b) $\int\frac{\cos(\log_8 5x+8)}{x}dx$ $\left|\begin{array}{l} u=\log_8 5x+8 \\ du=\frac{dx}{x\ln 8}\end{array}\right.$ $=$

$$=\int\cos u\cdot\ln 8\,du = \ln 8\int\cos u\,du = \ln 8\cdot\sin(\log_8 5x+8)+C$$

c) $\int\frac{2x+3}{\sqrt{15-6x-x^2}}dx = \int\frac{-(-2x-6)-3}{\sqrt{15-6x-x^2}}dx =$

$$= -\int \frac{-2x-6}{\sqrt{15-6x-x^2}} \left| \begin{array}{l} u = 15-6x-x^2 \\ du = (-2x-6)dx \end{array} \right| - 3\int \frac{dx}{\sqrt{24-(x+3)^2}} =$$

$$= -\int \frac{du}{\sqrt{u}} - 3\arcsin\left(\frac{x+3}{\sqrt{24}}\right) + C = -2\sqrt{15-6x-x^2} - 3\arcsin\left(\frac{x+3}{\sqrt{24}}\right) + C$$