

$$4. a) f(x) = \frac{x^2 + x}{|x|(x-1)}$$

$$\lim_{x \rightarrow 0+} \frac{x^2 + x}{|x|(x-1)} = \lim_{x \rightarrow 0+} \frac{\cancel{x}(x+1)}{\cancel{x}(x-1)} = \frac{0+1}{0-1} = -1$$

$$\lim_{x \rightarrow 0-} \frac{x(x+1)}{-x(x-1)} = 1$$

$-1 \neq 1 \Rightarrow$ jump disc.

$$\lim_{x \rightarrow 1+} \frac{x^2 + x}{|x|(x-1)} = \lim_{x \rightarrow 1+} \frac{\cancel{x}^2(1 + \frac{1}{x})}{\cancel{x}^2(1 - \frac{1}{x})} = \frac{2}{0} = +\infty \Rightarrow \text{a point of essential disc.}$$

$$b) f(x) = \begin{cases} -\frac{4}{x-5}, & x < 1 \\ 3x^2 + 5x - 4, & 1 < x \leq 9 \\ \frac{9}{x-9}, & x > 9 \end{cases}$$

$$\lim_{x \rightarrow 9+} \frac{9}{x-9} = \frac{9}{+0} = +\infty \Rightarrow \text{essential disc.}$$

$$\lim_{x \rightarrow 9-} 3x^2 + 5x - 4 = 3 \cdot 81 + 5 \cdot 9 - 4 = 281$$

$$8) a) \frac{1}{\sin x - \cos x}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\sin x - \cos x} = \frac{1}{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} = \frac{1}{0} = \infty \Rightarrow \text{essential disc.}$$

$$b) \cos \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \cos \frac{1}{x} \text{ is not defined} \Rightarrow \text{essential disc.}$$

$$c) x \sin \frac{1}{x^2}$$

$$-1 \leq \sin \frac{1}{x^2} \leq 1$$

$$-x \leq x \sin \frac{1}{x^2} \leq x$$

$$\lim_{x \rightarrow 0^\pm} x = 0 \Rightarrow \lim_{x \rightarrow 0^\pm} x \sin \frac{1}{x^2} = 0 \Rightarrow x=0 \text{ is a}$$

point of removable disc.

$$g) f(x) = \frac{x^2 - (k-2)x + 8}{x-k}$$

$$\begin{array}{r} x^2 - (k-2)x + 8 \mid x-k \\ x^2 - kx \\ \hline 0 + 2x + 8 \\ 2x - 8 \\ \hline 8 + 2k \end{array}$$

$$8 + 2k \Rightarrow 2k + 8 = 0 \Rightarrow 2k = -8 \Rightarrow k = -4$$

$$\lim_{x \rightarrow -4} \frac{x^2 + 6x + 8}{x + 4} = \lim_{x \rightarrow -4} \frac{(x+2)\cancel{(x+4)}}{\cancel{(x+4)}} = \lim_{x \rightarrow -4} (x+2) =$$

$$\begin{pmatrix} = \\ \lim_{x \rightarrow -4} \\ \lim_{x \rightarrow -4} \end{pmatrix} = -2 \text{ for } x \rightarrow -4^\pm \Rightarrow \text{removable disc.}$$

This homework was pretty short, so I decided to start a little challenge:

π , but digits double each week.

$$\pi = 3.141$$