1) $(1+i)^n = (1-i)^n$

$\left(\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)\right)^n = \left(\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right)^n$

$\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^n = \left(\sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}\right)^n$

$n=0$ is the only solution, as signs differ.

2) $|z|^2 = a^2+b^2 \in \mathbb{Z}$ $\Rightarrow$

$\left|\frac{1}{z}\right|^2 = \frac{1}{a^2+b^2} \in \mathbb{Z}$

$\Rightarrow a^2+b^2=\pm1$

Answer: $\left[\begin{array}{l} \begin{cases} a=\pm1 \\ b=0 \end{cases} \\ \begin{cases} a=0 \\ b=\pm1 \end{cases} \end{array}\right. \Rightarrow \left[\begin{array}{l} z=\pm i \\ z=\pm1 \end{array}\right.$

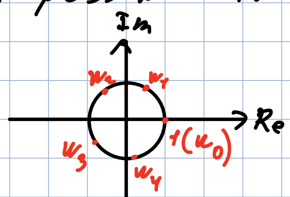
3) In hw8 I've proven that any invertible matrix has a $\det(A)=\pm1$.

In a) $A\in Mat_n(\mathbb{Z}[i])$ and $A^{-1}\in Mat_n(\mathbb{Z}[i])$

So, $\det(A)=\pm1$ which is in $Inv(\mathbb{Z}[i])$

b) If $\det(A)=\pm1 \in inv(\mathbb{Z}[i])$ then A is invertible and that is only possible if $A\in Mat_n(\mathbb{Z}[i])$, as proven in a)

4) $\sqrt[5]{1}=$



distance $= \frac{360°}{5} = 72°$

polar form $= e^{i\left(\frac{2\pi k}{5}\right)}$, $k\in\{0,1,2,3,4\}$

$1+w_1+w_2+w_3+w_4=0$

$$(1-e^{i\frac{2\pi}{5}})(1-e^{i\frac{4\pi}{5}})(1-e^{i\frac{6\pi}{5}})(1-e^{i\frac{8\pi}{5}})=$$

$$=(1-e^{i\frac{4\pi}{5}}-e^{i\frac{2\pi}{5}}+e^{i\frac{6\pi}{5}})(1-e^{\frac{8\pi}{5}}-e^{i\frac{6\pi}{5}}+e^{i\frac{14\pi}{5}})=$$

$$=1-e^{\frac{8\pi}{5}}-e^{i\frac{6\pi}{5}}+e^{i\frac{14\pi}{5}}-e^{i\frac{4\pi}{5}}+e^{i\frac{12\pi}{5}}+e^{i\frac{10\pi}{5}}-e^{i\frac{18\pi}{5}}-$$

$$-e^{i\frac{2\pi}{5}}+e^{i\frac{10\pi}{5}}+e^{i\frac{8\pi}{5}}-e^{i\frac{16\pi}{5}}+e^{i\frac{6\pi}{5}}-e^{i\frac{14\pi}{5}}-e^{i\frac{12\pi}{5}}+e^{i\frac{20\pi}{5}}=$$

$$=1-e^{i\frac{4\pi}{5}}+\underbrace{e^{i2\pi}}_{1}-e^{i\frac{8\pi}{5}}-e^{i\frac{2\pi}{5}}+\underbrace{e^{i2\pi}}_{1}-e^{i\frac{16\pi}{5}}+\underbrace{e^{i4\pi}}_{1}=$$

$$=4-e^{i\frac{4\pi}{5}}-e^{-i\frac{2\pi}{5}}-e^{i\frac{2\pi}{5}}-e^{-i\frac{4\pi}{5}}=$$

$$=4-(e^{i\frac{4\pi}{5}}+e^{-i\frac{2\pi}{5}}+e^{i\frac{2\pi}{5}}+e^{-i\frac{4\pi}{5}})=4-(-1)=5$$

5) $|a|=|b|=|c|=1 \Rightarrow |abc|=1$

$$|ab+bc+ac|\cdot|abc|=|a^2b^2c+b^2c^2a+a^2c^2b|=|a+b+c|$$

$$\Rightarrow \frac{|a+b+c|}{|a+b+c|}=1$$

6) $\sqrt[100]{1}=\{1,w,w^2\dots w^{99}\}$, $w=e^{i\frac{2\pi}{100}}$

$$\sum_{k=0}^{99} w_k^{10} = 1+(e^{i\frac{2\pi}{100}})^{10}+(e^{i\frac{4\pi}{100}})^{10}+\dots+(e^{i\frac{198\pi}{100}})^{10}=$$

$$=\frac{e^{i\frac{198\pi}{10}}\cdot e^{i\frac{2\pi}{10}}-1}{e^{i\frac{2\pi}{10}}-1}=\frac{e^{i\frac{200\pi}{10}}-1}{\underbrace{e^{i\frac{\pi}{5}}-1}_{\neq 0}}=\frac{1-1}{e^{i\frac{\pi}{5}}-1}=0$$

7) $z^3+2z^2-z+2=0$

$a+b+c=-2$

$ab+bc+ac=-1$

$abc=-2$

$(a+b+c)^2=(-2)^2$

$a^2+b^2+c^2+2ab+2bc+2ac=4$

$a^2+b^2+c^2+2(-1)=4$

$a^2+b^2+c^2=6$

$(ab+bc+ac)^2=(-1)^2$

$a^2b^2+b^2c^2+a^2c^2+2a^2bc+2ab^2c+2abc^2=1$

$a^2b^2+b^2c^2+a^2c^2=1-2(a^2bc+ab^2c+abc^2)$

$a^2b^2+b^2c^2+a^2c^2=1-2\underbrace{abc}_{-2}\underbrace{(a+b+c)}_{-2}$

$a^2b^2+b^2c^2+a^2c^2=1-8=-7$

$(a^2+b^2+c^2)^2=6^2$

$a^4+b^4+c^4+2a^2b^2+2a^2c^2+2b^2c^2=36$

$a^4+b^4+c^4=36-2(a^2b^2+a^2c^2+b^2c^2)$

$a^4+b^4+c^4=36-2(-7)=36+14=50$