1) $\left\{\frac{n+(-1)^n}{3n-1} : n \in \mathbb{N}\right\}$ bounded or not?

Solution: 1. $0 < 3n-1$

$0 \leq n+(-1)^n < 3n-1$

2. $0 \leq \frac{n+(-1)^n}{3n-1} < \frac{3n-1}{3n-1}$

$0 \leq \frac{n+(-1)^n}{3n-1} < 1$

Conclusion: the set is bounded

2) $X = \{n^2 : n \in \mathbb{N}\}$ is unbounded

Solution: for $\forall C \in \mathbb{R}$ $\exists x \in X$ such that $|x| > C$ where $x = n^2$ where $n = [|C|]+1$

Conclusion: the set is unbounded

3) $X = \left\{\frac{(-1)^n n}{n+1}\right\}$ is bounded and find inf X, sup X.

Solution: 1. let's notice that $|(-1)^n n| < |n+1|$, thus, $\forall |x_n| \in X < 1$

2. $(-1)^n$ in the numerator will change it from negative to positive values depending on n $\Rightarrow$ $\underset{\text{inf}}{-1} < \forall x_n \in X < \underset{\text{sup}}{1}$

Conclusion: X is a bounded set and $\inf = -1$, $\sup = 1$

4) $X = \{\sqrt{n+1} - \sqrt{n}\}$ is bounded and find inf X, sup X.

Solution: 1. Obviously, we get inf of X when $n \in \mathbb{N}$ is the lowest, $n=1$

$\sqrt{1+1} - \sqrt{1} = \underline{\sqrt{2} - 1}$ - supremum, because $X \downarrow$:

$$\sqrt{n+1}-\sqrt{n} = \frac{(\sqrt{n+1}-\sqrt{n})\cdot(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

which is monotone decreasing.

2. $\sqrt{n+1} > \sqrt{n}$, also $n+1>0$ and $n>0$ $\Rightarrow$ $\forall x \in X > 0$ $\Rightarrow$

$\Rightarrow$ X is getting closer to zero the higher the value of n $\Rightarrow$

$\Rightarrow$ 0 is infimum

Conclusion: X is bounded, $\inf = 0$; $\sup = \sqrt{2}-1$

5) $\left\{\frac{n+2}{5n+1}\right\}$ prove the sequence is monoton.

Solution: 1. Since $5n+1$ is monotone increasing for $n \in \mathbb{N}$

and $5n+1 > n+2$ $\Rightarrow$ X is monotone decreasing, trying to reach 0.

Conclusion: X is monotone decreasing

- Zinkin Zakhar

tg: Avgustine