(b) (HW) $\lim_{n\to\infty} \frac{(-1)^n}{5n-2} = 0;$

Solution by definition:

$\forall \varepsilon>0\ \exists N\in\mathbb{N}\ \ \forall n\geq N\ \ |x_n - L| < \varepsilon$

$$\left|\frac{(-1)^n}{5n-2}\right| < \varepsilon$$

$$\frac{1}{5n-2} < \varepsilon$$

$$5n-2 > \frac{1}{\varepsilon}$$

$$5n > \frac{1+2\varepsilon}{\varepsilon}$$

$$n > \frac{1+2\varepsilon}{5\varepsilon}$$

$$N = \left[\frac{1+2\varepsilon}{5\varepsilon}\right]+1$$

(c) (HW) $\lim_{n\to\infty} \frac{2n-1}{3n+5} = \frac{2}{3}.$

$$\left|\frac{2n-1}{3n+5} - \frac{2}{3}\right| < \varepsilon \quad \forall n\in\mathbb{N}:$$

$$\frac{2}{3} - \frac{2n-1}{3n+5} < \varepsilon$$

$$\frac{2(3n+5)-3(2n-1)}{3(3n+5)} < \varepsilon$$

$$\frac{6n+10-6n+3}{9n+15} < \varepsilon$$

$\frac{13}{9n+15} < \varepsilon$

$9n+15 > \frac{13}{\varepsilon}$

$9n > \frac{13-15\varepsilon}{\varepsilon}$

$n > \frac{13-15\varepsilon}{9\varepsilon}$

$N = \left[\frac{13-15\varepsilon}{9\varepsilon}\right]+1$

3. (HW) Prove that the sequence $\{(-1)^n n - 7\}$ is divergent.

Let's prove that X is unbounded, where X - the given sequence.

$\forall A>0\ \exists n \in \mathbb{N}: |(-1)^n n - 7| > A$

Consider $n>7$, then:

$|(-1)^n n - 7| > A$

$n-7>A$

$n>A+7$

Thus, $|X|$ - unbounded $\Rightarrow$ divergent, because every unbounded sequence is divergent by definition.

6. (HW) Suppose $\lim\limits_{n\to\infty} x_n = L < b$. Prove that $\exists N \in \mathbb{N}\ \forall n \geqslant N: \ x_n < b$.

Solution:

Definition of a limit: $\forall \varepsilon>0\ \exists N \in \mathbb{N}\ \forall n \geq N: |x_n - L| < \varepsilon$

If $L<b$, then $b-L>0 \Rightarrow \exists N_1 \in \mathbb{N}\ \forall n \geq N_1: |x_n - L| < b-L$

$|x_n| < b$

$x_n < b$

- Zakhar Zinkin

Here's a cookie for your checking!