2) a) $\int \frac{x}{2x^2-3x-2}dx = \int \frac{x}{(2x+1)(x-2)}dx = \frac{1}{5}\int \frac{dx}{(2x+1)} + \frac{2}{5}\int \frac{dx}{(x-2)} =$

$x = A(2x+1)+B(x-2) = 2xA + A + xB - 2B = (2A+B)x + A - 2B$

$\begin{cases} 2A+B=1 \\ A-2B=0 \end{cases} \Rightarrow \begin{cases} A=\frac{2}{5} \\ B=\frac{1}{5} \end{cases}$

$= \frac{1}{5}\int \frac{dx}{2x+1} \left| \begin{matrix} u=2x+1 \\ du=2dx \\ dx=\frac{dx}{2} \end{matrix} \right| + \frac{2}{5}\int \frac{dx}{x-2} \left| \begin{matrix} t=x-2 \\ dt=dx \end{matrix} \right| =$

$\frac{1}{5}\int \frac{du}{2u} + \frac{2}{5}\int \frac{dt}{t} = \frac{1}{10}\cdot \ln|2x+1| + \frac{2}{5}\cdot \ln|x-2| + C$

b) $\int \frac{x^4+x^3-x^2+x+1}{x^2+x-2}dx = \int x^2 dx + \int dx - \int \frac{dx}{x+2} + \int \frac{dx}{x-1} =$

$= \frac{x^3}{3} + x - \ln|x+2| + \ln|x-1| + C$

c) $\int \frac{4x^4-x+1}{x^3-x}dx = \int \left[4x + \frac{4x^2-x+1}{x^3-x}\right]dx = \int \left[4x + \frac{4x^2-x+1}{x(x-1)(x+1)}\right]dx =$

$\frac{4x^2-x+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+1)} \Rightarrow 4x^2-x+1 = A(x-1)(x+1) + B(x)(x+1) +$

$+ C(x)(x-1) = A(x^2-1) + B(x^2+x) + C(x^2-x) = (A+B+C)x^2 + (B-C)x$

$-A$

$\begin{cases} A+B+C=4 \\ B-C=-1 \\ -A=1 \end{cases} \Rightarrow \begin{matrix} A=-1 \\ B=2 \\ C=3 \end{matrix}$

$= \int \left[4x - \frac{1}{x} + \frac{2}{x-1} + \frac{3}{x+1}\right]dx = \int 4x\,dx - \int \frac{dx}{x} + 2\int \frac{dx}{x-1} + 3\int \frac{dx}{x+1} =$

$$=2x^2-\ln|x|+2\ln|x-1|+3\ln|x+1|+C$$

4) a) $$\int\frac{x^2-3x-4}{x^3-4x^2+4x}dx=\int\frac{x^2-3x-4}{x(x-2)(x-2)}dx=-\int\frac{dx}{x}+2\int\frac{dx}{x-2}-$$

$$-3\int\frac{dx}{(x-2)^2}=-\ln|x|+2\ln|x-2|-3\int\frac{du}{u^2}=$$

$$=-\ln|x|+2\ln|x-2|+\frac{3}{x-2}+C$$

b) $$\int\frac{2x^2-4x-8}{(x^2-x)(x^2+4)}dx=2\int\frac{x^2-2x-4}{x(x-1)(x^2+4)}dx=$$

$$x^2-2x-4=A(x-1)(x^2+4)+B(x)(x^2+4)+C(x)(x-1)=$$
$$=A(x^3-x^2+4x-4)+B(x^3+4x)+C(x^2-x)=$$
$$=(A+B)x^3+(-A+C)x^2+(4A+4B-C)x-4A$$

$$\begin{cases}A+B=0\\-A+C=1\\4A+4B-C=-2\\-4A=-4\end{cases}\Rightarrow\begin{cases}A=1\\B=-1\\C=2\end{cases}$$

$$=2\left(\int\frac{dx}{x}-\int\frac{dx}{x-1}+2\int\frac{dx}{x^2+4}\right)=2\left(\ln|x|-\ln|x-1|+\arctan\frac{x}{2}\right)=$$

$$=2\ln|x|-2\ln|x-1|+2\arctan\frac{x}{2}+C$$

6) a) $$\int\frac{2x+11}{x^2-6x+5}dx=\int\frac{2x+11}{(x-5)(x-1)}dx=$$

$$2x+11=A(x-1)+B(x-5)=(A+B)x+(-A-5B)$$

$$\begin{cases}A+B=2\\-A-5B=11\end{cases}\Rightarrow\begin{cases}A=\frac{21}{4}\\B=-\frac{13}{4}\end{cases}$$

$$= \frac{21}{4}\int\frac{dx}{x-5} - \frac{13}{4}\int\frac{dx}{x-1} = \frac{21}{4}\ln|x-5| - \frac{13}{4}\ln|x-1| + C$$

b) $$\int\frac{3x-7}{x^2+8x+19}\,dx = \int\frac{3x-7}{(x+4)^2+3}\,dx \left|\begin{array}{l} u=x+4 \\ du=dx \\ x=-4+u \end{array}\right| =$$

$$= \int\frac{3(u-4)-7}{u^2+3}\,du = \int\frac{3u-19}{u^2+3}\,du = 3\int\frac{u\,du}{u^2+3} - 19\int\frac{du}{u^2+3} =$$

$$= \frac{3}{2}\ln|u^2+3| - \frac{19}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}} + C =$$

$$= \frac{3}{2}\ln|x^2+8x+19| - \frac{19}{3}\arctan\frac{x+4}{\sqrt{3}} + C$$

c) $$\int\frac{5x+1}{\sqrt{1+2x-x^2}}\,dx = \int\frac{5x+1}{\sqrt{2-(x-1)^2}}\,dx \left|\begin{array}{l} t=x-1 \\ dt=dx \\ 5x+1=5t+6 \end{array}\right| =$$

$$= \int\frac{5t+6}{\sqrt{2-t^2}}\,dt = 5\int\frac{t}{\sqrt{2-t^2}}\,dt + 6\int\frac{dt}{\sqrt{2-t^2}} =$$

$$5\int\frac{t}{\sqrt{2-t^2}}\,dt \left|\begin{array}{l} u=2-t^2 \\ du=-2t\,dt \\ dt=-\frac{du}{2t} \end{array}\right| = 5\int\frac{t}{\sqrt{u}}\cdot\frac{-du}{2t} = -\frac{5}{2}\int\frac{du}{\sqrt{u}} =$$

$$= -\frac{5}{2}\cdot 2\sqrt{u} + C = -5\sqrt{u} + C = -5\sqrt{2-t^2} + C = -5\sqrt{1+2x-x^2} + C$$

$$= -5\sqrt{1+2x-x^2} + 6\arcsin\left(\frac{x-1}{\sqrt{2}}\right) + C$$