

$$2) f(x) = x^4 + 5x^3 + 11x^2 + 15x + 13$$

$$f(-2) = 3$$

$$f'(x) = 4x^3 + 15x^2 + 22x + 15$$

$$f'(-2) = -1$$

$$f''(x) = 12x^2 + 30x + 22$$

$$f''(-2) = 10$$

$$f'''(x) = 24x + 30$$

$$f'''(-2) = -18$$

$$f''''(x) = 24$$

$$\begin{aligned} f(x) &= f(-2) + \frac{f'(-2)}{1!} \cdot (x+2) + \frac{f''(-2)}{2!} \cdot (x+2)^2 + \\ &+ \frac{f'''(-2)}{3!} \cdot (x+2)^3 + \frac{f''''(-2)}{4!} \cdot (x+2)^4 = \\ &= (x+2)^4 - 3 \cdot (x+2)^3 + 5 \cdot (x+2)^2 - (x+2) + 3 \end{aligned}$$

$$3) f(x) = \frac{1}{2x+3}$$

$$\frac{1}{2x+3} = \frac{1}{3} \cdot \frac{1}{1+\frac{2x}{3}} = \frac{1}{3} \cdot \sum_{k=0}^n \frac{(-2)^k \cdot x^k}{3^k} + o(x^n) = \frac{1}{3} + \sum_{k=1}^n \frac{(-2)^k \cdot x^k}{3^{k+1}} +$$

$$+ o(x^n)$$

$$\begin{aligned} 4) f(x) &= \frac{2x+5}{x^2+5x+4} = \frac{1}{x+1} + \frac{1}{x+4} = \sum_{k=0}^n (-1)^k x^k + o(x^n) + \\ &+ \frac{1}{4} \cdot \sum_{k=0}^n \frac{(-1)^k x^k}{4^k} + o(x^n) = \sum_{k=0}^n (-1)^k x^k + o(x^n) + \frac{5}{4} + \sum_{k=1}^n \frac{(-1)^k x^k}{4^{k+1}} + o(x^n) = \end{aligned}$$

$$= \frac{5}{4} + \sum_{k=1}^n \left((-1)^k \cdot x^k \cdot \left(1 + \frac{1}{4^{k+1}} \right) \right) + o(x^n)$$

6) a) $f(x) = \ln(8 - 2x - x^2)$

$$f(0) = \ln(8)$$

$$f'(x) = \frac{2 + 2x}{-x^2 - 2x + 8}$$

$$f'(0) = -\frac{1}{4}$$

$$f''(x) = \frac{2x^2 + 4x + 20}{(-x^2 - 2x + 8)}$$

$$f''(0) = -\frac{20}{64} = -\frac{5}{16}$$

$$f'''(x) = \frac{4x^3 + 12x^2 + 120x + 112}{(-x^2 - 2x + 8)}$$

$$f'''(0) = -\frac{112}{512} = -\frac{7}{32}$$

$$f(x) = \ln 8 - \frac{1}{4}x - \frac{5}{32}x^2 - \frac{7}{192}x^3 + o(x^3)$$

b) $f(x) = (2x+1)\sqrt{1-x}$

$$f(0) = 1$$

$$f'(x) = \frac{3-6x}{2\sqrt{1-x}}$$

$$f'(0) = \frac{3}{2}$$

$$f''(x) = \frac{-9+6x}{\sqrt{1-x}(4-4x)}$$

$$f''(0) = -\frac{9}{4}$$

$$f'''(x) = \frac{6x-15}{8\sqrt{1-x}(1-x)^2}$$

$$f'''(0) = \frac{-15}{8}$$

$$f(x) = 1 + \frac{3}{2}x - \frac{9}{8}x^2 - \frac{5}{16}x^3 + o(x^3)$$