1. a) Suppose it is not a tautology:

True	False
$(((A \to B) \to A) \to A) \to A$	A
$(A \to B) \to A$	$((A \to B) \to A) \to A$
A	$A \to B$

Contradiction!

b) Suppose it is not a tautology:

True	False
$(A \to (B \to C))$	$((A \to B) \to (A \to C))$
$A \to B$	$A \to C$
A	C
B	
$B \to C$	
C	

Contradiction!

c) Suppose it is not a tautology:

True	False
$(A \to (C \wedge D)$	$(((A \to B) \wedge (E \to \neg D)) \to ((C \to B) \vee (D \wedge B \wedge \neg E)))$
C, D	$(C \to B) \vee (D \wedge B \wedge \neg E)$
$(A \to B) \wedge (E \to \neg D)$	$C \to B$
	$D \wedge B \wedge \neg E$
	B, A

No contradiction, as D, E can be any value.

2. a) $(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$\equiv$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

b) $(A \wedge B) \vee A \equiv A = A \wedge (A \vee B)$

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

$\equiv$

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

$=$

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

c) $\neg A \rightarrow B \equiv \neg B \rightarrow A$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

$\equiv$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Next page for problem #3. ↓

3)

a) 12 is divisible by x;

b) $x = 4$ or $x = 11$;

c) if x is even, then $x = 6$;

d) $4 \leq x \leq 6$;

e) 22 is divisible by x but $x < 22$;

f) $x = 7$ or $x = 12$.

1. Assume a is true, then b is also true, as $12:4=3$.
So, a cannot be true.
2. Assume b is true, then x must $= 11 \Rightarrow$ a, c, d, f are false.
e, on the other hand, is true, as $22:11=2$.
So, b cannot be true.
3. Assume d is true. $x=4$ or 6 contradicts with a.
$x=5$ contradicts with c, as (false $\rightarrow$ false) = true,
thus, two statements are true.
4. Assume e is true. 22 is divisible by 1, 2, 11. All numbers contradict with other statements.
So, e cannot be true.
5. Assume f is true. $x=12$ contradicts with a.
$x=7$ contradicts with c (as in example 3)
6. Assume c is true. $x=6$ contradicts with a, so there is only one case when c is true: (false $\rightarrow$ false) = True. Thus, $\neg(x:2) \wedge (x \neq 7) \wedge (x \neq 11)$

Conclusion: C is true when $\neg(x:2) \wedge (x \neq 7) \wedge (x \neq 11)$