2. a) $\int (x^2+x+4)\cos 3x\,dx \left| \begin{matrix} u=x^2+x+4 & dv=\cos 3x\,dx \\ du=(2x+1)dx & v=\frac{1}{3}\sin 3x \end{matrix} \right. =$

$= \frac{(x^2+x+4)\sin 3x}{3} - \int (2x+1)\cdot\frac{1}{3}\sin 3x\,dx \left| \begin{matrix} t=2x+1 & dy=\frac{1}{3}\sin 3x\,dx \\ dt=2dx & y=-\frac{1}{9}\cos 3x \end{matrix} \right. =$

$= \frac{(x^2+x+4)\sin 3x}{3} - \left(-\frac{(2x+1)\cos 3x}{9} + \int \frac{2}{9}\cos 3x\,dx\right) =$

$= \frac{(x^2+x+4)\sin 3x}{3} + \frac{(2x+1)\cos 3x}{9} - \frac{2}{27}\sin 3x + C$

b) $\int \frac{x}{\cos^2 x}dx \left| \begin{matrix} u=x & dv=\frac{1}{\cos^2 x}dx \\ du=dx & v=\tan x \end{matrix} \right. =$

$= x\tan x - \int \tan x\,dx = x\tan x - \int \frac{\sin x}{\cos x}dx \left| \begin{matrix} t=\cos x \\ dt=-\sin x\,dx \\ dx=-\frac{dt}{\sin x} \end{matrix} \right. =$

$= x\tan x - \int \frac{\sin x}{t}\cdot\left(-\frac{dt}{\sin x}\right) = x\tan x + \ln|\cos x| + C$

c) $\int x\ln(x+1)dx \left| \begin{matrix} u=\ln(x+1) & dv=x\,dx \\ du=\frac{dx}{x+1} & v=\frac{x^2}{2} \end{matrix} \right. =$

$= \frac{\ln(x+1)x^2}{2} - \frac{1}{2}\int \frac{x^2dx}{x+1} \left| \begin{matrix} t=x+1 \\ dt=dx \end{matrix} \right. = \frac{\ln(x+1)x^2}{2} - \frac{1}{2}\int \frac{(t-1)^2}{t}dt =$

$= \frac{\ln(x+1)x^2}{2} - \frac{1}{2}\left(\int t\,dt - \int 2dt + \int \frac{dt}{t}\right) =$

$= \frac{\ln(x+1)x^2}{2} - \frac{t^2}{4} + t - \frac{\ln|t|}{2} = \frac{\ln(x+1)x^2}{2} - \frac{(x+1)^2}{4} + (x+1) - \frac{\ln|x+1|}{2} + C$

d) $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx \left| \begin{matrix} u=\arcsin\sqrt{x} & dv=\frac{dx}{\sqrt{1-x}} \\ du=\frac{1}{2\sqrt{x-x^2}} & v=-2\sqrt{1-x} \end{matrix} \right. =$

$$= -2\sqrt{1-x}\arcsin\sqrt{x} + \int \sqrt{\frac{1-x}{x(1-x)}}\,dx =$$

$$= -2\sqrt{1-x}\arcsin\sqrt{x} + \int \frac{dx}{\sqrt{x}} = -2\sqrt{1-x}\arcsin\sqrt{x} + 2\sqrt{x} + C$$

e)

$$\int \arctan x\,dx \left|\begin{array}{ll} u = \arctan x & dv = dx \\ du = \frac{dx}{x^2+1} & v = x \end{array}\right. =$$

$$= \arctan x \cdot x - \int \frac{dx \cdot x}{x^2+1} \left|\begin{array}{l} t = x^2+1 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{array}\right. =$$

$$= \arctan x \cdot x - \frac{1}{2}\int \frac{dt}{t} = \arctan x \cdot x - \frac{1}{2}\ln|x^2+1| + C$$

F)

$$\int \sqrt{3-x^2}\,dx \left|\begin{array}{ll} u = \sqrt{3-x^2} & dv = dx \\ du = -\frac{x}{\sqrt{3-x^2}} & v = x \end{array}\right. =$$

$$= x\sqrt{3-x^2} - \int \frac{-x^2-3+3}{\sqrt{3-x^2}}dx = x\sqrt{3-x^2} - \int \frac{-3}{\sqrt{3-x^2}}dx - \int \frac{3-x^2}{\sqrt{3-x^2}}dx =$$

$$= x\sqrt{3-x^2} + 3\arcsin\left(\frac{x}{\sqrt{3}}\right) - \int \sqrt{3-x^2}\,dx + C;$$

$$\int \sqrt{3-x^2}\,dx = x\sqrt{3-x^2} + 3\arcsin\left(\frac{x}{\sqrt{3}}\right) - \int \sqrt{3-x^2}\,dx$$

$$2\int \sqrt{3-x^2}\,dx = x\sqrt{3-x^2} + 3\arcsin\left(\frac{x}{\sqrt{3}}\right)$$

$$\int \sqrt{3-x^2}\,dx = \frac{x}{2}\sqrt{3-x^2} + \frac{3}{2}\arcsin\left(\frac{x}{\sqrt{3}}\right) + C$$

g) $\int e^{2x} \cdot \sin 3x dx$ $\left| \begin{array}{ll} u = e^{2x} & dv = \sin 3x \\ du = 2e^{2x} & v = -\frac{1}{3}\cos 3x \end{array} \right.$ $=$

$= \frac{-e^{2x}\cos 3x}{3} + \frac{2}{3}\int e^{2x}\cos 3x$ $\left| \begin{array}{ll} t = e^{2x} & dw = \cos 3x \\ dt = 2e^{2x} & w = \frac{1}{3}\sin 3x \end{array} \right.$ $=$

$= \frac{-e^{2x}\cos 3x}{3} + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}\int e^{2x}\sin 3x + C$

$\int e^{2x} \cdot \sin 3x dx = \frac{-e^{2x}\cos 3x}{3} + \frac{2}{9}e^{2x}\sin 3x - \frac{4}{9}\int e^{2x}\sin 3x$

$\frac{13}{9}\int e^{2x} \cdot \sin 3x dx = \frac{-e^{2x}\cos 3x}{3} + \frac{2}{9}e^{2x}\sin 3x$

$13\int e^{2x} \cdot \sin 3x dx = -3e^{2x}\cos 3x + 2e^{2x}\sin 3x$

$\int e^{2x} \cdot \sin 3x dx = \frac{-3e^{2x}\cos 3x + 2e^{2x}\sin 3x}{13} + C$

4) a) $\int \frac{\ln(\ln x)}{x} dx$ $\left| \begin{array}{l} u = \ln x \\ du = \frac{dx}{x} \end{array} \right.$ $= \int \ln u \, du$ $\left| \begin{array}{ll} v = \ln u & dw = du \\ dv = \frac{du}{u} & w = u \end{array} \right.$ $=$

$dx = x du$

$= \ln u \cdot u - \int du = \ln u \cdot u - u = \ln(\ln x) \cdot \ln x - \ln x + C$

b) $\displaystyle\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\int \frac{2x}{1+(x^2)^2} \left| \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ dx = \frac{du}{2x} \end{array} \right. = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\arcsin x^2 + C$

c) $\displaystyle\int \sqrt{x}\sin\sqrt{x}\,dx \left| \begin{array}{l} u = \sqrt{x} \\ du = \frac{dx}{2\sqrt{x}} \\ dx = 2\sqrt{x}\,du \end{array} \right| =$

$\displaystyle = 2\int u^2 \sin u\,du \left| \begin{array}{ll} v = u^2 & dw = \sin u\,du \\ dv = 2u\,du & w = -\cos u \end{array} \right| =$

$\displaystyle = -2u^2\cos u + 4\int u\cos u\,du \left| \begin{array}{ll} z = u & dt = \cos u\,du \\ dz = du & t = \sin u \end{array} \right| =$

$\displaystyle = -2u^2\cos u + 4u\sin u - 4\int \sin u\,du =$

$= -2u^2\cos u + 4u\sin u + 4\cos u + C =$

$= -2x\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + 4\cos\sqrt{x} + C$

Task for you:

Prove $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} =$