2. (HW) Find the length of the indicated arc of the given curve:

(a) $y = x^2$, $1 \leqslant x \leqslant 2$; (b) $y = \frac{x^2}{2} - \frac{\ln x}{4}$, $1 \leqslant x \leqslant 3$.

a) $y' = 2x$

$$\int \sqrt{1+4x^2}\,dx \left| \begin{array}{ll} u = \sqrt{1+4x^2} & dv = dx \\ du = \frac{4x}{\sqrt{1+4x^2}} & v = x \end{array} \right| =$$

$$= x\sqrt{1+4x^2} - \int \frac{4x^2\,dx}{\sqrt{1+4x^2}} = x\sqrt{1+4x^2} - \int \frac{4x^2+1}{\sqrt{1+4x^2}}\,dx + \int \frac{dx}{\sqrt{1+4x^2}} =$$

$$= x\sqrt{1+4x^2} - \int \sqrt{1+4x^2}\,dx + \frac{\ln|2x+\sqrt{4x^2+1}|}{2}$$

$$2\int \sqrt{1+4x^2}\,dx = x\sqrt{1+4x^2} + \frac{\ln|2x+\sqrt{4x^2+1}|}{2}$$

$$\int_1^2 \sqrt{1+4x^2}\,dx = \frac{x}{2}\sqrt{1+4x^2} + \frac{\ln|2x+\sqrt{4x^2+1}|}{4}\Big|_1^2 =$$

$$= \sqrt{17} + \frac{\ln(4+\sqrt{17})}{4} - \frac{\sqrt{5}}{2} - \frac{\ln(2+\sqrt{5})}{4}$$

b) $y' = x - \frac{1}{4x} = \frac{4x^2-1}{4x}$

$$\int_1^3 \sqrt{1 + \frac{16x^4 - 8x^2 + 1}{16x^2}}\,dx = \int_1^3 \sqrt{\frac{16x^4+8x^2+1}{16x^2}}\,dx =$$

$$= \int_1^3 \frac{4x^2+1}{4x}\,dx = \frac{x^2}{2} + \frac{1}{4}\ln x \Big|_1^3 =$$

$$= \frac{9}{2} + \frac{1}{4}\ln 3 - \frac{1}{2} = 4 + \frac{1}{4}\ln 3$$

4. (HW) Find the arc length of the curve

$$x(t) = \sin^3(e^t), \quad y(t) = \cos^3(e^t), \quad \ln\frac{\pi}{4} \leqslant t \leqslant \ln\frac{\pi}{2}.$$

$$x(t)' = 3e^t \cdot \sin^2 e^t \cdot \cos e^t$$

$$y(t)' = -3e^t \cos^2 e^t \cdot \sin e^t$$

$$\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{2}} \sqrt{9e^{2t}\sin^4 e^t \cos^2 e^t + 9e^{2t}\cos^4 e^t \sin^2 e^t}\, dt =$$

$$= 3\int_{\ln\frac{\pi}{4}}^{\ln\frac{\pi}{2}} e^t \sqrt{\sin^4 e^t \cos^2 e^t + \cos^4 e^t \sin^2 e^t}\, dt \left| \begin{matrix} k = e^t \\ dk = e^t dt \end{matrix} \right| =$$

$$= 3\int_{\pi/4}^{\pi/2} (\sin^2 k \cos^2 k(\sin^2 k + \cos^2 k))\, dk = 3\int_{\pi/4}^{\pi/2} \sin k \cos k\, dk =$$

$$= \frac{3}{2}\int_{\pi/4}^{\pi/2} \sin 2k\, dk = -\frac{3}{4}\cos 2k \Big|_{\pi/4}^{\pi/2} = \frac{3}{4}$$

6. (HW) Find the area of the surface generated by revolving the arc of $x = y^3$ from $y = 0$ to $y = 1$ about the y-axis.

$$x' = 3y^2$$

$$2\pi \int_0^1 y^3 \sqrt{1 + 9y^4}\, dy \left| \begin{matrix} u = 1 + 9y^2 \\ du = 36y^3 dy \end{matrix} \right| = \frac{2\pi}{36}\int_{y=0}^{y=1} \sqrt{u}\, du =$$

$$= \frac{\pi}{27} u^{3/2} \Big|_{y=0}^{y=1} = \frac{\pi}{27}(1 - 9y^4)^{3/2} \Big|_0^1 = \frac{\pi}{27}(10\sqrt{10} - 1)$$

8. (HW) Find the area of the surface of solid obtained by revolving the arc of

$$x(t) = e^t \cos t, \quad y(t) = e^t \sin t$$

from $t = 0$ to $t = \pi$ about the x-axis.

$$x(t)' = e^t \cos t - e^t \sin t$$

$$y(t)' = e^t \sin t + e^t \cos t$$

$$2\pi \int_0^\pi e^t \sin t \sqrt{e^{2t}\cos^2 t - 2e^{2t}\sin t\cos t + e^{2t}\sin^2 t + e^{2t}\sin^2 t + 2e^{2t}\sin t\cos t + e^{2t}\cos^2 t}\, dt =$$

$$= 2\pi \int_0^\pi e^t \sin t \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t}\, dt =$$

$$= 2\pi \int_0^\pi e^t \sin t \cdot \sqrt{2} e^t = 2\sqrt{2}\pi \int_0^\pi e^{2t}\sin t\, dt =$$

I solved exactly the same one during the midterm, so let's use the "mind technique"

$$= 2\sqrt{2}\pi \cdot \frac{e^{2\pi}+1}{5} =$$

$$= \frac{2\sqrt{2}\pi e^{2\pi} + 2\sqrt{2}\pi}{5}$$

9*. (HW) Find the arc length of the curve

$$x(t) = 3t, \quad y(t) = \frac{2}{3}t^{3/2}, \quad z(t) = 0.5t^2, \quad 0 \leqslant t \leqslant 1.$$

$x(t)' = 3$

$y(t)' = \sqrt{t}$

$z(t)' = t$

$$\int_0^1 \sqrt{9+t+t^2}\,dt = \int_0^1 \sqrt{\left(t+\frac{1}{2}\right)^2 + \frac{35}{4}} =$$

$$= \frac{1}{2}\left(\left(t+\frac{1}{2}\right)\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{35}{4}} + \frac{35}{4}\ln\left|\frac{t+\frac{1}{2}+\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{35}{4}}}{\frac{35}{4}}\right|\right)$$

$$= \frac{3\sqrt{11}}{4} + \frac{35}{8}\ln\left(\frac{6+4\sqrt{11}}{35}\right) - \frac{3}{4} - \frac{35}{8}\ln\frac{2}{5} =$$

$$= \frac{3\sqrt{11}-3}{4} + \frac{35}{8}\ln\left(\frac{3+2\sqrt{11}}{7}\right)$$