1. Consider the schematic map of another city's downtown.

streels	~>	1	2	3	4	5
Avenues	1	H			S	
	2		HR		R	
	3	S	B S	S	S	SH
	4	B				B
	5	B		R		SR

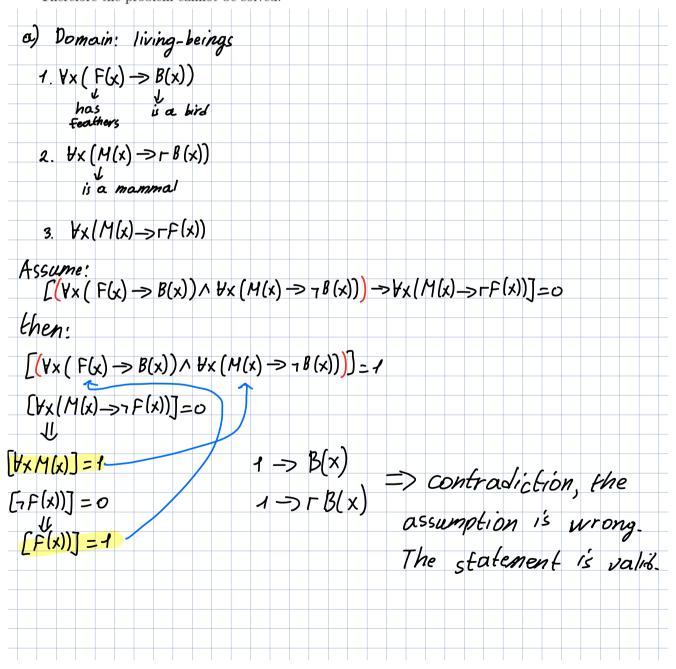
Translate the following statements into/from logical symbolism and check if they are true:

- a) There is a street with at least two different Bank offices.
- b) Only Restaurants and Supermarkets can share a crossing with a Hotel.
- c) Every avenue with a Supermarket has a Restaurant as well.
- d) If you mistake streets for avenues and vice versa, the map is still accurate.
- e)  $\exists m \forall n \, S(n,m) \to \forall n \exists m \, R(n,m)$ .
- f)  $\exists i \exists j \exists n \exists m (B(i,j) \land S(i,m) \land S(n,j) \land B(n,m)).$

a) 
$$\exists y \exists x_1 \exists x_2 (B(x_1, y) \land B(x_2, y))$$
 Such street does exist: street 4

- b)  $\exists x \exists y (H(x,y) \rightarrow R(x,y)) v (H(x,y) \rightarrow S(x,y)))$  That's true. A bank is never located on the same crossing as a hotel.
- c)  $\forall x \exists y_1 \exists y_2 (S(x,y_1) \land R(x,y_2))$  Not true. Avenue 1, For example, doesn't have a R.
- d) Let R, S, H, B = I (infrastructure)  $\forall x \forall y (I(x,y) = I(y,x))$ Not true, the map is completely different.
- e) If there exists a street all crossings of which have an S, then there is an avenue all crossings of which have an R. It's true, because the assumption is false.
- f) j b 5 There are 4 crossings, 2 of which have an S and the m S B other 2 have a B.

- 2. Put the following arguments in symbols and check their validity:
- a) Only birds have feathers. No mammal is a bird. Therefore each mammal is featherless.
- b) Everyone loves himself. Therefore someone is loved by somebody.
- c) Any mathematician can solve this problem if anyone can. Paul is a mathematician and cannot solve the problem. Therefore the problem cannot be solved.
- d) Anyone who can solve this problem is a mathematician. Paul cannot solve this problem. Therefore Paul is not a mathematician.
- e) Anyone who can solve this problem is a mathematician. No mathematician can solve this problem. Therefore the problem cannot be solved.



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b) 1 \fores who by whom
 2. Ix Iy (L(x,y))
Assume:
  [\forall x (L(x,x) \rightarrow \exists x \exists y (L(x,y))] = 0
 [ ( ( (x, x) ) = 1
[\exists x \exists y (L(x,y))] = 0
  If x=y, there's a contradiction => arguement is
  c) 1, \forall x (S(x) \rightarrow \forall y (H(y) \land S(y))
solve mathematician
  2. \(\frac{1}{2}\) \(\frac{1}{2}\)
           Paul
     3. ∀x (7S(x)
  Assume:
   \left[\left(\forall x \left(S(x) \rightarrow \forall y \left(H(y) \land S(y)\right) \land \forall x \left(P(x) \land \neg S(x)\right)\right) \rightarrow \forall x \left(\neg S(x)\right] = 0\right]
  then:
[(\forall x (S(x) \rightarrow \forall y (M(y) \land S(y)) \land \forall x (P(x) \land \neg S(x)))] = 1
[\tag{\chi} \(\angle \(\angle \)] =0
[ \ \ y ( M | y ) ] = P
                          => No contradiction, the argument is
[+x ( S(x)]=1
[S(y)]=1
                                         invalid.
[P(x)]=1
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