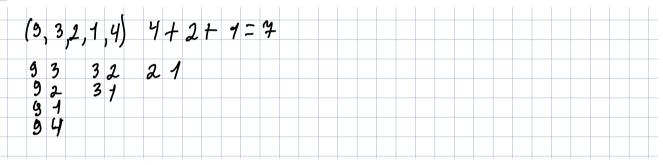
1. (1 point) Find the number of inversions in the following ordered line:

(9, 1, 3, 7, 8, 2, 5, 4, 6).

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2. (1 point) Find distinct positive integers a, b, c, d, and e, such that the ordered line (a, b, c, d, e) has seven inversions.



3. (0,5 points per item) Let

$$f = \begin{pmatrix} 2 & 3 & 5 & 1 & 4 \\ 4 & 1 & 2 & 3 & 5 \end{pmatrix} \quad \text{and} \quad g = \begin{pmatrix} 1 & 5 & 3 & 4 & 2 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}.$$

Then, find:

- (a) N(f)
- (b) N(g)
- (c) the cycle notation for f and g
- (d) sgn(f)
- (e) sgn(g)
- (f) $f \circ g$
- (g) $g \circ f$
- (h) $f^{-1} \circ g^{-1}$
- (i) f^{5}
- (i) a^5

Note: for Items (f)-(j) you can, at your discretion, represent the answer using either the cycle notation or the two-line notation.

a)
$$(1+1+2)+(3)=7$$

a)
$$(1+1+2)+(3)=4$$

b) $(3+1+1)+(4+3)=12$
c) $f=(245)(31); g=(15423)$
d) $sgn(f)=(-1)^{4}=-1$

d)
$$sgn(f) = (-1)^{4} = -1$$

$$f) \ Fog = \begin{pmatrix} 1 & 5 & 3 & 4 & 2 \\ 2 & 5 & 3 & 4 & 1 \end{pmatrix}$$

h)
$$f^{-1} = (41235) g^{-1} = (54123) (23514) g^{-1} = (54123)$$

i)
$$f^5 = ((245)(13))^5 = (245)^2(13)^7 = (254)(13)$$

j) $g^5 = (15423)^5 = (1)(5)(4)(2)(3)$

4. (3 points) Let

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 8 & 11 & 7 & 12 & 9 & 1 & 6 & 3 & 2 & 13 & 5 & 4 & 10 \end{pmatrix}.$$

Then, find f^{2077} .

[hint: 1) use the cycle notation (do not even try to solve this problem using two-line notation or direct calculation); 2) remember that the independent cycles commute, that it, for instance, if f = (12)(345) then $f^k = (12)^k (345)^k$, for any integer k; 3) $(a, b, c, d, e)^5 = ?$

$$f^{20\%4} = ((183\%6)(21159)(412)(59211)(1013))^{20\%4} =$$

$$= (183\%6)^{2} (21159)^{1} (412)^{1} (1013)^{2} =$$

$$= (1368\%)(21159)(412)(1013)$$