1) a) $f(x) = x^3$

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3 - \cancel{x^3}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + \Delta x^2 = 3x^2$$

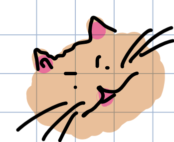
b) $F(x) = 2\sqrt{x+4}$, $x_0 = 5$

$$\left(2\sqrt{x+4}\right)' = \lim_{\Delta x \to 0} \frac{2\sqrt{(x+\Delta x)+4} - 2\sqrt{x+4}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\left(2\sqrt{x+\Delta x+4} - 2\sqrt{x+4}\right)\left(2\sqrt{x+\Delta x+4} + 2\sqrt{x+4}\right)}{\Delta x\left(2\sqrt{x+\Delta x+4} + 2\sqrt{x+4}\right)} =$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{4x} + 4\cancel{\Delta x} + \cancel{16} - \cancel{4x} - \cancel{16}}{\cancel{\Delta x}\left(2\sqrt{x+\Delta x+4} + 2\sqrt{x+4}\right)} = \lim_{\Delta x \to 0} \frac{4}{2\sqrt{x+\Delta x+4} + 2\sqrt{x+4}} =$$

$$= \frac{4}{2\sqrt{x+4} + 2\sqrt{x+4}} \overset{x=5}{=} \frac{4}{2\cdot 3 + 2\cdot 3} = \frac{1}{3}$$



c) $f(x) = \cos x$

$$\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos x \cos\Delta x - \sin x \sin\Delta x - \cos x}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\cos\Delta x - 1}{\Delta x}\cos x - \frac{\sin\Delta x}{\Delta x}\sin x = 0\cdot\cos x - \sin x = -\sin x$$

4) a) $f(x) = |\sin x|$

$|\sin x|$ is differentiable at any point except 0.

Consider $x = \pi k$.

$$\lim_{\Delta x \to 0+} \frac{|\sin(\pi k + \Delta x)| - |\sin(\pi k)|}{\Delta x} = \lim_{\Delta x \to 0+} \frac{|\sin \Delta x|}{\Delta x} = 1$$

$$\lim_{\Delta x \to 0-} \frac{|\sin(\pi k + \Delta x)| - |\sin(\pi k)|}{\Delta x} = \lim_{\Delta x \to 0+} \frac{|\sin \Delta x|}{\Delta x} = -\lim_{\Delta x \to 0-} \frac{\sin \Delta x}{\Delta x} = -1$$

b) 1) $x > 0$ $f(x) = x^2 \Rightarrow$ differentiable at any point.

2) $x < 0$ $f(x) = -x^2 \Rightarrow$ differentiable at any point.

$$\left.\begin{array}{l} \lim\limits_{\Delta x \to 0+} \frac{x \cdot \cancel{x}}{\cancel{x}} = 0 \\ \lim\limits_{\Delta x \to 0-} -\frac{x \cdot x}{x} = 0 \end{array}\right\} \Rightarrow \text{differentiable at } x = 0$$

6) $f(x) = \begin{cases} ax + b, & x > 1 \\ bx^2 + 2, & x \leq 1 \end{cases}$

$$\lim_{\Delta x \to 0-} \frac{b(x+\Delta x)^2 + 2 - bx^2 - 2}{\Delta x} = \lim_{\Delta x \to 0-} \frac{\cancel{bx^2} + 2x\Delta x b + \Delta x^2 b \cancel{+2} \cancel{-bx^2} \cancel{-2}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0-} \frac{\Delta x(2xb + \Delta x b)}{\Delta x} = 2xb \Rightarrow \text{if } f'(1) \Rightarrow 2b \text{ and } a+b \text{ has to be cont. so } b = 1$$

$$\lim_{\Delta x \to 0+} \frac{a(x+\Delta x) + b - ax - b}{\Delta x} = \lim_{\Delta x \to 0+} \frac{ax + a\Delta x - ax}{\Delta x} = a$$

So, $a = 2$, $b = 1$

8) a) $f(x) = 5^{\cos x} \cdot \ln x + \frac{x^2+\sin x}{\sqrt{5x^2+3x-7}}$

$5^{\cos x} =$

$f'(x) = \left(5^{\cos x} \cdot \ln x\right)' + \left(\frac{x^2+\sin x}{\sqrt{5x^2+3x-7}}\right)' = \left(\left(5^{\cos x}\right)'(\ln x) + \left(5^{\cos x}\right)(\ln x)'\right) +$

$+ \left(\frac{(x^2+\sin x)'(\sqrt{5x^2+3x-7}) - (x^2+\sin x)(\sqrt{5x^2+3x-7})'}{5x^2+3x-7}\right) =$

$= \left(\left(\ln 5 \cdot 5^{\cos x} \cdot (-\sin x)\right)(\ln x) + \left(5^{\cos x}\right)\left(\frac{1}{x}\right)\right) +$

$+ \left(\frac{(2x+\cos x)(\sqrt{5x^2+3x-7}) - (x^2+\sin x)\left(\frac{1}{2\sqrt{5x^2+3x-7}}\right)(10x+3)}{5x^2+3x-7}\right)$

b) $f(x) = (x^2+3) \cdot \tan\sqrt{x} + \frac{5^x}{7x-\ln x}$

$f'(x) = \left((x^2+3) \cdot \tan\sqrt{x}\right)' + \left(\frac{5^x}{7x-\ln x}\right)' =$

$= \left((2x)(\tan\sqrt{x}) + (x^2+3)\left(\frac{1}{\cos^2\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}\right)\right) +$

$+ \left(\frac{(5^x \cdot \ln 5)(7x-\ln x) - (5^x)(7-\frac{1}{x})}{(7x-\ln x)^2}\right)$

10) a) $f(x) = (\arctan x)^{\cos^2 x}$

$f'(x) = e^{\cos^2 x \cdot \ln(\arctan x)} \cdot \left(\frac{1}{\arctan x} \cdot \frac{1}{1+x^2} \cdot \cos^2 x + \ln(\arctan x \cdot\right.$

$\left. \cdot 2\cos x \cdot (-\sin x)\right) =$

$$= e^{\ln(\arctan x)^{\cos^2 x}} \cdot \left(\frac{\cos^2 x - \arctan x\,(1+x^2)\cdot \ln(\arctan x \cdot \sin 2x)}{\arctan x \cdot (1+x^2)} \right)$$

$$= \arctan x^{\cos^2 x} \cdot \left(\frac{\cos^2 x - \arctan x\,(1+x^2)\cdot \ln(\arctan x \cdot \sin 2x)}{\arctan x \cdot (1+x^2)} \right)$$

b) $$f'(x) = \frac{e^{\arccos x}(x+7)^9}{(1+x^2)^4} =$$

$$= \frac{\left(e^{\arccos x}(x+7)^9\right)'\left((1+x^2)^4\right) - e^{\arccos x}(x+7)^9\left((1+x^2)^4\right)'}{\left((1+x^2)^4\right)^2} =$$

$$= \frac{\left(e^{\arccos x}\left(-\frac{1}{\sqrt{1-x^2}}\right)(x+7)^9 + e^{\arccos x}\left(9(x+7)^8\right)\right)(1+x^2)^4}{\left((1+x^2)^4\right)^2} -$$

$$- \frac{e^{\arccos x}(x+7)^9\left(4(1+x^2)^3\cdot 2x\right)}{\left((1+x^2)^4\right)^2}$$

$\pi = 3{,}1415926$

Let me know if I need to simplify 8) and 10) further.