2) a) $\displaystyle\int \frac{15x+3}{2\sqrt{x+3}}dx \left| \begin{array}{l} y=\sqrt{x+3} \\ y^2=x+3 \\ x=y^2-3 \\ dx=2y\,dy \end{array}\right. = \int \frac{15y^2-42}{2y}\cdot 2y\,dy =$

$= 15\int y^2 dy - 42\int dy = 5(x+3)^{\frac{3}{2}} - 42\sqrt{x+3} + C$

b) $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}+1}dx \left| \begin{array}{l} y=\sqrt{x} \\ y^2=x \\ dx=2y\,dy \end{array}\right. = \int \frac{y}{y+1}\cdot 2y\,dy = 2\int \frac{y^2}{y+1}\,dy =$

$$\begin{array}{r|l} -\,y^2 & y+1 \\ \underline{y^2+y} & y-1 \\ -\,-y & \\ \underline{-y-1} & \\ 1 & \end{array}$$

$= 2\int\left(y-1+\frac{1}{y+1}\right)dy = y^2-2y+\ln|y+1|+C =$

$= x-2\sqrt{x}+2\ln|\sqrt{x}+1|+C$

c) $\displaystyle\int \frac{\sqrt{x}\,dx}{x-\sqrt[3]{x^2}} \left| \begin{array}{l} y=\sqrt[3]{x} \\ y^3=x \\ dx=3y^2dy \end{array}\right. = 3\int \frac{y^{\frac{3}{2}}}{y^3-y^2}\,y^2dy =$

$= 3\displaystyle\int \frac{\sqrt{y^3}}{y-1}\,dy \left| \begin{array}{l} t=\sqrt{y} \\ t^2=y \\ dy=2t\,dt \end{array}\right. = 6\int \frac{t^3}{t^2-1}\,t\,dt =$

$$\begin{array}{r|l} -\,t^4 & t^2-1 \\ \underline{t^4-t^2} & t^2+1 \\ -\,t^2 & \\ \underline{t^2+1} & \\ 1 & \end{array}$$

$= 6\int\left(t^2+1+\frac{1}{t^2-1}\right)dt = 2t^2+6t+3\ln\left|\frac{t-1}{t+1}\right|+C =$

$t=x^{1/6}$

$F(x) = 2\sqrt{x}+6\sqrt[6]{x}+3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right|+C$

d) $\displaystyle\int \frac{\sqrt{x}-1}{\sqrt{x}+1}dx \left| \begin{array}{l} y=\sqrt{x} \\ y^2=x \\ dx=2y\,dy \end{array}\right. = 2\int \frac{y-1}{y+1}\,dy \left| \begin{array}{l} t=y+1 \\ dt=dy \end{array}\right. =$

$$=2\int \frac{t-2}{t}\,dt = 2t - 4\ln|t| + C = 2(\sqrt{x}+1) - 4\ln|\sqrt{x}+1| + C$$

e) $$\int \frac{dx}{\sqrt[3]{x}-\sqrt[4]{x}} \left| \begin{array}{l} y = x^{\frac{1}{4}} \\ y^4 = x \\ dx = 4y^3dy \end{array} \right. = 4\int \frac{y^3dy}{y^{4/3}-y} = 4\int \frac{y^2dy}{\sqrt[3]{y}-1} \left| \begin{array}{l} u = y^{\frac{1}{3}} \\ u^3 = y \\ dy = 3u^2du \end{array} \right. =$$

$$= 12\int \frac{u^8\,du}{u-1} = 12\left[\int \frac{u^8-1}{u-1}\,du + \int \frac{1}{u-1}\,du\right] =$$

$$= 12\left(\frac{u^8}{8} + \frac{u^4}{4} + \frac{u^6}{6} + \frac{u^2}{2} + \frac{u^7}{7} + \frac{u^3}{3} + \frac{u^5}{5} + u + \ln|u-1|\right) =$$

$u = x^{1/12}$

$$= \frac{3}{2}x^{2/3} + 3x^{1/3} + 2x^{1/2} + 6x^{1/6} + \frac{12}{7}x^{7/12} + 4x^{1/4} + \frac{12}{5}x^{5/12} + 12x^{1/12} +$$
$$+ 12\ln|x^{1/12}-1|$$

4) a) $$\int x^2\sqrt{1-x^2}\,dx \left| \begin{array}{l} x = \sin t \\ dx = \cos t\,dt \end{array} \right. = \int \sin^2 t\sqrt{1-\sin^2 t}\cos t\,dt =$$

$$= \int \sin^2 t\cos^2 t\,dt = \int (\sin t\cos t)^2\,dt = \frac{1}{4}\int \sin^2 2t\,dt =$$

$$= \frac{1}{8}\int (1-\cos 4t)\,dt = \frac{t}{8} - \frac{1}{32}\sin 4t + C \Big|^{t=\arcsin x} =$$

$\sin 4x = 2\sin 2x\cos 2x = 4\sin x\cos x(\cos^2 x - \sin^2 x)$

$$= \frac{\arcsin x}{8} - \frac{1}{32}\left(4x\sqrt{1-x^2}(1-x^2-x^2)\right) + C =$$

$$= \frac{\arcsin x}{8} - \frac{1}{8}x\sqrt{1-x^2}(1-2x^2) + C$$

b) $$\int \sqrt{x^2+9}\,dx \left| \begin{array}{l} x = 3\sinh t \\ dx = 3\cosh t\,dt \end{array} \right. = 9\int \sqrt{\sinh^2 t+1}\cosh t\,dt =$$

$$= 9\int \cosh^2 t\,dt = \frac{9}{2}\int (\cosh 2t + 1)\,dt = \frac{9}{4}\sinh 2t + \frac{9}{2}t + C =$$

$$= \frac{9}{2}\sinh t\cosh t + \frac{9}{2}t + C \Big|^{t=\operatorname{arcsinh}\frac{x}{3}} =$$

$$= \frac{3}{2}x\sqrt{\frac{x^2}{9}+1} + \frac{9}{2}\operatorname{arcsinh}\frac{x}{3} + C$$

5) a) $$\int \frac{\sin x}{1+\sin x}\,dx = \int \frac{\sin x+1}{\sin x+1}dx - \int \frac{dx}{\sin x+1} \left|\begin{array}{l} x = 2\arctan t \\ \sin x = \frac{2t}{1+t^2} \\ dx = \frac{2\,dt}{1+t^2} \end{array}\right. =$$

$$= x - \int \frac{2dt}{1+t^2}\cdot\frac{1+t^2}{2t+t^2+1} = x - \int \frac{2dt}{2t+t^2+1} =$$

$$= x - 2\int \frac{dt}{(t+1)^2} \left|\begin{array}{l} u = t+1 \\ du = dt \end{array}\right. = x - 2\int \frac{du}{u^2} = x + \frac{2}{t+1} =$$

$$= x + \frac{2}{\tan\frac{x}{2}+1} + C$$

b) $$\int \frac{(x+3)(x+4)}{(x-2)(x-6)^2}dx = \int \frac{15dx}{8(x-2)} - \int \frac{7dx}{8(x-6)} + \int \frac{45dx}{2(x-6)^2} =$$

$$= \frac{15}{8}\ln|x-2| - \frac{7}{8}\ln|x-6| - \frac{45}{2(x-6)} + C$$

c) $$\int \cos x\cos 2x\cos 3x\,dx = \frac{1}{2}\int (2\cos(-x)\cos 2x)\cos 3x\,dx =$$

$$= \frac{1}{2}\int (\cos 3x + \cos x)\cos 3x\,dx = \frac{1}{2}\int (\cos^2 3x + \cos x\cos 3x)dx =$$

$$= \frac{1}{4}\int (\cos 6x + 1 + \cos 4x + \cos 2x)dx =$$

$$= \frac{1}{24}\sin 6x + \frac{1}{16}\sin 4x + \frac{1}{8}\sin 2x + \frac{x}{4} + C$$

d) $$\int \frac{\ln x\cos(\ln x)}{x}dx \left|\begin{array}{l} t = \ln x \\ dt = \frac{dx}{x} \end{array}\right. = \int t\cos t\,dt \left|\begin{array}{ll} y = t & dw = \cos t\,dt \\ dy = dt & w = \sin t \end{array}\right. =$$

$-\int \sin t\,dt = \cos t$

$= \ln x \sin(\ln x) + \cos(\ln x) + C$

e) $\int \frac{3x^2-1}{x\sqrt{x}} \arctan x\,dx \quad \left| \begin{array}{l} u = \arctan x \\ du = \frac{dx}{1+x^2} \end{array} \right. \quad dw = \frac{3x^2-1}{x\sqrt{x}} \quad w = \frac{2x^2+2}{\sqrt{x}} \quad =$

$\int \frac{3x^2-1}{x\sqrt{x}} = \int 3\sqrt{x}\,dx - \int \frac{dx}{x^{3/2}} = 2x\sqrt{x} + \frac{2}{\sqrt{x}} + C$

$= \arctan x \left(\frac{2x^2+2}{\sqrt{x}} \right) - \int \frac{2(1+x^2)}{\sqrt{x}} \cdot \frac{1}{1+x^2}\,dx =$

$= \arctan x \left(\frac{2x^2+2}{\sqrt{x}} \right) - 4\sqrt{x}\,dx$