2) a) $\int (x+1)(3x-2)dx = \int (3x^2+x-2)dx = x^3+\frac{x^2}{2}-2x+C$

$\left(x^3+\frac{x^2}{2}-2x+C\right)' = 3x^2+x-2 = (x+1)(3x-2)$

b) $\int \frac{4}{6+x^2}dx = 4\int \frac{dx}{\sqrt{6}^2+x^2} = \frac{4}{\sqrt{6}}\operatorname{arctg}\left(\frac{x}{\sqrt{6}}\right)+C$

$\left(\frac{4}{\sqrt{6}}\operatorname{arctg}\left(\frac{x}{\sqrt{6}}\right)+C\right)' = \frac{4}{6+x^2}$

c) $\int \frac{\cos 2x}{\sin^2 x\cdot\cos^2 x}dx = \int \frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x}dx = \int\frac{dx}{\sin^2 x} - \int\frac{dx}{\cos^2 x} =$

$= -\cot x - \tan x + C$

$(-\cot x-\tan x+C)' = \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} = \frac{\cos^2 x-\sin^2 x}{\sin^2 x\cos^2 x} = \frac{\cos 2x}{\sin^2 x\cos^2 x}$

d) $\int x^{7/8}(2x^{3/5}-3x^{-2}+\sqrt{x})dx = \int 2x^{59/40}-3x^{-9/8}+x^{11/8}dx =$

$2x^{3/5+7/8}-3x^{-2+7/8}+x^{1/2+7/8} = 2x^{59/40}-3x^{-9/8}+x^{11/8}$

$= \frac{80}{99}x^{99/40}+24x^{-\frac{1}{8}}+\frac{8}{19}x^{19/8}+C$

$\left(\frac{80}{99}x^{99/40}+24x^{-\frac{1}{8}}+\frac{8}{19}x^{19/8}+C\right)' = 2x^{59/40}-3x^{-9/8}+x^{11/8} =$

$= x^{7/8}(2x^{3/5}-3x^{-2}+\sqrt{x})$

e) $\int \frac{\sqrt{x}-x^3e^x+x^2}{x^3}dx = \int\left[\frac{x^{\frac{1}{2}}}{x^3}-\frac{x^3e^x}{x^3}+\frac{x^2}{x^3}\right]dx =$

$= \int\left[x^{-\frac{5}{2}}-e^x+x^{-1}\right]dx = -\frac{2}{3}x^{-\frac{3}{2}}-e^x+\ln|x|+C$

$\left(-\frac{2}{3}x^{-\frac{3}{2}}-e^x+\ln|x|+C\right)' = x^{-\frac{5}{2}}-e^x+x^{-1} = \frac{x^{\frac{1}{2}}}{x^3}-\frac{x^3e^x}{x^3}+\frac{x^2}{x^3} =$

$= \frac{\sqrt{x}-x^3e^x+x^2}{x^3}$

f) $\int \frac{x\sin 2x + \sqrt[5]{x^2}\cos x}{x\cos x}\,dx = \int \left[\frac{x\sin 2x}{x\cos x} + \frac{x^{\frac{2}{5}}\cos x}{x\cos x}\right] dx =$

$= \int [2\sin x + x^{-\frac{3}{5}}]\,dx = -2\cos x + \frac{5}{2} x^{\frac{2}{5}} + C$

$\left(-2\cos x + \frac{5}{2} x^{\frac{2}{5}} + C\right)' = 2\sin x + x^{-\frac{3}{5}} = \frac{x\sin 2x}{x\cos x} + \frac{x^{\frac{2}{5}}\cos x}{x\cos x} =$

$= \frac{x\sin 2x + \sqrt[5]{x^2}\cos x}{x\cos x}$

g) $\int \frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{\sqrt{9-x^4}}\,dx = \int \frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{\sqrt{(3-x^2)(3+x^2)}}\,dx =$

$= \int \left[\frac{1}{\sqrt{3-x^2}} - \frac{1}{\sqrt{3+x^2}}\right] dx = \arcsin\left(\frac{x}{\sqrt{3}}\right) - \ln|x + \sqrt{x^2+3}| + C$

$\left(\arcsin\left(\frac{x}{\sqrt{3}}\right) - \ln|x + \sqrt{x^2+3}| + C\right)' = \frac{1}{\sqrt{3-x^2}} - \frac{1}{\sqrt{3+x^2}} = \frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{\sqrt{(3-x^2)(3+x^2)}} =$

$= \frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{\sqrt{9-x^4}}$

h) $\int (\tan x + \cot x)^2\,dx = \int \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)^2 dx =$

$= \int \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)^2 dx = \int \left(\frac{\sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x}{\sin^2 x\cos^2 x}\right) dx =$

$= \int \left[\frac{\sin^2 x}{\cos^2 x} + 2 + \frac{\cos^2 x}{\sin^2 x}\right] dx = \int [\tan^2 x + \cot^2 x + 2]\,dx =$

$= \int \left[\frac{1}{\cos^2 x} - 1 + \frac{1}{\sin^2 x} - 1 + 2\right] dx = \int \left[\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right] dx =$

$= \tan x - \cot x + C$

$(\tan x - \cot x + C)' = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \ldots = (\tan x + \cot x)^2$

i) $\displaystyle\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int\left[\frac{1}{x^2(1+x^2)} + \frac{2x^2}{x^2(1+x^2)}\right]dx = \int\left[\frac{1}{x^2} - \frac{1}{1+x^2} + \frac{2}{1+x^2}\right]dx =$

$\displaystyle= \int\left[\frac{1}{x^2} + \frac{1}{1+x^2}\right]dx = -\frac{1}{x} + \arctan x + C$

$\displaystyle\left(-\frac{1}{x} + \arctan x + C\right)' = \frac{1}{x^2} + \frac{1}{1+x^2} = \dots = \frac{1+2x^2}{x^2(1+x^2)}$

4) a) $\displaystyle\int \frac{4x+5}{2x^2+5x-6}dx \left|\begin{array}{l} u = 2x^2+5x-6 \\ du = (4x+5)dx \\ dx = \frac{1}{4x+5}du \end{array}\right| = \int \frac{4x+5}{2x^2+5x-6}\cdot\frac{1}{4x+5}du =$

$\displaystyle= \int \frac{1}{u}\cdot du = \ln|u| + C = \ln|2x^2+5x-6| + C$

b) $\displaystyle\int \frac{\cos 2x}{\sin x\cos x}dx \left|\begin{array}{l} u = \sin x\cos x \\ du = \cos 2x\cdot dx \\ dx = \frac{1}{\cos 2x}du \end{array}\right| = \int \frac{\cos 2x}{u}\cdot\frac{1}{\cos 2x}du =$

$\displaystyle= \int \frac{1}{u}\cdot du = \ln|u| + C = \ln|\sin x\cos x| + C$, where $\ln(2) \in C$

$\displaystyle= \int \frac{2\cos 2x}{\sin 2x} \left|\begin{array}{l} u = \sin 2x \\ du = 2\cos 2x\cdot dx \\ dx = \frac{1}{2\cos 2x}\cdot du \end{array}\right| = \int \frac{1}{u}\cdot du = \ln|\sin 2x| + C$

c) $\displaystyle\int \frac{x^3}{\sqrt{1-x^8}}dx = \int \frac{x^3}{\sqrt{1-(x^4)^2}}dx \left|\begin{array}{l} u = x^4 \\ du = 4x^3\cdot dx \\ dx = \frac{1}{4x^3}\cdot du \end{array}\right| = \frac{1}{4}\int \frac{du}{\sqrt{1-u^2}} = \frac{\arcsin u}{4} + C =$

$\displaystyle= \frac{\arcsin x^4}{4} + C$

d) $\displaystyle\int \frac{\arctan x}{1+x^2}dx = \int \arctan x\cdot\frac{1}{1+x^2}dx \left|\begin{array}{l} u = \arctan x \\ du = \frac{1}{1+x^2}\cdot dx \\ dx = (1+x^2)du \end{array}\right| =$

$$= \int u \cdot du = \frac{u^2}{2} + C = \frac{\arctan^2 x}{2} + C$$

e) $$\int e^{\cos x} \cdot \sin x \, dx \left| \begin{array}{l} u = \cos x \\ du = -\sin x \cdot dx \\ dx = \frac{1}{-\sin x} \cdot du \end{array} \right| = -\int e^u \cdot du = -e^u + C =$$

$$= -e^{\cos x} + C$$

f) $$\int \frac{\cos(\ln x)}{x} dx \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x} \cdot dx \\ dx = x \cdot du \end{array} \right| = \int \cos u \cdot du = \sin(\ln x) + C$$

6) a) $$\int e^{7x-2} dx \left| \begin{array}{l} u = 7x-2 \\ du = 7dx \\ dx = \frac{1}{7} du \end{array} \right| = \int e^u \cdot \frac{du}{7} = \frac{1}{7} \int e^u \cdot du = \frac{e^{7x-2}}{7} + C$$

b) $$\int (4-3x)^{7/2} dx \left| \begin{array}{l} u = 4-3x \\ du = -3 \cdot dx \\ dx = \frac{du}{-3} \end{array} \right| = \int u^{7/2} \cdot \frac{du}{-3} = -\frac{1}{3} \int u^{7/2} \cdot du =$$

$$= -\frac{1}{3} \cdot \frac{2}{9} \cdot u^{9/2} + C = -\frac{2}{27} (4-3x)^{9/2} + C$$

c) $$\int \frac{dx}{3 + (2x+5)^2} \left| \begin{array}{l} u = 2x+5 \\ du = 2dx \\ dx = \frac{du}{2} \end{array} \right| = \int \frac{du}{6 + 2u^2} = \frac{1}{2} \int \frac{du}{3 + u^2} =$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{3}} \cdot \arctan\left(\frac{u}{\sqrt{3}}\right) + C = \frac{\arctan\left(\frac{2x+5}{\sqrt{3}}\right)}{2\sqrt{3}} + C$$