1. (1 point) Find the number of inversions in the following ordered line:

$$(9, 1, 3, 7, 8, 2, 5, 4, 6).$$

9 1   3 2   7 2   8 2   5 4
9 3   7 5   8 5
9 7   7 4   8 4
9 8   7 6   8 6
9 2
9 5
9 4
9 6

$8 + 2 + 8 = 18$

2. (1 point) Find distinct positive integers a, b, c, d, and e, such that the ordered line (a, b, c, d, e) has seven inversions.

$(9, 3, 2, 1, 4)$ $4 + 2 + 1 = 7$

9 3   3 2   2 1
9 2   3 1
9 1
9 4

3. (0,5 points per item) Let

$$f = \begin{pmatrix} 2 & 3 & 5 & 1 & 4 \\ 4 & 1 & 2 & 3 & 5 \end{pmatrix} \quad \text{and} \quad g = \begin{pmatrix} 1 & 5 & 3 & 4 & 2 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}.$$

Then, find:

(a) $N(f)$
(b) $N(g)$
(c) the cycle notation for f and g
(d) $\operatorname{sgn}(f)$
(e) $\operatorname{sgn}(g)$
(f) $f \circ g$
(g) $g \circ f$
(h) $f^{-1} \circ g^{-1}$
(i) f^5 †
(j) g^5

Note: for Items (f)-(j) you can, at your discretion, represent the answer using either the cycle notation or the two-line notation.

a) $(1+1+2)+(3)=7$

b) $(3+1+1)+(4+3)=12$

c) $f=(245)(31);\ g=(15423)$

d) $sgn(f)=(-1)^{7}=-1$

e) $sgn(g)=(-1)^{12}=1$

f) $f\circ g=\begin{pmatrix}1&5&3&4&2\\2&5&3&4&1\end{pmatrix}$

g) $g\circ f=\begin{pmatrix}2&3&5&1&4\\2&5&3&1&4\end{pmatrix}$

h) $f^{-1}=\begin{pmatrix}4&1&2&3&5\\2&3&5&1&4\end{pmatrix}$ $g^{-1}=\begin{pmatrix}5&4&1&2&3\\1&5&3&4&2\end{pmatrix}$

$f^{-1}\circ g^{-1}=\begin{pmatrix}5&4&1&2&3\\3&4&1&2&5\end{pmatrix}$

i) $f^{5}=((245)(13))^{5}=(245)^{2}(13)^{1}=(254)(13)$

j) $g^{5}=(15423)^{5}=(1)(5)(4)(2)(3)$

4. (3 points) Let

$$f=\begin{pmatrix}1&2&3&4&5&6&7&8&9&10&11&12&13\\8&11&7&12&9&1&6&3&2&13&5&4&10\end{pmatrix}.$$

Then, find f^{2077}.
[**hint:** 1) use the cycle notation (do not even try to solve this problem using two-line notation or direct calculation); 2) remember that the independent cycles commute, that it, for instance, if $f=(12)(345)$ then $f^k=(12)^k(345)^k$, for any integer k; 3) $(a,b,c,d,e)^5=?$]

$f^{2077}=((18376)(21159)(412)(59211)(1013))^{2077}=$

$=(18376)^{2}(21159)^{1}(412)^{1}(1013)^{1}=$

$=(13687)(21159)(412)(1013)$