Introduction :

The term 'Operations Research' was coined in 1940 in England. During World war II, the military management of UK called on scientists from various disciplines and organized them into teams to assist it in solving strategic and tactical problems relating to air and land defence of the country. They were required to formulate specific proposals and plans for aiding the military commands to arrive at decisions on optimal utilization of military resources and efforts and also to implement the decision effectively. This is called Operations Research. This new technique is called OR. Hence OR can be termed as "an art of winning war without actually fighting it".

Scope of OR :

There is a great scope for economists, statisticians, administrative working as a team to solve the problems of defence by using OR approach. Besides this OR is using in

(i) Agriculture (ii) Industry (iii) Finance (iv) Marketing (v) Personal management (vi) Production management.

Phases of OR:

The procedure to be followed in the study of OR involves

(i) Formulating the problem
(ii) Constructing a mathematical model
(iii) Deriving the sol^n from the model
(iv) Testing the model and its sol^n
(v) Controlling the sol^n
(vi) Implementation

Uses and Limitations of OR :

Uses: (i) Its provides logical and systematic approach to problem
(ii) It suggests all the alternate courses of action for the same management.
(iii) It facilitates improved quality of decision.

Limitations :

(i) Models are the only idealized representations of reality and cannot be regarded as absolute in any case.
(ii) OR requires huge calculations which cannot be handled manually and require computers, resulting in heavy costs.
(iii) As it is a new field, there is a resistance from the employees to the new proposals.

Linear Programing Problems

□ **Linear Programing Problems:** A problems of the following type is called linear programing problems.

Find $x_1, x_2, ----, x_n$ which maximize the objective function

$$Z = c_1x_1 + c_2x_2 + ---- + c_nx_n$$

$$\text{s.t } a_{11}x_1 + a_{12}x_2 + --- + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + --- + a_{2n}x_n = b_2$$
$$-----------$$
$$-----------$$
$$a_{m1}x_1 + a_{m2}x_2 + --- + a_{mn}x_n = b_n$$

where $x_1, x_2, x_3, ---, x_n \geq 0$

which is in the matrix form can be written as

$$\text{Max } Z = \underset{\sim}{c}x$$
$$\text{s.t } A\underset{\sim}{x} = \underset{\sim}{b} \text{ and } \underset{\sim}{x} \geq 0$$

where $\underset{\sim}{c} = (c_1 \; c_2 \; --- \; c_n)$

$$\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } A = \begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ - & - & - & - \\ - & - & - & - \\ a_{m1} & a_{m2} & --- & a_{mn} \end{bmatrix}$$

and $\underset{\sim}{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$ and $\underset{\sim}{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

Ex.1: A manufacture of a line of patent medicine, prepairing a production plan on medicine A and B. There are sufficient in gradient available to make 20000 bottles of A and 40000 bottles of B medicine but there are only 45000 bottles into which either of the medicine can be put. Further more if takes 3 hours to prepair enough meterial to fill 1000 bottles of A, it takes 1 hour to prepair enough meterial to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is Rs-8 per bottles for A and Rs-7 per bottles of B. Formulate the LPP to maximize the profit.

Ans

A	20000	3	8
B	40000	1	7
	45000	66	

Let x_1 bottles of A and x_2 bottles of B are to be prepared.

Then total profit $Z = 8x_1 + 7x_2$

We have to maximize $Z = 8x_1 + 7x_2$

Since there are 45000 bottles into which either of the medicine can put.

$$\therefore \quad x_1 + x_2 \le 45000$$

Now the time required to produces x_1 bottles of A is $\frac{3x_1}{1000}$ hours

and " " " " " " x_2 " " B is $\frac{x_2}{1000}$ hours

Then Obviosly

$$\frac{3x_1}{1000} + \frac{x_2}{1000} \le 66$$

$$\text{or,} \quad 3x_1 + x_2 \le 66000$$

Moreover there are ingradients to produces 20000 bottles of A and 40000 bottles of B medicine

$$\therefore \quad x_1 \le 20000$$
$$x_2 \le 40000$$

Hence the problem is to max $Z = 8x_1 + 7x_2$

$$\text{s.t.} \quad x_1 + x_2 \le 45000$$
$$3x_1 + x_2 \le 66000$$
$$x_1 \le 20000$$
$$x_2 \le 40000$$
$$\text{and} \quad x_1, x_2 \geq 0$$

Thus is the LPP formulation of the given problem.

Ex. 2: A resourceful home decoreter manufacture two types of lamps say A and B. Both lamps go through two technitions first a cutter and second a finisher. Lamp A and B requires two hours and 1 hour of the cutters time and 1 hour & 2 hours of the finisher's time. The cutter has 104 hours and the finisher has 76 hours available in each month. Profit one lamp of A is Rs- 6 and one lamp of B is Rs- 11. Assuming that he can sell all that he produces, how many of each type lamps should be manufactured to obtained the best return.

Let the manufacture produces x_1 and x_2 no's of A and B lamp's resp.

	Cutter	Finishers	
A	2	1	6
B	1	2	11
	104	76	

Then the profit is

$$Z = 6x_1 + 11x_2$$

We have to maximize $Z = 6x_1 + 11x_2$

Since A and B lamps takes 2 hours & 1 hour of cutters time per each lamps

and since there are only 104 hours of cutters time.

$\therefore \; 2x_1 + x_2 \leq 104$

Illy there are only 76 hours of finsher's time

$$x_1 + 2x_2 \leq 76$$

where $x_1, x_2 \geq 0$

Hence the LPP formulation is $\max Z = 6x_1 + 11x_2$

s.t $2x_1 + x_2 \leq 104$

$x_1 + 2x_2 \leq 76$

and $x_1, x_2 \geq 0$

Ex.3: A firm can produces three types of clothes say A, B and C. Three kinds of wool are required for it, say Red wool, green wool and blue wool. 1 unit of type A clothes needs two yards of Red wool, 3 yards of blue wool and 1 unit of type B clothes needs 3 yards of Red wool and 2 yards of green wool, 2 yards of blue wool and 1 unit of type C clothes needs 5 yards of green wool and 4 yards of ~~Red~~ blue wool. Firm has only stocks of 8 yards of Red wool, 10 yards of green wool, 15 yards of blue wool. It is assume that the income obtain 1 unit of type A clothes is Rs-3, of type B clothes is Rs.-5, of type C clotes is Rs-4. Determine how the firm cloud use the available metarials so as to maximize the profit.

Let x_1 nos of A, x_2 nos of B & x_3 nos of C wool required.

	Red	green	blue	
A	2		3	3
B	3	2	2	5
C		5	4	4
	8	10	15	

Then the total profit is

$$Z = 3x_1 + 5x_2 + 4x_3$$

We have to maximize the profit $Z = 3x_1 + 5x_2 + 4x_3$.

Since there are 8 yards of Red wool, 10 yards of green wool and 15 yards of blue wool.

$$\therefore \; 2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

and $3x_1 + 2x_2 + 4x_3 \leq 15$

where $x_1, x_2, x_3 \geq 0$

This is the LPP formulation so as the profit is maximum.

Ex. Egg contains 6 unit of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paisa per gram. Milk contains 8 unit of vitamin A per gram and 12 unit of vitamin B per gram and costs 20 paisa per gram. The daily minimum requirements for vitamin A and vitamin B are 100 unit and 120 unit. Find the minimum amount of egg and milk. Formulate the LPP.

$$\text{Min } Z = 12x_1 + 20x_2$$
$$\text{s.t.} \quad 6x_1 + 8x_2 \geq 100$$
$$7x_1 + 12x_2 \geq 120$$
$$x_1 \geq 0, \; x_2 \geq 0$$

Ex. A firm manufactures products A and B and sells them at a profit of Rs 2 and Rs 3 resp. Each product is processed on two machines I and II. Type A requires 1 minute processing on machine I and 2 minutes on machine II. Type B takes 1 minute in both machines. Machine I is available for not more than 6 hour and 40 minutes, while machine II is available for 10 hour any day. Formulate LPP.

$$\text{Max } Z = 2x_1 + 3x_2$$
$$\text{s.t.} \quad x_1 + x_2 \leq 400$$
$$2x_1 + x_2 \leq 600, \quad x_1;\, x_2 \geq 0$$

Ex. A firm makes two types of furniture chairs and table. Two contribution to project for each product as calculated by the accounting department is Rs 20 per chair and Rs 30 per table. Both products are to be processed on three machines M_1, M_2, M_3. The time required in hours by each product and total time available in hours per week on each machine is as follows

	Chair	Table	Available Time
M_1	3	3	36
M_2	5	2	50
M_3	2	6	30

Formulate LPP.

$$\text{Max } Z = 20x_1 + 30x_2$$
$$\text{s.t.} \quad 3x_1 + 3x_2 \leq 36, \quad 2x_1 + 6x_2 \leq 60$$
$$5x_1 + 2x_2 \leq 50 \quad\quad x_1 \geq 0;\; x_2 \geq 0$$

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Ex.4: Four different metals namely Iron, coppur, Zinc and manganize are required to produces three commudities A, B and C. To produce one unit of A 40 Kg Fe, 30 Kg Cu, 7 Kg Zn and 4 Kg Mn are needs. Similarly to produces 1 unit of B 70 Kg Fe, 14 Kg Cu, 9 Kg Mn are needs and for producing 1 unit of C 50 Kg Fe, 18 Kg Cu, 8 Kg Zn are required. The total available quantities of metals are 1 metric ton Iron, 5 quintals of Cu, 2 quintals of Zn and mn each. The profit are Rs-300, Rs-200, Rs-100 in selling per unit of A, B and C respectively.
Formulate the problem mathematically.

Ans

	Fe	Cu	Zn	Mn	Profit
A	40	30	7	4	300
B	70	14	0	9	200
C	50	18	8	0	100
Availability →	1000	500	200	200	

Let x_1 unit of A, x_2 unit of B and x_3 unit of C are required.

Then $Z = 300x_1 + 200x_2 + 100x_3$

We have to maximize $Z = 300x_1 + 200x_2 + 100x_3$

s.t

$$40x_1 + 70x_2 + 50x_3 \le 1000$$
$$30x_1 + 14x_2 + 18x_3 \le 500$$
$$7x_1 + 8x_3 \le 200$$
$$4x_1 + 9x_2 \le 200$$

where $x_1, x_2, x_3 \geq 0$

This is the LPP so as maximize the profit.

Ex.5:

☐ **Basic Solution:** If $Ax = b$ be a system of m equⁿˢ with n-unknowns then a Solⁿ obtained by seting $(n-m)$ variables to zero is called a basic solution provided the determinant of the co-efficient of the remaining m variables is not zero.

☐ **Theorem:** The total number of basic Solⁿ is finite in number for the system of equations $A\underline{x} = \underline{b}$.

Proof: If the given system of equⁿˢ $Ax = \underline{b}$ contains n-variables and m ~~contains~~ equations then the total no. of basic Solⁿ is at most $^nc_m = \frac{\lfloor n}{\lfloor n-m \lfloor m}$, which is finite for finite values of n and m.

Hence proved.

□ Feasible Solution : If all the component of a solution set are non-negative quantities then the solution is called feasible solution of L.P.P.

□ Basic Feasible Solution : The solution set of an L.P.P which is feasible as well as basic is called basic feasible solution.
i.e, A basic Solⁿ of an LPP which satisfies the feasibility condition (non-negative quantities [redacted] except basic variable) is called basic feasible solution of the LPP.

□ Degenarate basic Solⁿ : A basic feasible solution of an LPP is said to be degenarate basic feasible Solution if at least one of the basic variables is zero.

□ Non-degenarate basic Solⁿ : A basic feasible solution of an LPP is said to non-degenarate basic feasible solution if none of the basic variables is zero.

□ Basic Variables : The variables attached to the independent column vectors of the basis matrix are known as basic variables and the remaining $(n-m)$ variables whose values are assumed to be zero are known as non-basic variables.

□ Basis Matrix : The square matrix formed with any independent column vectors taken from the original co-efficient matrix is known as basis matrix, which is denoted by B.

Ex.1: Find all the basic solution of the following system of equⁿ

$$x_1+2x_2+x_3=4$$
$$2x_1+x_2+5x_3=5$$

Also Find the B.F.S if any and degenarate B.F.S if any.

The given system of equⁿs is
$$x_1+2x_2+x_3=4 \quad \text{---(1)}$$
$$2x_1+x_2+5x_3=5 \quad \text{---(2)}$$

Putting $x_3=0$ in (1) & (2)
$$x_1+2x_2=4$$
$$2x_1+x_2=5$$

Now we see that $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3 \neq 0$

So solving the above equⁿs $x_1=2, x_2=1$.

∴ One basic Solⁿ is $(2,1,0)$

Now putting $x_2=0$ in (1) & (2)
$$x_1+x_3=4$$
$$2x_1+5x_3=5$$

Now we see that $\begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = 3 \neq 0$

Solving above eqn we get. $x_1 = 5, x_3 = -1$

So another basic soln is $(5, 0, -1)$

Also putting $x_1 = 0$ in ① & ② we get

$$2x_2 + x_3 = 4$$
$$x_2 + 5x_3 = 5$$

We see that $\begin{vmatrix} 2 & 1 \\ 1 & 5 \end{vmatrix} = 9 \neq 0$

Solving above eqns we get $x_2 = \frac{5}{3}, x_3 = \frac{2}{3}$

$\therefore (0, \frac{5}{3}, \frac{2}{3})$ is also a basic soln.

Thus we get basic solns $(2,1,0)$, $(5,0,-1)$, $(0, \frac{5}{3}, \frac{2}{3})$ amongst which $(2,1,0)$ and $(0, \frac{5}{3}, \frac{2}{3})$ are B.F.S.

Obviously the given system has no degenerate B.F.S.

Ex.2: Prove that $x_1 = 2, x_2 = 0, x_3 = 1$ is a basic solution to the set of eqns

$$2x_1 + x_2 - x_3 = 3$$
$$x_1 + 2x_2 + 3x_3 = 5.$$

Given eqns are
$$2x_1 + x_2 - x_3 = 3$$
$$x_1 + 2x_2 + 3x_3 = 5.$$

Putting $x_2 = 0$ in the above eqns we get

$$2x_1 - x_3 = 3$$
$$x_1 + 3x_3 = 5$$

Now we see that $\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 7 \neq 0$

So the non-zero variables (basic variables) are linearly independent.

So $x_1 = 2, x_2 = 0, x_3 = 1$ is a basic soln.

Ex.3: Prove that $x_1 = 2$, $x_2 = -1$, $x_3 = 0$ is a soln set but not a basic soln to the set of eqns

$$3x_1 - 2x_2 + x_3 = 8$$
$$9x_1 - 6x_2 + 4x_3 = 24.$$

The given set of eqns are
$$3x_1 - 2x_2 + x_3 = 8$$
$$9x_1 - 6x_2 + 4x_3 = 24$$

Putting $x_3 = 0$ in the above eqns we get

$$3x_1 - 2x_2 = 8$$
$$9x_1 - 6x_2 = 24$$

Now we see that $\begin{vmatrix} 3 & -2 \\ 9 & -6 \end{vmatrix} = -18 + 18 = 0$

So the variables x_1, x_2, x_3 is a solution set but not a basic soln.

Ex4: Find the basic solutions of the set of equations

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$
$$4x_1 - 2x_2 - x_3 + 2x_4 = 10$$