IE-402: Optimization

Optimization in Finance

Optimizing Asset Liability Management using Linear Programming

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Abstract

The aim of Asset Liability management in Banks is to effectively allocate proportions of total funds to assets, so as to ensure that the liability outflow is met, at the same time, to ensure no asset is overused, and that the net return is optimised. In this study, we have considered 9 assets and liabilities each, given their weights, maturities, and total present and future values, we have tried to optimise weight division among the assets so as to maximise the total returns we get.

The model used for the analysis the Macauley duration model. The Macauley duration is given by,

$$\frac{\sum_{t=1}^{n}\left(\frac{tXC}{(1+y)^t}\right)+\frac{nXM}{(1+y)^n}}{CurrentBondPrice}$$

There are two techniques used for the solution: Analytical solution is done using Penalty method of Linear Programming, and a simulated and complete solution is obtained using SOLVER optimization tool in Excel.

The report thus, finds, given a set of constraints, the allocation of assets, that can maximize profit for the bank.

Contents

1 Introduction to Asset Liability Management[1]

1. **Asset**
   Things that are resources owned by a company and which have future economic value that can be measured and can be expressed in dollars. Examples include cash, investments, accounts receivable, inventory, supplies, land, buildings, equipment, and vehicles.

2. **Liability**
   Liabilities are defined as a company's legal financial debts or obligations that arise during the course of business operations. They can be limited, or unlimited liability. Liabilities are settled over time through the transfer of economic benefits including money, goods, or services. Recorded on the right side of the balance sheet, liabilities include loans, accounts payable, mortgages, deferred revenues, earned premiums, unearned premiums, and accrued expenses.

3. **Price**
   In accounting, cost or price is defined as the cash amount (or the cash equivalent) given up for an asset. Cost includes all costs necessary to get an asset in place and ready for use. For example, the cost of an item in inventory also includes the item's freight-in cost. The cost of land includes all costs to get the land ready for its use.

4. **Yield**
   In finance, the yield on a security is the amount of cash (in percentage terms) that returns to the owners of the security, in the form of interest or dividends received from it. Normally, it does not include the price variations, distinguishing it from the total return.

5. **Duration**
   1 Duration measures how long it takes, in years, for an investor to be repaid the bond's price by the bond's total cash flows. At the same time, duration is a measure of sensitivity of a bond's or fixed income portfolio's price to changes in interest rates.

6. **Coupon**
   A coupon, in relation to bond instruments, is an interest payment made to the bondholder during the term of the bond. It is used to compensate the holder for lending their money. Coupon rate is the stated interest rate on a fixed income security like a bond. In other words, it's the rate of interest that bondholders receive from their investment. It's based on the yield as of the day the bond is issued.

7. **Maturity**
   In finance, maturity or maturity date refers to the final payment date of a loan or other financial instrument, at which point the principal (and all remaining interest) is due to be paid. The term fixed maturity is applicable to any form of financial instrument under which the loan is due to be repaid on a fixed date. A serial maturity is when bonds are all issued at the same time but are divided into different classes with different, staggered redemption dates.

Asset-liability management (ALM) may be defined as the simultaneous planning of all asset and liability positions on the bank's balance sheet under consideration of the different bank management objectives and legal, managerial and market constraints, for the purpose of enhancing the value of the bank, providing liquidity, and mitigating interest rate risk.

Aims of ALM:
An efficient asset-liability management system aims to manage the volume, mix, maturity, rate sensitivity, quality and liquidity of the assets and liabilities as a whole, so as to earn a predetermined, acceptable risk/reward ratio.

Parameters for framework:
The framework of asset-liability management broadly covers area of interest rate risk, liquidity risk, exchange risk and credit risk.
ALM can be defined as an operation for assessing the above mentioned risks, actively altering the asset-liability portfolio, and for strategically taking actions and managing risks with the objective of maximizing profits. ALM is not limited to on balance sheet assets and liabilities such as deposits and lending's only, but also includes off-balance sheet activities such as swaps, futures and options. The objective of ALM is to make banks fully prepared to face the emerging challenges.

2 Problem Statement

The present study is a Macaulay duration model for asset-liability management. The input variables consist of a list of assets, which are expected to yield beneficial return, along with their rates of return, and a list of liabilities, which represents outflow.

The most basic and indispensable constraint, is that none of these can be a value less than zero, since, at any given time, a bank is assumed to have assets and liabilities. The most crucial constraint is that duration of assets must equal the Macaulay duration of liabilities. Duration, in finance, refers to change in returns, against a unit change in rate. The Macaaulay duration is given by,

$$\frac{\sum_{t=1}^{n}\left(\frac{tXC}{(1+y)^t}\right)+\frac{nXM}{(1+y)^n}}{CurrentBondPrice}$$

The money covered through assets must be enough to satiate the money being used up through liabilities.

2.1 Algorithm for solution:

1. Find Present values of liabilities from past values and maturity dates.
2. Given the coupon rates, maturity and yield values, we calculate the Macaulay duration for the assets by the aforementioned formulae.
3. We assume default weights as our decision variables, and calculate the matrix product of the weights and the durations of the assets.
4. The sum of these should equate to the sum of duration of the liabilities. We use this as our objective, with decision variables, that is, those which can be modified, being the weights of the assets, and constraints being that the sum of weights should add upto 100%.

We are given a Future Values(FVs) and Yields(total return generated from investing or holding financial service over bond duration) of liabilities. We need to calculate present value and weights for each liabilities from specific formulas. Now for a given values of Maturity, rate of interest, Price and Yields of each assets, we optimize bond duration and weights of each assets so that we have total bond duration of assets and liabilities will be equal and same for total value.

3 Variables

There are two categories, assets and liabilities.

3.1 Input variables

Input variables for the study are those, whose values are taken frm Input variables are:

- List of assets with their:
  - Coupon rates, $C[i]$
  - maturity dates, $M[i]$
  - price, $P[i]$
  - yield rate, $Y[i]$
  - Weights
- List of liabilities with their:
  - Present values, $PV[j]$
  - Future values, $FV[j]$
  - Maturity, $M[j]$
  - Yields, $Y[j]$
  - Weights, $W[i]$
  - Duration, $D[i]$

3.2 Derived variables

- Duration of assets and liabilities
- Present worth of assets
- Face value
- Cost
- Shift value, $S[i]$
- Weights, $W[j]$
- Duration, $D[j]$

4 Mathematical Formulation

4.1 Objective Function

$$\sum_{i \in Assets} (W[i] * D[i]) = \sum_{j \in Liabilities} (W[j] * D[j])$$

4.2 Constraints

$$\sum_{i \in Assets} W[i] = \sum_{j \in Liabilities} W[j] = 100\%$$

$$\sum_{i \in Assets} P[i] = \sum_{i \in Liabilities} PV[j]$$

$$\sum_{i \in Assets} S[i] >= \sum_{i \in Liabilities} S[j]$$

5 Solution via Hand Analysis : Penalty Method

5.1 Limitations on solution via Linear Programming methods

Macaulay duration is calculated, keeping in check, constraints of maturity and coupon rates, which are not feasible to replicate by hand, given the complex nature of the Macaulay equation.
Thus, for the purpose of the Analytical Solution, we take in values of duration calculated via EXCEL functions, and work towards optimizing the objective.

5.2 Variables and constraints

$$\text{Maximize } 6.87x_1{+}8.58x_2{+}8.56x_3$$

$$\text{subject to}$$

$$0 \leq x_i \leq 1$$

$$0 \leq x_1 \leq 0.1$$

$$0 \leq x_2 \leq 0.4$$

$$0 \leq x_3 \leq 0.3$$

$$Tradedate \geq MaturityDate$$

$$x_1{+}S_1{=}0.7$$

$$x_2{+}S_2{=}0.4$$

$$x_3{+}S_3{=}0.3$$

$$x_1{+}x_2{+}x_3{+}A_1{=}1$$

5.3 Solution by simplex method

Step 1									
0	**0**	**-6.87**	**-8.58**	**-8.56**	**0**	**0**	**0**	**M**	**0**
Y_B	C_B	x_1	x_2	x_3	S_1	S_2	S_3	A_1	b
S_1	0	1	0	0	1	0	0	0	0.7
S_2	0	0	1	0	0	1	0	0	0.4
S_3	-8.56	0	0	1	0	0	1	0	0.3
A_1	M	1	1	0	0	0	-1	1	0.7
$c_j - z_j$	$--$	$-6.87+M$	$-8.58+M$	8.56	0	0	$8.56+M$	0	$-$

x_2 enters.
S_2 leaves.

Step 2									
0	**0**	**-6.87**	**-8.58**	**-8.56**	**0**	**0**	**0**	**M**	**0**
Y_B	C_B	x_1	x_2	x_3	S_1	S_2	S_3	A_1	b
S_1	0	1	0	0	1	0	0	0	0.7
x_2	-8.58	0	1	0	0	1	0	0	0.4
x_3	-8.56	0	0	1	0	0	1	0	0.3
A_1	M	1	0	0	0	-1	-1	1	0.3
$c_j - z_j$	$--$	$-6.87+M$	0	0	0	$8.58+M$	$8.56+M$	0	$-$

x_1 enters.
A_1 leaves.

Step 3								
0	**0**	**-6.87**	**-8.58**	**-8.56**	**0**	**0**	**0**	**0**
Y_B	C_B	x_1	x_2	x_3	S_1	S_2	S_3	b
S_1	0	0	0	0	1	1	1	0.4
x_2	-8.58	0	1	0	0	1	0	0.4
x_3	-8.56	0	0	1	0	0	1	0.3
x_1	-6.87	1	0	0	0	-1	-1	0.3
$c_j - z_j$	$--$	0	0	0	0	$8.58-6.87$	$8.56-6.87$	$-$

Solution:
x_1=0.3
x_2=0.4

x_3=0.3
S_1=0.4
Z_{max}=8.061

6 SOLVER solution

6.1 Brief discussion about Solver and the functions used

Solver is an add-in available in MS Excel and Google Sheets distributions.
Solver is an optimization tool that, given an objective function to maximize/minimize or equate to, a set of constraints, and the list of values that can be modified, uses either of a variety of techniques(Linear programming in our case), to find the most optimal allocation of weights to the values, so as to optimize the objective function.
Functions of Excel used:

- **MDuration**: The Excel Mduration function calculates the Modified Macaulay Duration of a security that pays periodic interest, assuming a par value of $100.
  Syntax:
  MDURATION(settlement, maturity, coupon, yld, frequency, [basis])
  Arguments:

  1. settlement - The settlement date of the security (i.e. the date that the coupon is purchased).

  2. maturity - The maturity date of the security (i.e. the date that the coupon expires).

  3. coupon - The security's annual coupon rate.

  4. yld - The security's annual yield.

  5. frequency - The number of coupon payments per year. This must be one of the following:
     1 - Annually
     2 - Semi-Annually
     4 - Quarterly

  6. basis - An optional integer argument which specifies the financial day count basis that is used by the security.

- PRICE The Excel PRICE function returns the price per $100 face value of a security that pays periodic interest.
  Purpose: Get price per $100 face value - periodic interest Return value : Bond price
  Syntax =PRICE (sd, md, rate, yld, redemption, frequency, [basis]) Arguments
  sd - Settlement date of the security.

md - Maturity date of the security.
rate - Annual coupon rate.
yld - Annual required rate of return.
redemption - Redemption value per $100 face value.
frequency - Coupon payments per year (annual = 1, semiannual = 2; quarterly = 4. basis - [optional] Day count basis (see below, default =0).

- MMULT The Excel MMULT function returns the matrix product of two arrays. The array result contains the same number of rows as array1 and the same number of columns as array2. If returning multiple results in an array on the worksheet, enter as an array formula with control + shift + enter.

  Purpose : Perform matrix multiplication
  Return value : The matrix product of two arrays
  Syntax =MMULT (array1, array2)
  Arguments
  array1 - The first array to multiply.
  array2 - The second array to multiply.

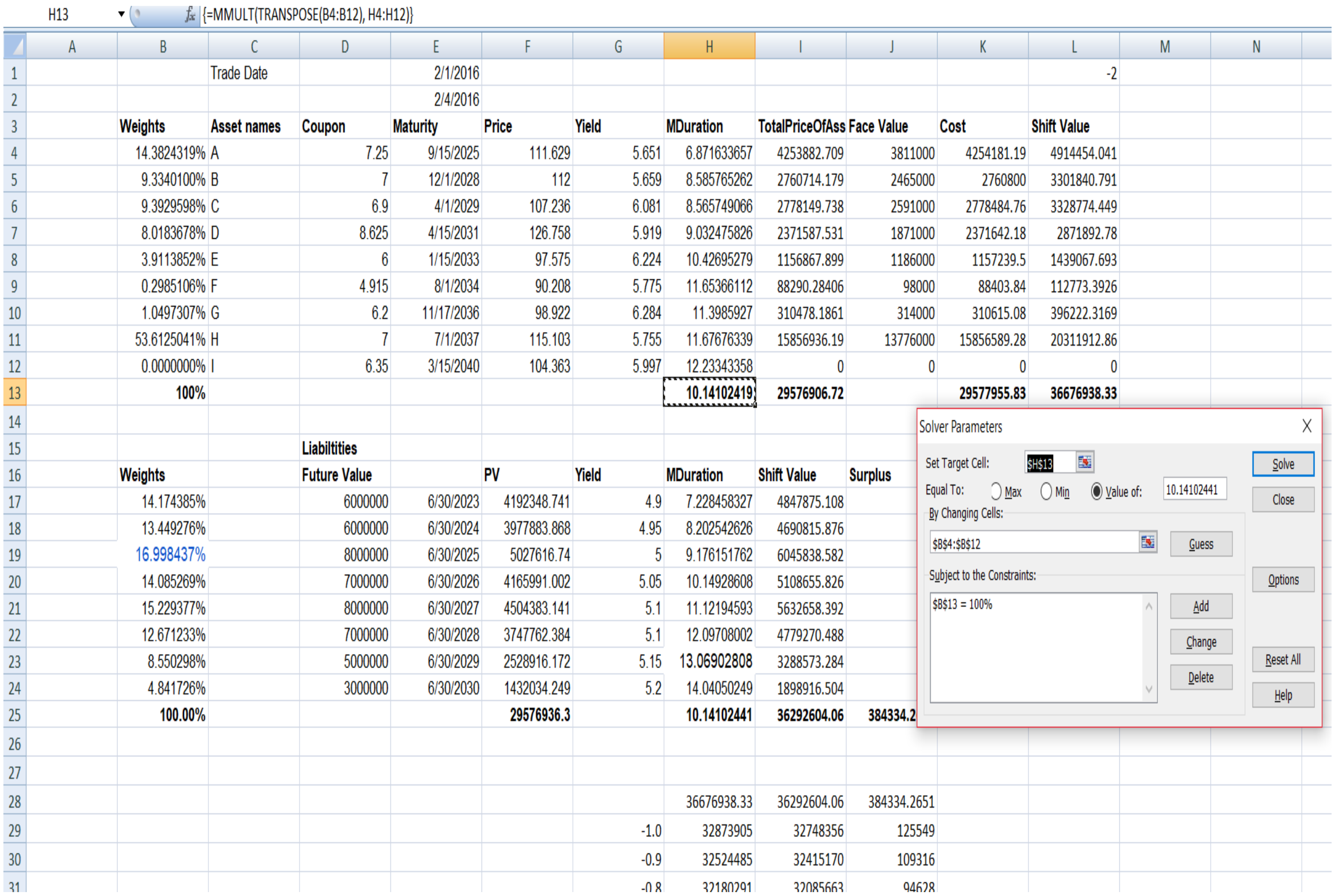
- ROUND The Excel ROUND function returns a number rounded to a given number of digits. The ROUND function can round to the right or left of the decimal point.
  Purpose : Round a number to a given number of digits
  Return value : A rounded number.
  Syntax =ROUND (number, num-digits) Arguments number - The number to round. num-digits - The number of digits to which number should be rounded.

6.2 Graphical Representation

H10 =MDURATION($E$2,E10,D10/100,G10/100,2,0)

	A	B	C	D	E	F	G	H	I	J	K	L
1			Trade Date		2/1/2016							-2
2					2/4/2016							
3		**Weights**	**Asset names**	**Coupon**	**Maturity**	**Price**	**Yield**	**MDuration**	**TotalPriceOfAss**	**Face Value**	**Cost**	**Shift Value**
4		14.3824319%	A	7.25	9/15/2025	111.629	5.651	6.871633657	4253882.709	3811000	4254181.19	4914454.041
5		9.3340100%	B	7	12/1/2028	112	5.659	8.585765262	2760714.179	2465000	2760800	3301840.791
6		9.3929598%	C	6.9	4/1/2029	107.236	6.081	8.565749066	2778149.738	2591000	2778484.76	3328774.449
7		8.0183678%	D	8.625	4/15/2031	126.758	5.919	9.032475826	2371587.531	1871000	2371642.18	2871892.78
8		3.9113852%	E	6	1/15/2033	97.575	6.224	10.42695279	1156867.899	1186000	1157239.5	1439067.693
9		0.2985106%	F	4.915	8/1/2034	90.208	5.775	11.65366112	88290.28406	98000	88403.84	112773.3926
10		1.0497307%	G	6.2	11/17/2036	98.922	6.284	11.3985927	310478.1861	314000	310615.08	396222.3169
11		53.6125041%	H	7	7/1/2037	115.103	5.755	11.67676339	15856936.19	13776000	15856589.28	20311912.86
12		0.0000000%	I	6.35	3/15/2040	104.363	5.997	12.23343358	0	0	0	0
13		**100%**						**10.14102419**	**29576906.72**		**29577955.83**	**36676938.33**
14												
15				**Liabiltities**								
16		**Weights**		**Future Value**		**PV**	**Yield**	**MDuration**	**Shift Value**	**Surplus**		
17		14.174385%		6000000	6/30/2023	4192348.741	4.9	7.228458327	4847875.108			
18		13.449276%		6000000	6/30/2024	3977883.868	4.95	8.202542626	4690815.876			
19		16.998437%		8000000	6/30/2025	5027616.74	5	9.176151762	6045838.582			
20		14.085269%		7000000	6/30/2026	4165991.002	5.05	10.14928608	5108655.826			
21		15.229377%		8000000	6/30/2027	4504383.141	5.1	11.12194593	5632658.392			
22		12.671233%		7000000	6/30/2028	3747762.384	5.1	12.09708002	4779270.488			
23		8.550298%		5000000	6/30/2029	2528916.172	5.15	13.06902808	3288573.284			
24		4.841726%		3000000	6/30/2030	1432034.249	5.2	14.04050249	1898916.504			
25		**100.00%**				**29576936.3**		**10.14102441**	**36292604.06**	**384334.2651**		
26												

Figure 1: Data



H13 | {=MMULT(TRANSPOSE(B4:B12), H4:H12)}

	A	B	C	D	E	F	G	H	I	J	K	L
1			Trade Date		2/1/2016							-2
2					2/4/2016							
3		Weights	Asset names	Coupon	Maturity	Price	Yield	MDuration	TotalPriceOfAss	Face Value	Cost	Shift Value
4		14.3824319%	A	7.25	9/15/2025	111.629	5.651	6.871633657	4253882.709	3811000	4254181.19	4914454.041
5		9.3340100%	B	7	12/1/2028	112	5.659	8.585765262	2760714.179	2465000	2760800	3301840.791
6		9.3929598%	C	6.9	4/1/2029	107.236	6.081	8.565749066	2778149.738	2591000	2778484.76	3328774.449
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8		3.9113852%	E	6	1/15/2033	97.575	6.224	10.42695279	1156867.899	1186000	1157239.5	1439067.693
9		0.2985106%	F	4.915	8/1/2034	90.208	5.775	11.65366112	88290.28406	98000	88403.84	112773.3926
10		1.0497307%	G	6.2	11/17/2036	98.922	6.284	11.3985927	310478.1861	314000	310615.08	396222.3169
11		53.6125041%	H	7	7/1/2037	115.103	5.755	11.67676339	15856936.19	13776000	15856589.28	20311912.86
12		0.0000000%	I	6.35	3/15/2040	104.363	5.997	12.23343358	0	0	0	0
13		100%						10.14102419	29576906.72		29577955.83	36676938.33
14												
15				Liabiltities								
16		Weights		Future Value		PV	Yield	MDuration	Shift Value	Surplus		
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24		4.841726%		3000000	6/30/2030	1432034.249	5.2	14.04050249	1898916.504			
25		100.00%				29576936.3		10.14102441	36292604.06	384334.2		
26												
27												
28								36676938.33	36292604.06	384334.2651		
29							-1.0	32873905	32748356	125549		
30							-0.9	32524485	32415170	109316		
31							-0.8	32180291	32085663	94628		

Solver Parameters

Set Target Cell: $H$13

Equal To: Max Min Value of: 10.14102441

By Changing Cells: $B$4:$B$12

Subject to the Constraints: $B$13 = 100%

Solve Close Guess Options Add Change Delete Reset All Help

Figure 2: Solver Analysis

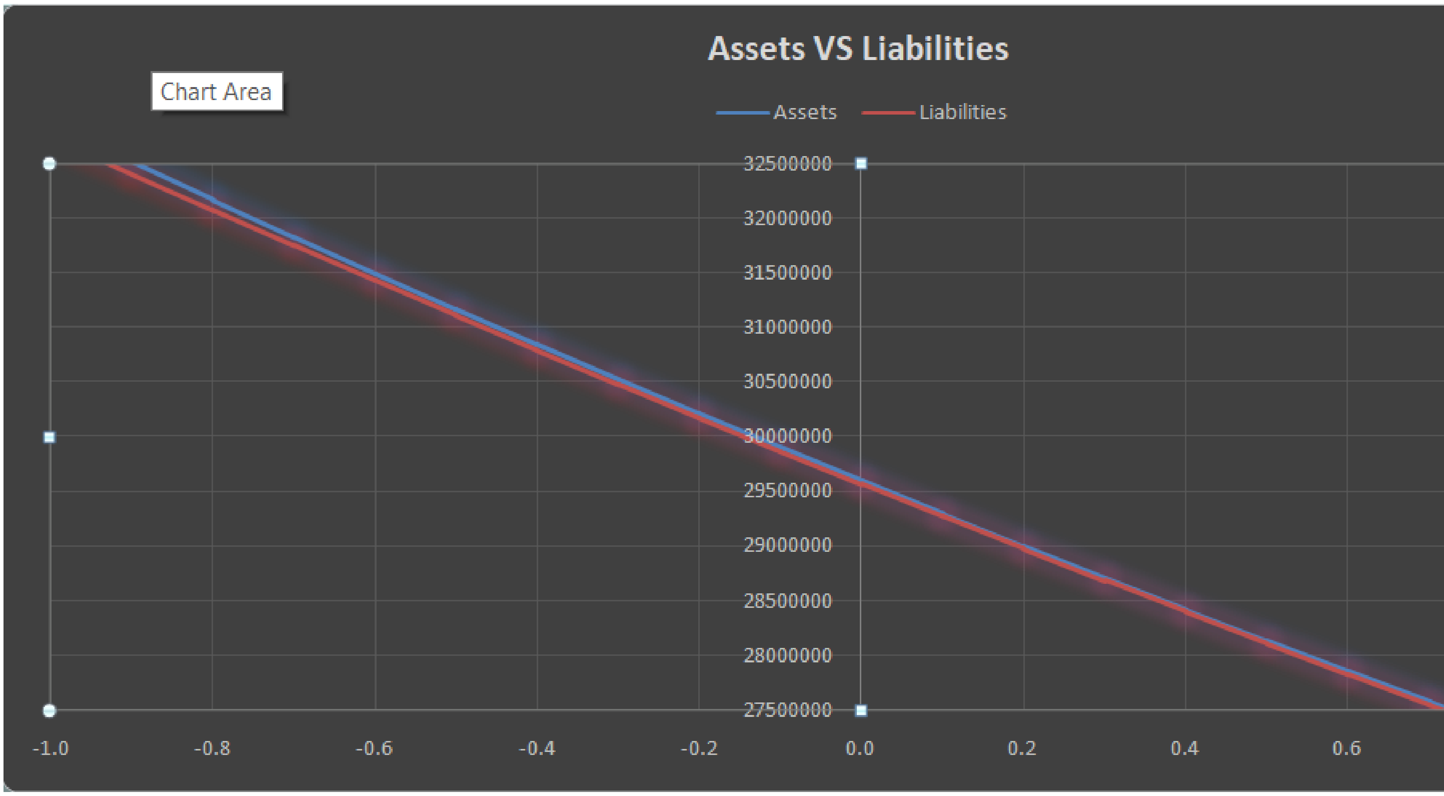

Figure 3: Graph 1

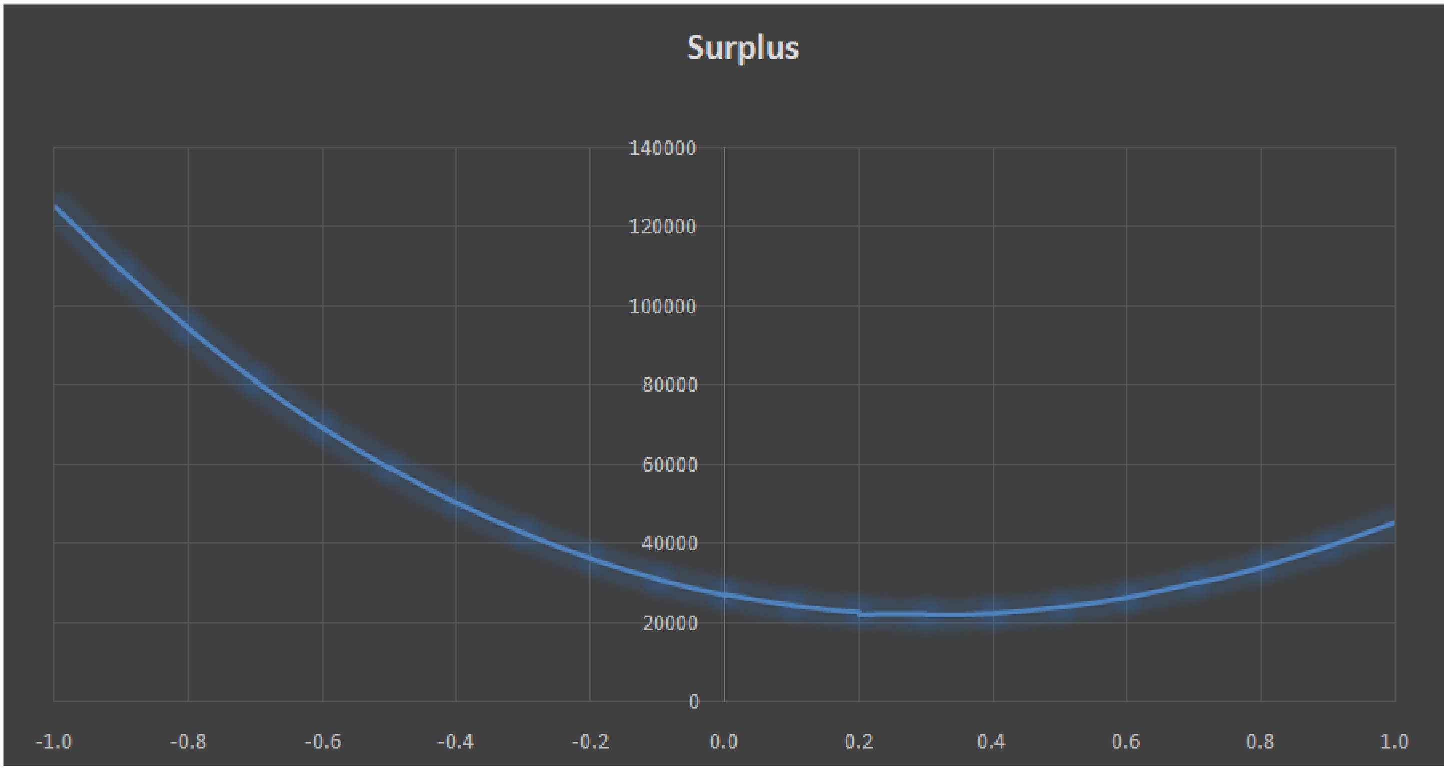

Figure 4: Graph 2

6.3 Interpretations from the graph

Graph 1 shows the Assets and Liabilities lines, which are almost coincident, thus satisfying our objective, that the total payoff to liabilities should be managed by the total income generated from interest on assets.
Graph 2 shows

7 Conclusions

The aim for the project was that the total amount expected to pay of the liabilities born out of durations was to be availed through the total amount earned through the assets.
The constraints taken into consideration were non negative weights of the assets and liabilities, a non zero surplus amount and the total price of assets should be greater than or equal to total present value of the liablities.
This is the method that is employed by most banks to allocate funds and weights to the assets substantially so as to recoginze the limitations of assets and make sure that all liablities are paid off.

8 Acknowledgements

We are thankful to Prof Nabinkumar Sahu for guiding for first introducing us to optimization in a seemingly disparate topic like Finance and his guidance throughout the project. We are also grateful to the teaching assistants for their guidance for the project.

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