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PROBABILITY FOR COMPUTING

* UNIT-1

⇒ Introduction to probability

- Sample space: Set of all possible outcomes of an event. It is denoted by 'S'

1.) Flipping of a coin

$$S = \{H, T\}$$

2.) Rolling a die

$$S = \{1, 2, 3, 4, 5, 6\}$$

3.) Flipping two coins

$$S = \{(H,H), (T,T), (H,T), (T,H)\}$$

4.) Lifetime of a car

$$S = [0, \infty)$$

- Event: Any subset of the sample space 'S' is called an event. It is denoted by 'E'

1.) $E = \{H\}$, then E is the event that a head appears on the flip of the coin.

2.) $E = \{2, 4, 6\}$, then E is the event that even numbers appears on the roll of die.

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⇒ After rolling a die

$S = \{1,2,3,4,5,6\}$
$E = \{2,4,6\}$
$F = \{1,3,5\}$
$K = \{1,2,3\}$

- if either F $\boxed{\text{or}}$ K occurs :- union
  $F \cup K = \{1,2,3,5\}$

- if F $\boxed{\text{and}}$ K occurs :-
  ↳ intersection
  $F \cap K = \{1,3\}$

- $E \cap F = \phi$ ⇒ null event
  ⇒ mutually exclusive

Union : $\bigcup_{n=1}^{\infty} E_n$ Intersection : $\bigcap_{n=1}^{\infty} E_n$

- $E^c / E' = \{1,3,5\}$
  ↳ complement of E

- since the experiment must result in some outcome

$$\boxed{S' \,|\, S^c = \phi}$$

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⇒ Probability defined on events

1. $0 \le P(E) \le 1$
2. $P(S) = 1$
3. For any sequence of events $E_1, E_2 \ldots$ that are mutually exclusive

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n)$$

For eg:

(1.) Tossing a coin

$P(\{H\}) = \frac{1}{2}$ and $P(\{T\}) = \frac{1}{2}$

- Properties

(1.) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

(2.) $P(E \cup F) = P(E) + P(F)$ [if E and F are mutually exclusive]

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⇒ CONDITIONAL PROBABILITIES

- Let A and B are two events associated with some random experiment. Then the probability of occurence of event A when event B has already occured is called the conditional probability.

- $P(A|B)$: Probability of event A given that B has already occured.
- $P(B|A)$ = Probability of event B given that A has already occured

Formula:

$$\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$$

Eg Two coins are tossed

$S = \{(H,H), (H,T), (T,H), (T,T)\}$

Let A = Exact two head will come

B = Atleast 1 head.

$A = \{(H,H)\}$, $B = \{(H,H), (H,T), (T,H)\}$

$A\cap B = \{H,H\}$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$= \frac{1/4}{3/4} = \frac{1}{3}$$

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⇒ Independent events

- Two events associated with a random experiment are said to be independent if the occurence of one event does not effect the probability of occurence of other.

So for two independent events -

$P(A|B) = P(A)$ and $P(B|A) = P(B)$

⇒ BAYES' THEOREM

- If $E_1, E_2, \ldots E_n$ are n mutually exclusive and exhaustive events, such that $P(E_i) > 0$ for each i and A is an arbitrary event for which $P(A) \neq 0$, then for each $1 \leq i \leq n$, the conditional probability of occurence of E_i, given that A has occured is given by the formula:-

$$P(E_i/A) = \frac{P(E_i)\,P(A|E_i)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2) \ldots P(E_n)P(A|E_n)}$$

Ques. Three urns containing 2 red, 3 black; 3 red, 2 black and 4 red, 1 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red, find the probability that it is drawn from the second urn.

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Sol^n.

Urn A	Urn B	Urn C
2R 3B	3R 2B	4R 1B

$P(A) = 1/3$, $P(B) = 1/3$, $P(C) = 1/3$

$P(E/A) = 2/5$, $P(E/B) = 3/5$, $P(E/C) = 4/5$

E = getting a red ball

$$P(B/E) = \frac{P(B)P(E/B)}{P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)}$$

$$P(B/E) = \frac{1/3 \times 3/5}{1/3 \times 2/5 + 1/3 \times 3/5 + 1/3 \times 4/5}$$

$$P(B/E) = \frac{3/15}{2/15 + 3/15 + 4/15}$$

$$= \frac{3/15}{9/15} = \frac{3}{9} = \frac{1}{3}$$

$$P(B/E) = \frac{1}{3}$$

$\therefore$ Probability of getting a red ball from second urn is $\frac{1}{3}$.

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Ques-2 Given 3 identical boxes I, II, and III, each containing two coins. In box I, both coins are gold coins; in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold.

Solⁿ

Box I (A)	Box II (B)	Box III (C)
2G	2S	1G 1S

$P(A) = 1/3$ $\quad P(B) = 1/3$ $\quad P(C) = 1/3$

E – getting a gold coin

$P(E/A) = 1$

$P(E/B) = 0$

$P(E/C) = 1/2$

$$P(A/E) = \frac{P(A).P(E/A)}{P(A).P(E/A) + P(B).P(E/B) + P(C).P(E/C)}$$

$$P(A/E) = \frac{1/3 \times 1}{1/3 + 0 + 1/6} = \frac{1/3}{3/6} = \frac{2}{3}$$

∴ Probability that the other coin in the box is also of gold is $\frac{2}{3}$.

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* UNIT-2 - RANDOM VARIABLES

⇒ RANDOM VARIABLES

- It is a real valued function whose domain is the sample space associated with a random experiment and range is the real line.

Eg:
A coin is tossed 2 times

$S = \{HH, HT, TH, TT\}$

$X \rightarrow$ Random variable
↳ 'no. of heads'

$X = \{0, 1, 2\}$

- Distribution [Probability distribution]

	X_1	X_2	X_3
X_i	0	1	2
$P(X_i)$	1/4	2/4	1/4
	P_1	P_2	P_3

Note: $P_1 + P_2 + P_3 = 1$

So,

$$\Sigma P_i = 1 \quad \{0 \le P_i \le 1\}$$

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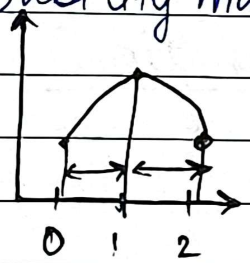
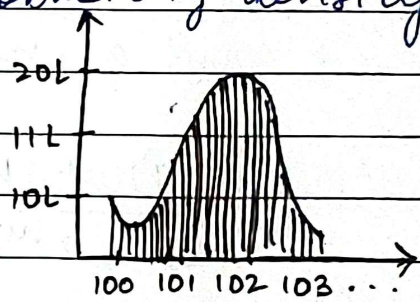
⇒ TYPES OF RANDOM VARIABLE

Discrete random variable	Continuous random variable.
• No. of experiments are finite	• Data is large and no. of experiments are infinite
eg:- two coins are tossed, two die are rolled	eg:- height of persons from 100cm to 110cm
• Probability mass fnc. $P(X)$	• Probability density fnc. $F(x)$





⇒ Cumulative distribution function also known as continuous distribution function.

$$F_x(x) = P(x \leq x)$$

eg: Rolling a six-sided die.

X	$P(X)$
0	0
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

$F_X(0) = 0$

$F_X(1) = P_X(0) + P_X(1) = 0 + 1/6 = 1/6$

$F_x(2) = P_x(0) + P_x(1) + P_x(2) = 0 + \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

⋮

$F_x(6) = P_x(0) + P_x(1) \ldots P_x(6)$

Cummulative – adding previous Prob.

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* DISCRETE RANDOM VARIABLE

- Binomial distribution

Condition:

→ There should be finite no. of trials
→ Trials should be finite (independent)
→ Exact two outcomes → p (success)
↘ q (fail)
→ finite outcome → small (countable:
→ Probability should be same (success/fail)
→ p and q → nearly same value (trial)

→ FORMULA (Probability mass function)

→ $P(X=r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$

n → no. of trials; ^nC_r → combinations; p → success; q → fail; r → random variable

→ Mean (μ) = np

n → no. of trials; p → probability of x

→ Variance (σ^2) = npq

→ standard deviation (σ) = $\sqrt{npq}$

⇒ Note: if $p = q = \frac{1}{2}$ it is called symmetrical binomial distribution otherwise skewed binomial distribution.

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- Bernoulli distribution
- It is a discrete PD which has only two possible outcomes
  success - 1
  failure - 0

→ **Probability mass function**

$P(\text{success}) = p$
$P(\text{failure}) = 1-p$

$f(x) = p^{x}(1-p)^{1-x}$ → random variable

→ Mean : $(\mu) = 0(1-p) + p \times 1 = \underline{\underline{p}}$

→ Variance : $(\sigma)^2 = p(1-p) = pq$

→ Standard deviation : $(\sigma) = \sqrt{p(1-p)} = \sqrt{pq}$

- Poisson distribution
  Condition :
  → no. of trials are very large
  → p is very small

→ **Probability mass function**

$P(X=r) = \dfrac{e^{-m} m^{r}}{r!}$ → mean $\boxed{e = 2.718}$

random variable

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→ Mean : $np = m$
→ Variance : $np = m$
→ standard deviation : $\sqrt{np}$

- Multinomial distribution

→ Probability mass function

$$P(X_1, X_2, X_3 ..) = \frac{n!}{X_1! \, X_2! \, X_3! ... X_n!} \cdot P_1^{x_1}, P_2^{x_2}, P_3^{x_3} ... P_n^{x_n}$$

→ Mean : np

→ Variance : $npq = np(1-p)$

→ Standard deviation : $\sqrt{npq} = \sqrt{np(1-p)}$

- Geometric distribution

→ Probability mass function

$$P(X=x) = q^{x-1} \cdot p$$

Ques- what is the probability of getting the 1st "6" on the 4th roll using a six-sided die?

Ans. $x = 4$ $p = \frac{1}{6}$ $q = \frac{5}{6}$

$$P(X=4) = q^{4-1} \cdot p$$

$$P(X=4) = \left(\frac{5}{6}\right)^3 \cdot \left(\frac{1}{6}\right) = \underline{\underline{0.09645}}$$

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→ Mean $= \mu = \frac{1}{p}$

→ Variance $= \sigma^2 = \left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right)$

→ standard deviation $= \sqrt{\frac{1}{p}\left(\frac{1}{p} - 1\right)}$

Ques 2% of all tires produced by company XYZ has a defect. A random sample of 100 tires is tested for quality assurance.

(a) What is the probability that the 8th tire selected has a defect?

(b) What is the probability that the first defect is identified among the first 5 samples?

(c) What is the probability that the first defect is detected among the first 10 samples?

(d) How many tires would you expect to test until you find the first defective one?

(e) find $P(X > 12)$

Ans (a) $p = 0.02 \qquad q = 0.98$

$X = 8$

$P(X=8) = q^{x-1} \cdot p$

$= (0.98)^7 \cdot 0.02$

$= 0.0174$

$= 1.74\%$

(b) $X \leq 5$

$P(X \leq 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$P(X \leq 5) = q^0 \cdot p^1 + q^1 p^1 + q^2 p^1 + q^3 p^1 + q^4 p^1$

$= p^1 (q^0 + q^1 + q^2 + q^3 + q^4)$

$= (0.02)(1 + 0.98 + (0.98)^2 + (0.98)^3 + (0.94)^4)$

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$p(x \leq 5) = 0.02(1 + 0.98 + 0.9604 + 0.941192 + 0.9223\}$

$p(x \leq 5) = 0.02(4.8038)$

$= 0.09607$

$= 9.6\%$

since this method is very time consuming, we have a formula for this:–

$$\underset{\downarrow}{F(x)} = P(X \leq X) = 1 - q^{X}$$

cumulative geometric distribution function

⇒ solving using this formula:–

$P(X \leq 5) = 1 - (0.98)^5$

$= 1 - 0.9039$

$= 0.096$

$= 9.6\%$

(c) for first 10 samples

$P(X \leq 10) = 1 - q^{10}$

$= 1 - (0.98)^{10}$

$= 0.1829$

$= 18.29\%$

(d) Mean $(\mu) = \frac{1}{p}$

$= \frac{1}{0.02} = 50$

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∴ we expect to test 50 tiles to find first defect.

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(e) $P(X > 12)$

formula for this is :-

$$P(X > x) = q^x$$

$$P(X > 12) = (0.98)^{12}$$
$$= 0.7847$$
$$= 78.5\%$$

* CONTINOUS RANDOM VARIABLES

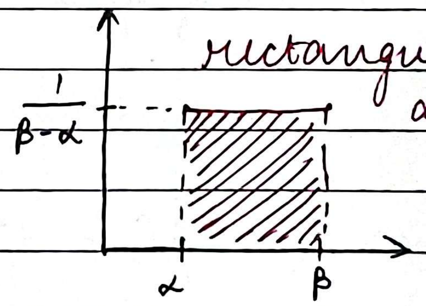
- Uniform distribution

→ Probability distribution ~~mass~~ function

$$f(x) = \frac{1}{b-a} \quad \text{if} \quad a < x < b, \; 0, \text{ else where}$$

→ Mean $= \dfrac{a+b}{2}$

rectangular distribution



→ variance $= \dfrac{(b-a)^2}{12}$

→ standard deviation $= \dfrac{b-a}{2\sqrt{3}}$

Ques: X is uniformly distributed in (2, 12) find the value of $P(0 < x < 10)$, Mean and variance

Ans: $f(x) = \dfrac{1}{\beta - \alpha} = \dfrac{1}{10}, \; 2 < x < 12$ $\quad \beta = 12, \; \alpha = 2$

i) $P(0 < x < 10) = \int_0^2 f(x)\,dx + \int_2^{10} f(x)\,dx$

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$$P(0<x<10) = \int_0^2 0\,dx + \int_2^{10} \frac{1}{10}\,dx$$

$$= 0 + \left[\frac{1}{10}x\right]_2^{10} = \frac{10}{10} - \frac{2}{10} = \frac{8}{10}$$

(ii) Mean $= \frac{\alpha+\beta}{2} = \frac{2+12}{2} = 7$

(iii) Variance $= \frac{(\beta-\alpha)^2}{12} = \frac{100}{12}$

Ques Bus is uniformly late between 2 and 10 min. How long can you expect to wait? What is standard deviation? if its >7 min late, you will be late for work. What is the probability of you being late?

Ans $\alpha = 2$

$\beta = 10$

Mean $\mu = \frac{\alpha+\beta}{2} = \frac{12}{2} = 6$ minutes

Standard deviation $= \cancel{(\alpha+\beta)^2}\ \frac{\beta-\alpha}{\sqrt{12}} = \sqrt{5.33}$

$= 2.31$ min

$$P(7<x<10) = \frac{d-c}{\beta-a} = \frac{10-7}{10-2} = \frac{3}{8} = 0.375$$

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- Exponential distribution

→ Probability density function

$$P(X = r) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$0 \leq x < \infty$$

→ Commulative distribution function

$$F(x) = 1 - e^{-\lambda a}$$

→ Mean $= \frac{1}{\lambda}$

→ Variance $= \frac{1}{\lambda^2}$

→ Standard deviation $= \frac{1}{\lambda}$

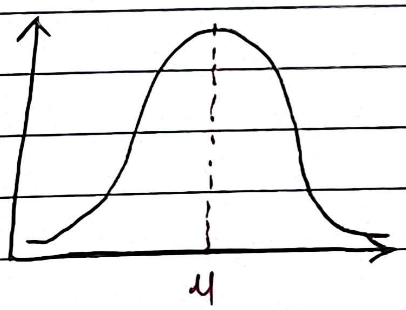
- Normal distribution

→ Probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad (-\infty < x < \infty)$$

μ → Mean

σ → standard deviation



Bell shaped curve.

* Mean = Median = Mode

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→ Standard Normal distribution

Mean = 0 Variance = S.D = 1

$\frac{x-\mu}{\sigma}$ → standard normal variate → z

* EXPECTATION OF A RANDOM VARIABLE