Additional type rules to "Extensional Normalisation and Type-Directed Partial Evaluation for Typed Lambda Calculus with Sums"

$$\frac{}{\cdot\ ctx}\ ctx\text{-}EMP$$

Empty context

Making "lower" variables

$$\frac{\Gamma \vdash A\ type}{\Gamma :: (x : A)\ ctx}\ ctx\text{-}EXT1$$

concatenation

variable not in Γ

making type variables

$$\frac{\Gamma\ ctx}{\Gamma :: (A\ type)\ ctx}\ ctx\text{-}EXT2$$

A context Γ is hence a list of membership judgements $b : B$ and type judgements $A\ type$

corresponds to membership judgment $A : U_0$ in HoTT

0th level universe

$$\frac{\Gamma \vdash A : \mathrm{type} \qquad \Gamma \vdash B : \mathrm{type}}{\Gamma \vdash A \times B : \mathrm{type}}$$ $\times$ intro

$$\frac{}{\Gamma \vdash 1 : \mathrm{type}}$$ unit intro

$$\frac{}{\Gamma \vdash 0 : \mathrm{type}}$$ empty type intro

$$\frac{\Gamma \vdash A : \mathrm{type} \qquad \Gamma \vdash B : \mathrm{type}}{\Gamma \vdash A + B : \mathrm{type}}$$ sum type intro

$$\frac{\Gamma \vdash A : \mathrm{type} \qquad \Gamma \vdash B : \mathrm{type}}{\Gamma : A \to B \ \mathrm{type}}$$ function type intro

$$\frac{\Gamma :: (x:A) :: \Gamma' \ \mathrm{ctx}}{\Gamma :: (x:A) :: \Gamma' \ [illegible]}$$ Vble1

$$\Gamma :: (x:A) :: \Gamma' \vdash x:A$$

$$\frac{\Gamma :: (B \text{ type}) :: \Gamma' \text{ ctx}}{\Gamma :: (B \text{ type}) :: \Gamma' \vdash B \text{ type}} \text{ Vble2}$$

optional

We may also want to include the following substitution & weakening rules from the HoTT book

$$\frac{\Gamma \vdash A \text{ type} \qquad \Gamma :: \Delta \vdash b : B}{\Gamma :: (x:A) :: \Delta \vdash b : B} \text{ Wkg}_1$$

$$\frac{\Gamma \vdash A \text{ type} \qquad \Gamma :: \Delta \vdash K \text{ type}}{\Gamma :: (x:A) :: \Delta \vdash K \text{ type}} \text{ Wkg}_2$$

$$\frac{\Gamma \vdash a : A \qquad \Gamma :: (x:A) :: \Delta \vdash b : B}{\Gamma :: \Delta[a/x] \vdash b[a/x] : B} \text{ Subst}_1$$

replace x with a in term b

$$\Gamma \vdash J \text{ type} \qquad \Gamma :: (L \text{ type}) :: \Delta \vdash b : B$$

$$\frac{}{\Gamma :: \Delta[J/L] \vdash b[J/L]}\ \mathrm{Subst}_2$$

$$\frac{\Gamma \vdash a : A \qquad \Gamma :: (x : A) :: \Delta \vdash V \text{ type}}{\Gamma :: \Delta[a/x] \vdash V \text{ type}}\ \mathrm{Subst}_3$$

$$\frac{\Gamma \vdash J \text{ type} \qquad \Gamma :: (L \text{ type}) :: \Delta \vdash V \text{ type}}{\Gamma :: \Delta[J/L] \vdash b[J/L]}\ \mathrm{Subst}_4$$