NUMBERING SYSTEMS: BINARY, DECIMAL & HEXADECIMAL

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NUMBERING SYSTEMS: A BRIEF HISTORY

- ***Unary*** System (left to right):
- used lines to represent value.
  - 1 was represented by |
  - 2 was represented by ||
  - 5 was represented by |||||
  - 7 was represented by ||||| ||

NUMBERING SYSTEMS: A BRIEF HISTORY

- Roman Numeral System (left to right, then left, then right again):
  - used lines and introduced *representation* for major values (5, 10, 100, etc)
  - 1 was represented by |
  - 3 was represented by |||
  - However, for the value 5 it was *represented* by V 10 represented by X and 100 by C
  - Numbers to the left of the representation are the value minus the numbers.
    - 4 is represented by |V which is the same as 5-1=4
  - Numbers to the right of the representation are the value plus the numbers
    - 7 is represented by V|| which is the same as 5+2=7
  - 21 is represented by XX| and 91 is represented by XC| (100-10+1)

QUICK QUIZ

- What year was this super bowl?

- Who won?

NUMBERING SYSTEMS: A BRIEF HISTORY

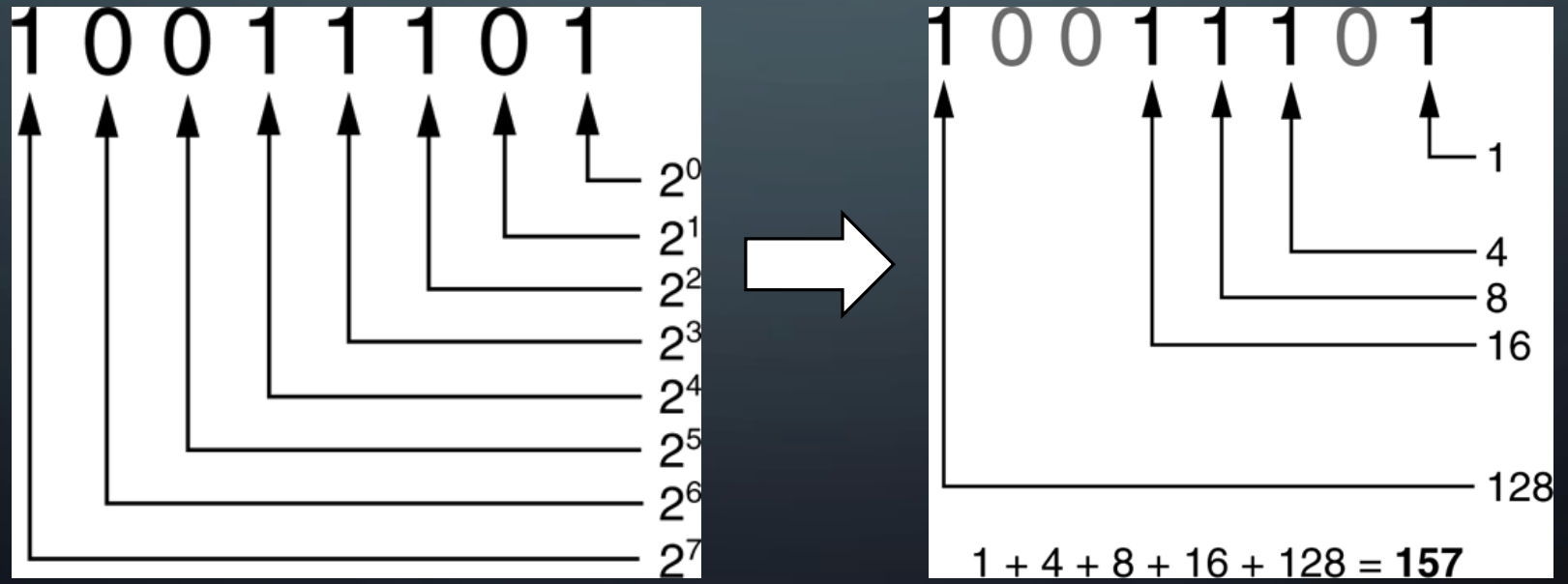
- ***Decimal*** System:
  - The system we know and love. Introduced by the Arabs and Hindus
  - Base 10 so only had 10 representations from 0 to 9.
  - Each value had its on representation from 0 to 9 (10 representations) and then repeated
  - Increases from right to left
  - Used the concept of Most Significant Digit (MSD) and Least Significant Digits (LSD) go beyond the value 9.
    - 10 has the left most digit, the number 1, as the MSD and 0 as the LSD.
    - 9,325 has the 9 as the MSD (thousands) and the number 5 as the LSD
  - Base 10 by the powers:
    - $10^3 = 1{,}000$
    - $10^2 = 100$
    - $10^1 = 10$
    - $10^0 = 1$

BINARY SYSTEM

- Introduced by Gottfried Wilhelm Leibniz
- Increases from right to left
- The MSD is still left most digit and LSD is the right most digit.
- Base 2 thus only has two numbers, 0 and 1. Can be used to represent any number.
  - 1 is repressed by 1
  - 2 is represent by 10
  - 3 is represented by 11
  - 4 is resented by 100
  - 5 is represented by 101
  - and so forth

BY THE POWERS OF 2

Power	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^0
Base									
Max Value	128 =	64 =	32 =	16 =	8 =	4 =	2 =	1 =	0 =
location	1	0	0	0	0	0	0	0	0



QUICK QUIZ

- What decimal number power is needed to reach 100?
- What are the following Decimal numbers in Binary:
  - 4
  - 7
  - 10
  - 16
  - 33
- What are the following Binary number in Decimal
  - 10
  - 110
  - 1110
  - 10101

ADDITION & SUBTRACTION RULES

- The basic Rules of addition
  - 0 + 0 = 0
  - 1+ 0 = 1
  - 1 + 1 = 10 (1 is carried to next MSD, value is two)
- The basic rules of subtraction
  - 0-0=0
  - 1-0=1
  - 1-1=0
  - 10-1=1 Key concept to binary subtraction
  - 110-1 = need to borrow from the next closest MSD:
    - 110-1 =
    - $1\cancel{1}^{0}\cancel{0}^{10}$-1 = 101
- Decimal Analogy
  - 0 + 0 = 0
  - 1+ 0 = 1
  - 1 + 1 = 2
  - 9 + 1 = 10 (the 1 is carried to the next MSD, value is ten)
  - 28-9 = the 8 needs to borrow from next closest MSD
    - 28-9 =
    - $\cancel{2}^{1}\cancel{8}^{18}$-9= 19

BINARY ADDITION

- binary additions

```
  1     0          1
 +0    +0         +1
  1     0          1
```

Carried to 2nd bit

```
   1   (1)
 +10   (2)
 ----------
  11 = 3 in decimal
```

Does 3 + 6 = 9 in binary as well? Lets see..

```
  3     1  11
 +6    +110
 ----------
  9     001
```

BINARY SUBTRACTION

1	0	1	10
- 0	- 0	- 1	- 1
1	0	0	?

Borrow from 2nd bit

~~10~~^{10} (two)
- 1 (one)
1 = one in decimal

11110 (30)
- 1011 -(11)
Should = 19

1110~~10~~^{10}
- 1011

1110~~10~~^{10}
- 101 1
1

110~~10~~^{10}~~10~~^{10}
- 10 11
0 11

110~~10~~^{10}~~10~~^{10}
- 10 11
100 11 = 19

BIT, NIBBLE, BYTE

- Bit (0 or 1)
  - Largest number of 1
- Nibble (4bits)
- Byte (8 Bits)
  - largest number of 15 (hexadecimal)
- Kilobyte (1024 Bytes)
- Megabyte (1024 kilobytes))
- Gigabyte (1024 megabyte)
- Terabyte (1024 Gig)
- Petabyte (1024 TB)

HEXADECIMAL

- 16 values, from 0-15
- Values from 0-9 are represented the same as decimal
- Values from 10-15 are resented using letters
  - A = 10
  - B = 11
  - C = 12
  - D = 13
  - E = 14
  - F =15
- Benefits:
  - maps to binary (one hex digit is four bits, or, a nibble)
  - Fit more in less space and memory
  - Easier to read than binary
    - In hex: RGB (FF00EE) or Red(FF) Green (00) and Blue (EE)
    - Dec to bin: (255,0,255) => (11111111, 0, 11111111)

CONVERTING FROM/TO HEX

- Convert: A3

LSD
MSD

- A^{16^1} 3^{16^0}
- Reminder: A in hex = 10
- $(16^0)X\ 3 = 1\ X\ 3 = 3$
- $(16^1)\ X\ 16 = 10\ X\ 16 = 160$
- 3 + 160 = 163

HEXADECIMAL EXAMPLE

- What is 14, 16 and 29 in hex?

DECIMAL VS BINARY VS HEX

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	26
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	1A
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111	10000	11010
Highest Bin Power	0	2^0	2^1	2^1	2^2	2^2	2^2	2^2	2^3	2^3	2^3	2^3	2^3	2^3	2^3	2^3	2^4	2^4
Highest Dec Power	0	10^0	10^0	10^0	10^0	10^0	10^0	10^0	10^0	10^0	10^1	10^1	10^1	10^1	10^1	10^1	10^1	10^1

QUESTIONS