

Derivation of the Gravitational Constant from Electric Interaction

Gravitation is universally attractive and acts between all macroscopic bodies, independent of their composition or electric neutrality.

Despite the absence of net electric charge in matter, the gravitational interaction exhibits the same inverse-square dependence on distance as the Coulomb force.

This work explores whether the gravitational constant G must be treated as a fundamental parameter, or whether it can instead emerge from a known interaction through an appropriate mass–charge scaling.

Without introducing additional forces or modifying Newtonian gravity, we examine whether the observed gravitational law can be obtained from an electric interaction framework by identifying the corresponding conversion constant.

1. Starting Point: Newtonian Gravitation

The classical Newtonian gravitational force between two masses is

$$F = G \frac{M_1 M_2}{R^2}$$

where: - F is the force, - M_1, M_2 are the interacting masses, - R is the separation distance, - G is the gravitational constant.

2. Electric Force Between Two Charges

The Coulomb force between two electrons separated by a distance R is

$$F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2}$$

Define the electric constant

$$K_e \equiv \frac{e^2}{4\pi\epsilon_0}$$

so the force becomes

$$F = \frac{K_e}{R^2}$$

3. Many-Particle Generalization

For two systems containing N_1 and N_2 identical charges respectively, the force scales with the number of interacting charge pairs:

$$F = \frac{2K_e N_1 N_2}{R^2}$$

The factor of 2 accounts for the bidirectional nature of the interaction between the two systems.

4. From Particle Count to Mass

Instead of counting particles explicitly, define a mass–conversion scale μ such that

$$N = \frac{M}{\mu}$$

Substituting into the force expression gives

$$F = \frac{2K_e}{R^2} \left(\frac{M_1}{\mu} \right) \left(\frac{M_2}{\mu} \right)$$

which simplifies to

$$F = \frac{2K_e}{\mu^2} \frac{M_1 M_2}{R^2}$$

5. Identification of the Gravitational Constant

Comparing directly with the Newtonian form,

$$F = G \frac{M_1 M_2}{R^2}$$

leads to the identification

$$G \equiv \frac{2K_e}{\mu^2}$$

Thus, within this framework, the gravitational constant is not fundamental but emerges from the electric interaction strength K_e and a characteristic mass scale μ .

6. Solving for the Required Mass Scale

Using the experimentally measured value of G :

$$G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

and

$$K_e = \frac{e^2}{4\pi\epsilon_0} \approx 2.307 \times 10^{-28} \text{ N}\cdot\text{m}^2$$

solve for μ :

$$\mu = \sqrt{\frac{2K_e}{G}}$$

Numerically,

$$\mu \approx 2.63 \times 10^{-9} \text{ kg}$$

7. Final Form of the Gravitational Law

The gravitational force can therefore be written as

$$F = \frac{2K_e}{\mu_g^2} \frac{M_1 M_2}{R^2}$$

with

$$\mu_g = 2.63 \times 10^{-9} \text{ kg}$$

and

$$G = \frac{2K_e}{\mu_g^2}$$

Equivalent Normalization Using the Planck Mass

The same force law may be written using the Planck mass m_p by introducing an effective coupling factor η :

$$F = \frac{2\eta K_e}{m_p^2} \frac{M_1 M_2}{R^2}$$

where

$$\eta = \left(\frac{\mu_g}{m_p} \right)^2 \approx 1.4 \times 10^{-2}$$

This form makes explicit that the use of μ_g absorbs the effective coupling into the mass scale, while the Planck-mass formulation displays this normalization separately.

8. Conclusion

In this formulation: - Gravitation emerges in the same mathematical form as an electric interaction. - The gravitational constant G is not fundamental, but a derived quantity. - Its value is fixed by the ratio between the electric interaction constant and a specific mass scale μ_g .

This provides a direct, quantitative origin for G within the defined framework.