Emergence of the Fine-Structure Constant from a Classical Electric Field

Abstract

This paper presents a derivation of the fine-structure constant $\alpha \approx 1/137$ from purely classical field considerations. Starting with the energy stored in a spherically symmetric electric field and comparing it to the energy of a periodic field configuration $E = \hbar\omega$, we demonstrate that the dependence on the spatial scale cancels naturally, leaving a dimensionless ratio. By invoking Maxwell's equations and relativistic causality, we identify the angular frequency ω with the fastest allowable field response, c/R. This leads directly to the known expression for α without appealing to quantum assumptions, boundary conditions, or quantization rules. The result suggests that α may originate from structural properties of classical electromagnetism, offering a new perspective on its fundamental role.

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1. Energy of an Electric Field

In classical electrodynamics, the energy stored in an electric field is given by:

$$E = \frac{1}{2}\varepsilon_0 \int E^2 \, dV$$

For a spherically symmetric radial field of charge Q, integrating from an effective radius R to infinity yields:

$$E = \frac{Q^2}{8\pi\varepsilon_0 R}$$

This energy is positive and independent of the sign of the charge.

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2. Periodic Field Energy and Scale Substitution

The energy of a periodic field is:

$$E = \hbar\omega$$

Substituting from the electric field energy:

$$\hbar\omega = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Assuming a maximum field response speed:

$$\omega = \frac{c}{R}$$

Substitute and cancel R:

$$\hbar \cdot \frac{c}{R} = \frac{Q^2}{8\pi\varepsilon_0 R} \Rightarrow \hbar c = \frac{Q^2}{8\pi\varepsilon_0}$$

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3. Derivation of the Fine-Structure Constant

Rewriting the above:

$$\alpha = \frac{Q^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$$

This shows that the inverse fine-structure constant emerges from a ratio between electric field strength and a relativistic dynamical scale.

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4. Clarification of the Scale ( )

From the wave equation:

$$\nabla^2 E = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Over scale R:

$$\frac{E}{R^2} \sim \mu_0\varepsilon_0 \frac{E}{\tau^2} \Rightarrow \tau = \sqrt{\mu_0\varepsilon_0} R = \frac{R}{c}$$

Hence:

$$\omega = \frac{1}{\tau} = \frac{c}{R}$$

5. Interpretation of (c) and ( )

The speed of light is not a spacetime postulate but a consequence of the field's finite response rate:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Therefore, α is a structural feature of classical field theory — not inherently quantum.

References

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