CSE 330 Numerical Methods

SUMMER 2022

Quiz 4

ANSWER ALL THE QUESTIONS Time: 20 mins

Name_____ ID_____

Section _____ Theory Faculty Initial: _____

1) Using the QR decomposition method to find the values of a_0 and a_1. [10 marks]

$$a_0 + 100\, a_1 = 70$$

$$a_0 + 220\, a_1 = 180$$

$$a_0 + 430\, a_1 = 300$$

$$A \cdot x = b$$

$$\begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 70 \\ 180 \\ 300 \end{bmatrix}$$

$u_1 \quad u_2$

$$p_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$p_2 = u_2 - \frac{u_2 \cdot p_1}{p_1 \cdot p_1} p_1$$

$$= \begin{bmatrix} 100 \\ 220 \\ 430 \end{bmatrix} - \frac{(100\times1)+(220\times1)+(430\times1)}{(1\times1)+(1\times1)+(1\times1)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 100 \\ 220 \\ 430 \end{bmatrix} - 250 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -150 \\ -30 \\ 180 \end{bmatrix}$$

$$q_1 = \frac{p_1}{|p_1|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$q_2 = \frac{p_2}{|p_2|} = \frac{1}{30\sqrt{62}} \begin{bmatrix} -150 \\ -30 \\ 180 \end{bmatrix} = \begin{bmatrix} -5\sqrt{62}/62 \\ -\sqrt{62}/62 \\ 3\sqrt{62}/31 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{3} & -5\sqrt{62}/62 \\ 1/\sqrt{3} & -\sqrt{62}/62 \\ 1/\sqrt{3} & 3\sqrt{62}/31 \end{bmatrix}$$

$$R = Q^T A$$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix}$$

$$R \cdot x = Q^T b$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 70 \\ 180 \\ 300 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix}^{-1} \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$= \begin{bmatrix} 1175/93 \\ 127/186 \end{bmatrix}$$

$$= \begin{bmatrix} 12.634 \\ 0.683 \end{bmatrix}$$ Ans