CSE 330 Numerical Methods

SUMMER 2022

Makeup Quiz

ANSWER ALL THE QUESTIONS Time: 20 mins

Name____________________________ ID______________

Section __________ Theory Faculty Initial: ______________

1) Consider the function $f(x) = e^{2x} + \frac{1}{5}x^2$

For the above function, within the interval [-1,0]:

a) Calculate the actual integral. [2 marks]
b) Calculate the approximate value of the integral using the Trapezium rule. [2 marks]
c) Calculate the approximate value of the integral using the Simpson's rule. [2 marks]
d) Calculate the approximate value of the integral using Composite Newton Cotes with 3 segments. [3 marks]
e) Calculate the percentage error for part (b), (c) and (d). [1 mark]

a) $\int_{-1}^{0}\left(e^{2x} + \frac{1}{5}x^2\right)dx$

$= \left[\frac{e^{2x}}{2} + \frac{1}{5}\cdot\frac{x^3}{3}\right]_{-1}^{0}$

$= 0.5 - (1.000\times10^{-3})$

$= 0.499$

b) $I_1(f) = \frac{b-a}{2}\left[f(a) + f(b)\right]$

$= \frac{0-(-1)}{2}\left[f(-1) + f(0)\right]$

$= \frac{1}{2}\left[0.335 + 1\right]$

$= 0.668$

c) $I_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$

$= \frac{0-(-1)}{6}\left[f(-1) + 4f(-0.5) + f(0)\right]$

$= \frac{1}{6}\left[0.335 + 4(0.418) + 1\right]$

$= \cancel{0.292}\ 0.501$

(d) $h = \frac{b-a}{m} = \frac{0-(-1)}{3} = \frac{1}{3}$

$x_0 = a = -1$

$x_1 = x_0 + h = -2/3$

$x_2 = x_1 + h = -1/3$

$x_3 = x_2 + h = 0$

$C_{1,3}(f) = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + f(x_3)\right]$

$= \frac{1/3}{2}\left[f(-1) + 2f\left(\frac{-2}{3}\right) + 2f\left(\frac{-1}{3}\right) + f(0)\right]$

$= \frac{1/3}{2}\left[0.335 + 2(0.352) + 2(0.536) + 1\right]$

$C_{1,3}(f) = \cancel{0.5185}\ 0.519$

~~(d)~~

(e) (i) $\frac{|I(f) - I_1(f)|}{|I(f)|} \times 100 = \frac{|0.499 - 0.668|}{0.499} \times 100 = 33.9\%$

(ii) $\frac{|I(f) - I_2(f)|}{|I(f)|} \times 100 = \frac{|0.499 - \overset{0.501}{\cancel{0.292}}|}{0.499} \times 100 = \cancel{41.5\%}\ 0.40\%$

(iii) $\frac{|I(f) - C_{1,3}(f)|}{|I(f)|} \times 100 = \frac{|0.499 - 0.519|}{0.499} \times 100 = 4.01\%$