CSE 330 Numerical Methods

SUMMER 2022

Quiz 3

ANSWER ALL THE QUESTIONS

Time: 20 mins

Name______________________________ ID______________

Section ________ Theory Faculty Initial: ____________

1. Consider the following system of linear equation:

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$

$$3x_1 + 5x_2 + 3x_3 = 4$$

a) Represent the above system in matrix form A.x = b [1 mark]
b) From the Frobenius matrices using matrix A. [3 marks]
c) Decompose A into LU form. [3 marks]

2. Consider the following set of column vectors:

$$S = \{ (1, 2, 3)^T, (2, -1, 0)^T\}$$

a) Show that set S is an orthogonal set. [1 mark]
b) Convert set S into an orthonormal set. [2 marks]

1(a)

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}}_{b}$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \quad \begin{matrix} R_2' \to R_2 - \left(\frac{4}{1}\right) R_1 \\ R_3' \to R_3 - \left(\frac{3}{1}\right) R_1 \end{matrix}$$

$$F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 2 & 0 \end{bmatrix} \quad R_3' \to R_3 - \left(\frac{2}{-1}\right) R_2$$

$$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & -10 \end{bmatrix}$$

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(c) $L = \left(F^{(1)}\right)^{-1} \cdot \left(F^{(2)}\right)^{-1}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix}} \cdot \underset{U}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & -10 \end{bmatrix}}$$

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2 (a) $S = \left\{ \underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{s_1}, \underbrace{\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}}_{s_2} \right\}$

$s_1 \cdot s_2 = (1 \times 2) + (2 \times -1) + (3 \times 0) = 0$

Since dot product $= 0$, s_1 and s_2 are orthogonal.

(b) $\hat{s}_1 = \frac{s_1}{|s_1|} = \frac{1}{\sqrt{1^2+2^2+3^2}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$

$\hat{s}_2 = \frac{s_2}{|s_2|} = \frac{1}{\sqrt{2^2+(-1)^2+0^2}} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \\ 0 \end{bmatrix}$

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