CSE 330 Section 02 & 05

Hermite Interpolation

1. Use Hermite polynomial that agrees with the data listed below to find the approximation of f(1.5)

k	x_k	$f(x_k)$	$f'(x_k)$
0	1.3	0.6200860	-0.5220232
1	1.6	0.4554022	-0.5698959
2	1.9	0.2818186	-0.5811571

2.

x	$\sin x$	$D_x \sin x = \cos x$
0.30	0.29552	0.95534
0.32	0.31457	0.94924
0.35	0.34290	0.93937

a. Use the following values and five-digit rounding arithmetic to construct the Hermite interpolating polynomial to approximate sin 0.34
b. Determine an error bound for the approximation in part (a), and compare it to the actual error.
c. Add sin 0.33=0.32404 and cos 0.33=0.94604 to the data, and redo the calculations.

Lagrange

1. Find the value of y for $x=2.1$ using a 2^{nd} order Lagrange polynomial with the appropriate data sets from the table below. Also find the change of Y between X=1.5 and 2.

Sl.	x	Y
1	-1	2.2
2	0	10.6
3	1	17.0
4	2	22.4
5	3	25.8

2. What is Round off error and Truncation true error in numerical method?

3. Let's assume you have a dataset as given below. Perform second order Lagrange interpolation and Newton's divided difference interpolation to find the f(x) for x=1.5. Also comment on the results that you are getting from the two methods.

x	1	1.3	1.6	1.9	2.2
f(x)	0.1411	−0.6878	−0.9962	−0.5507	0.3115

(b) For the given data, fit a Lagrange interpolating polynomial of order four. Use it to estimate the value of the function at $x = 0.65$.

x	-1	-0.5	0	0.5	1
$f(x)$	9	0.625	-5	0	-5

4. What is the advantage and disadvantage of using Lagrange interpolating polynomials? And how can we solve that problem?

5. The function $f(x)=\frac{1}{x+1}$ can be approximated by using a polynomial $p_n(x)$ according to Weierstrass Approximation Theorem in the interval [-2,2]. True or False.

FLOATING POINT ARITHMETIC

8. a) Given β=2, m=4, e_{min} = -2, e_{max} = 0.

1. Find out the values that each group represents for the sets of e, and plot them on a number line starting from 0.
2. Calculate the machine epsilon for the problem ϵ_m.

b) Write down the Mathematical notation for **Fixed-point Representation** and **Floating-Point Representation** and explain each term.

9. Suppose the domain of X is given as [0.101,0.111]. If x_6=0.1101. What will it be rounded to?

Differentiation

9. Calculate the derivative of $log_e x$numerically at $x_0 = 2$and find the truncation error for the following values of step size using Forward Difference:

i) h=1, ii) h=0.1, iii) h=0.01, iv) h=0.001, v) Based on the calculations, comment on the error and the step size.

a) If $f(x) = sinx + 5e^{5.6x}$, then find the forward, backward and central difference approximation for $f'(3)$ $with$ $\Delta x = 0.3$. Also compare the results.

b) What will be the error term for a central differentiation with higher order derivative ($f''(x)$)?

10. The upward velocity of a rocket is given as a function of time (t) as:

s	**m/s**
0	**0**
7	**101**
15	**197**
22	**280**
27	**360**
32	**460**

a) Determine the value of the velocity at t=16 seconds with fourth order polynomial interpolation using Newton's divided difference polynomial method.

b) Using the third order polynomial interpolant for velocity, find the distance covered by the rocket from t=11s to t=16s.

c) Using the third order polynomial interpolant for velocity, find the acceleration of the rocket at t=16s.

11. . Find the value of y for $x=1.8$ using a *3rd* order Newton's divided difference polynomial with the appropriate data sets from the table below.

Sl.	**x**	**Y**
1	-1	2.5
2	0	12.6
3	1	19.0
4	2	22.4
5	3	27.8

12.

1.

(a) Given the data

x	1.6	2	2.5	3.2	4	4.5
$f(x)$	2	8	14	15	8	2

Calculate $f(2.8)$ using Newton's interpolating polynomials of order 1 and 3. Choose the sequence of the points for your estimates to attain the best possible accuracy.

13. The truncation error of the forward difference method varies according to which of the following rules?