Practice Problems

1.

f(2)=3 , f(4)=5, f(5)=8

Prepare an overdetermined system and then solve using

1. Least Square Method
2. QR Decomposition

2.

f(18)=22 , f(19)=35, f(20)=60

Prepare an overdetermined system and then solve using

3. Least Square Method
4. QR Decomposition

3.

f(18)=22 , f(19)=35, f(20)=60, f(25)=80

Prepare an overdetermined system using quadratic Polynomial and then solve using

5. Least Square Method
6. QR Decomposition

4

Using Gauss elimination method solve the below system:

$$3x_1 + 5x_2 + 7x_3 + 9x_4 = 1.4$$

$$7x_1 + 3x_2 + 11x_3 + 4x_4 = 1.8$$

$$2x_1 + 5x_2 + 3x_3 + 2x_4 = 2.7$$

$$8x_1 + 7x_2 + 7x_3 + 4x_4 = 3.4$$

Solve using LU Decomposition as well

5

Using Gauss elimination method solve the below system:

$$x_1 + 3x_2 + 2x_3 + 4x_4 = 1.4$$

$$2x_1 + x_2 + x_3 + 3x_4 = 1.8$$

$$2x_1 + 5x_2 + x_3 + x_4 = 2.7$$

$$3x_1 + 4x_2 + 2x_3 + 5x_4 = 3.4$$

Solve using LU Decomposition as well

6

Using Gauss elimination method solve the below system:

$$12x_1 + 10x_2 - 7x_3 = 15$$

$$6x_1 + 5x_2 + 3x_3 = 14$$

$$24x_1 - x_2 + 5x_3 = 28$$

Solve using LU Decomposition as well

7

Consider the following set of vectors $S = \{v_1, v_2, v_3\}$ in R^3 where

$$\vec{v}_1 = \begin{pmatrix} \frac{3}{5} \\ 0 \\ \frac{4}{5} \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \vec{v}_3 = \begin{pmatrix} -\frac{3}{5} \\ 0 \\ \frac{4}{5} \end{pmatrix}.$$

a. Check if S is an orthonormal set.
b. Let's say the following matrix below is an orthonormal matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

Compute the value of $(A^T A)^{-1}$

8

a)
$$\begin{aligned} -4x_1 + 7x_2 - 2x_3 &= 2 \\ x_1 - 2x_2 + x_3 &= 3 \\ 2x_1 - 3x_2 + x_3 &= -4 \end{aligned}$$

a. Construct $F^{(1)}$ and $F^{(2)}$
b. Find Lower Triangular Matrix
c. Solve the System using LU Decomposition

9

State two situations for which the Simpson's rule is limited.

Find the exact value of the following integral

$$\int_0^{0.8} f(x)\,dx$$

where $f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$.

Use multi segment Trapezoidal rule with $m = 4$ to approximate the integral in the previous part. And also find the actual relative error.

10

Compute the following integration

$$\int_0^2 f(x)\,dx$$

numerically by using Trapezoidal and Simpson's rules if the functions $f(x)$ are given as: (i) $f(x) = \sqrt{1+x^2}$, (ii) $f(x) = \sin(x)$ and (iii) $f(x) = e^x$.