LONG-RUN TOTAL COST CURVE

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FROM EXPANSION PATH TO LONG-RUN TOTAL COST

In chapter 7 we learned that an **expansion path** maps out the cost-minimizing input choices as Q increases.

We can take those input choices, multiply them by the relevant prices, and form the **long-run total cost curve**.

> A curve that shows how total cost varies with output, holding input prices fixed and **choosing all inputs to minimize costs**. Denoted by TC.

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DERIVING THE LONG-RUN TOTAL COST CURVE (Graphically)

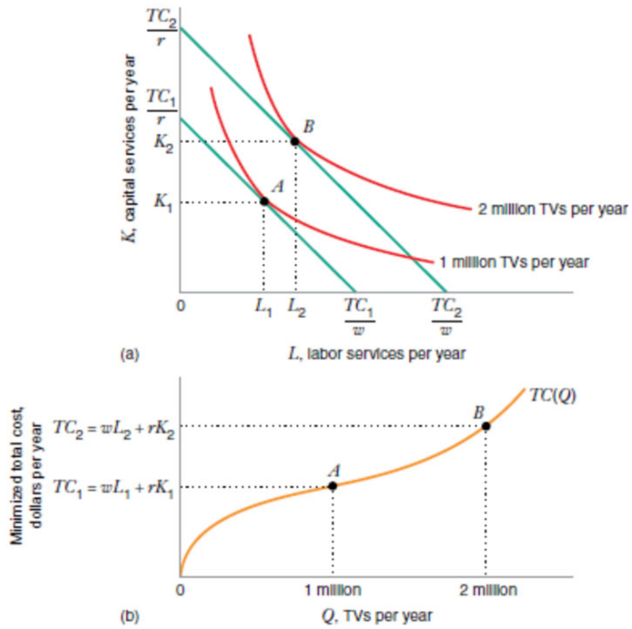
Point A

- L_1 and K_1 are chosen to produce 1 million TVs
- The associated $TC_1 = wL_1 + rK_1$

Point B

- L_2 and K_2 are chosen to produce 2 million TVs
- The associated $TC_2 = wL_2 + rK_2$

(Very similar to the process of finding the Engel curve in consumer theory)



DERIVING THE LONG-RUN TOTAL COST CURVE (Mathematically)

We can use the idea of cost minimization to derive a ***function*** that describes long-run cost curves!

Basic Process…

1. Use cost minimizing process to
   a. Find Labor demand: $L^*(w, r, q)$ and
   b. Find Capital demand: $K^*(w, r, q)$
2. Plug input demands into isocost line to find Total Cost: $TC = wL^* + rK^*$

LONG-RUN COST EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

We previously determined the input demand curves.

$$L^*(r,w,q) = \left(\frac{qr}{4w}\right)^{\frac{2}{3}}$$

$$K^*(r,w,q) = 2\left(\frac{w}{r}\right)^{\frac{1}{3}}\left(\frac{q}{4}\right)^{\frac{2}{3}}$$

We are going to determine the total cost curve when $w = 8$ and $r = 4$.

Process to follow…

Step 1: Update the input demand curves

$$L^*(q) = \left(\frac{q \times 4}{4 \times 8}\right)^{\frac{2}{3}} = \frac{q^{\frac{2}{3}}}{4} = 0.25q^{\frac{2}{3}}$$

$$K^*(q) = 2\left(\frac{8}{4}\right)^{\frac{1}{3}}\left(\frac{q}{4}\right)^{\frac{2}{3}} = q^{\frac{2}{3}}$$

Step 2: Plug input demands into total cost formula

$$TC(q) = wL^* + rK^*$$

$$TC(q) = 8\left(0.25q^{\frac{2}{3}}\right) + 4\left(q^{\frac{2}{3}}\right)$$

$$TC(q) = 6q^{\frac{2}{3}}$$

LONG-RUN COST EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$.

1. $L^*(r,w,q) = 0.25q^{\frac{2}{3}}$
2. $K^*(r,w,q) = q^{\frac{2}{3}}$
3. $TC(q) = 6q^{\frac{2}{3}}$

Sketch the long run total cost when $w = 8$ and $r = 4$.

Investigate the shape…

$TC(0) = 0$

The total cost curve starts at the origin.

$$\frac{dTC}{dq} =$$

Positive sign tells us total cost is increasing as q increases→ TC(q) is upward sloping

$$\frac{d^2TC}{dq^2} =$$

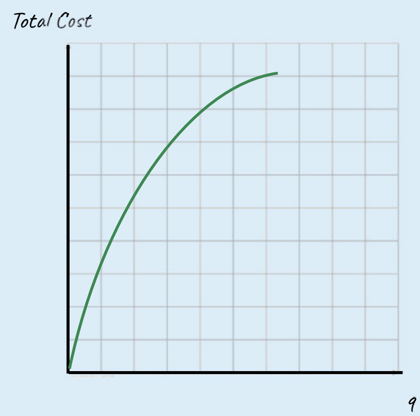
Negative sign tells us the slope of the total cost curve is flattening as q increases.

LONG-RUN COST EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$.

1. $L^*(r,w,q) = 0.25q^{\frac{2}{3}}$
2. $K^*(r,w,q) = q^{\frac{2}{3}}$
3. $TC(q) = 6q^{\frac{2}{3}}$

Sketch the long run total cost when $w = 8$ and $r = 4$.



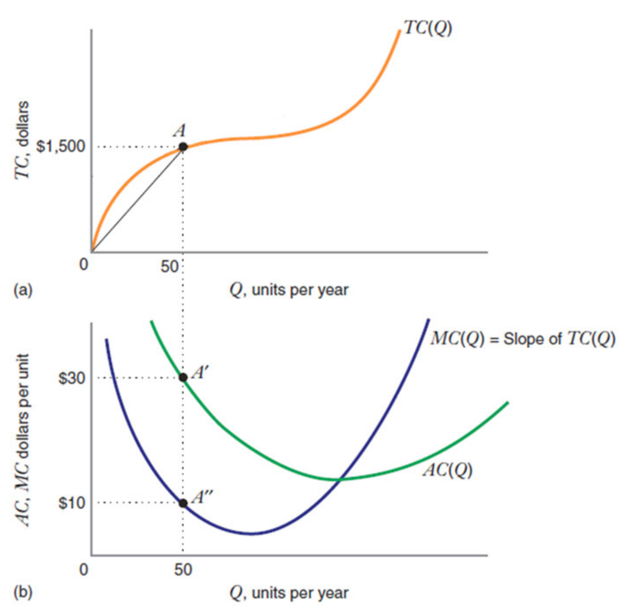
LONG-RUN AVERAGE AND MARGINAL COSTS

Average Total Cost (AC) – Firm's total cost per unit of output.

$$AC(Q) = \frac{TC}{Q}$$

Marginal cost (MC) – rate at which long-run total cost changes with respect to changes in output.

$$MC(Q) = \frac{dTC}{dq}$$



LONG-RUN COST EXAMPLE (Continued)

Suppose that the firm's production function is $Q = 2L^{0.5}K,\ w = 8\ \&\ r = 4.$

We know from previous work that…

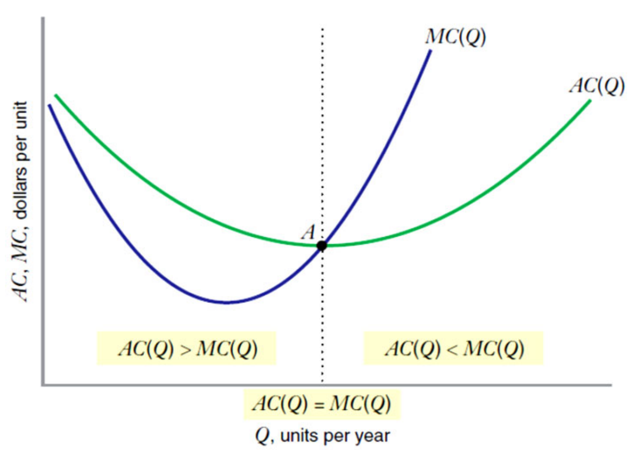
$$TC(q) = 6q^{\frac{2}{3}}$$

Compute the firm's average total cost and marginal cost.

RELATIONSHIP BETWEEN AC(Q) AND MC(Q)

When…

- $AC(Q) > MC(Q)$, then $AC(Q)$ is decreasing.
- $AC(Q) = MC(Q)$, then $AC(Q)$ is not changing.
- $AC(Q) < MC(Q)$, then $AC(Q)$ is increasing.



ECONOMIES AND DISECONOMIES OF SCALE

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ECONOMIES AND DISECONOMIES OF SCALE

As output increases average cost may…

Initially decline for the following reasons:

1. Scale allows workers to specialize.
2. Scale provides flexibility
3. Scale allows for cost savings on production inputs via bulk purchases.

Eventually rise for the following reasons:

1. Limited factory space or machinery might make it difficult for workers to be effective
2. Management becomes difficult
3. At some point, supply of key inputs may be limited

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ECONOMIES AND DISECONOMIES OF SCALE

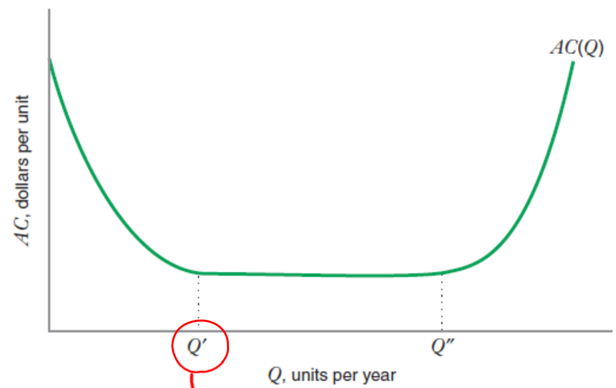
Economies of scale

A characteristic of production in which average cost decreases as output increases. **Where $Q < Q'$.**

Diseconomies of scale

A characteristic of production in which average cost increases as output increases. **Where $Q > Q''$.**

Determined under the assumption that inputs are selected to minimize a firm's costs for a given output



Minimum efficient scale (MES) – the smallest output at which the long-run average cost curve attains its minimum point

ECONOMIES OF SCALE VS. RETURNS TO SCALE

Economies/diseconomies of scale assumes inputs are selected to minimize a firm's costs (not equally scaled) and examines the **impact on average cost**.

Returns to scale assumes all inputs are scaled equally and examines the **impact on output**.

Despite these differences, there is a relationship between the two!

Production functions with…

- **increasing returns to scale** generate production with **economies of scale**.

The production function $Q = LK$ has **increasing returns to scale** and generates a **downward sloping average cost** curve.

ECONOMIES OF SCALE VS. RETURNS TO SCALE

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Production functions with…

- **constant returns to scale** generate production with **neither economies or diseconomies of scale**.

The production function $Q = L^{0.5}K^{0.5}$ has **constant returns to scale** and generates a **horizontal average cost** curve.

ECONOMIES OF SCALE VS. RETURNS TO SCALE

Economies/diseconomies of scale assumes inputs are selected to minimize a firm's costs (not equally scaled) and examines the **impact on average cost**.

Returns to scale assumes all inputs are scaled equally and examines the **impact on output**.

Despite these differences, there is a relationship between the two!

Production functions with…

- **decreasing returns to scale** generate production with **diseconomies of scale**.

The production function $Q = L^{0.4}K^{0.3}$ has **decreasing returns to scale** and generates an **upward sloping average cost** curve.

LONG-RUN COST EXAMPLE (Continued)

Suppose that the firm's production function is $Q = 2L^{0.5}K,\ w = 8\ \&\ r = 4.$

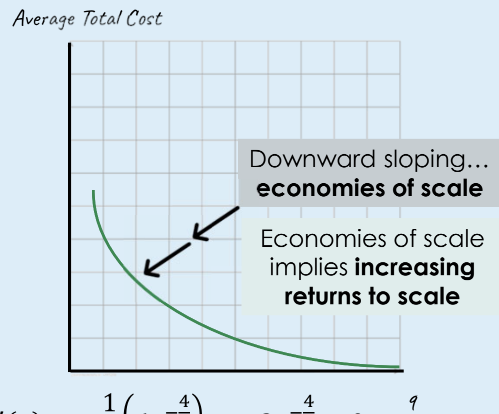
We know from previous work that…

$$TC(q) = 6q^{\frac{2}{3}}$$

$$AC(q) = 6q^{-\frac{1}{3}}$$

Sketch the firm's average total cost.

- Identify whether this firm's production exhibits economies or diseconomies of scale.
- Identify whether this production function has increasing, decreasing, or constant returns to scale.



$$AC'(q) = -\frac{1}{3}\left(6q^{-\frac{4}{3}}\right) = -2q^{-\frac{4}{3}} < 0$$

$$AC''(q) = -\frac{4}{3}\left(-2q^{-\frac{7}{3}}\right) = \frac{8}{3}q^{-\frac{7}{3}} > 0$$

OUTPUT ELASTICITY OF TOTAL COST

The percentage change in total cost per one percent change in output.

Used to measure the extent of economies /diseconomies of scale!

$$E_C = \frac{\%\Delta TC}{\%\Delta Q}$$

A quick derivation (available in the book) gets us the more useful version…

$$\boldsymbol{E_C = \frac{MC}{AC}}$$

The closer E_C is to zero, the larger the economies of scale.

Economies: $E_C < 1 \implies MC < AC$

implies AC is falling

Diseconomies: $E_C > 1 \implies MC > AC$

implies AC is rising

SHORT-RUN TOTAL COST CURVES

THE SHORT-RUN COST CURVES

Short-run total cost (STC)

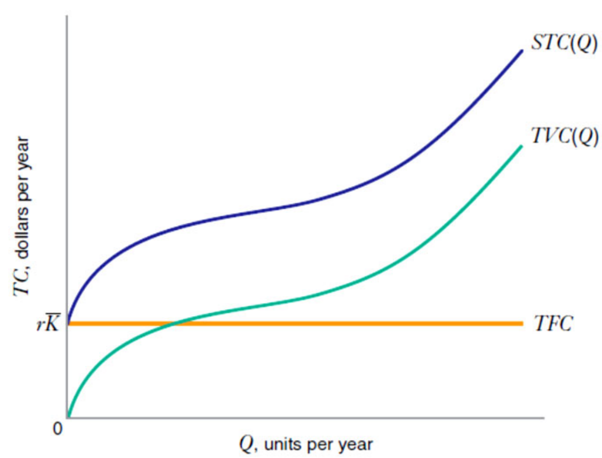
A curve that shows the minimized total cost of producing any given output when at least one input is fixed.

Total variable cost (TVC)

A curve that shows the sum of expenditures on variable inputs at the short-run cost-minimizing output.

Total fixed cost (TFC)

A curve that shows the cost of fixed inputs and does not vary with output.



SHORT-RUN TOTAL COST

Suppose that a firm's production function is $Q = 2L^{0.5}K$. Their output goal is **q units**.

The firm's capital is **fixed at $\bar{K}$**.

The price of a labor is $\boldsymbol{w = 8}$, and the price of capital is $\boldsymbol{r = 4}$.

1. What is the short run total cost curve for this production function when capital is fixed at a level $\bar{K}$?
2. Decompose total costs into variable and fixed costs.
3. Why are there fixed costs in this example?

From previous work we know that…

$$L^*(q,\bar{K}) = \left(\frac{q}{2\bar{K}}\right)^2$$

Evaluate $STC(q)$ at $\bar{K}$ and L^*

$$STC(q) = w \times L^*(q,\bar{K}) + r \times \bar{K}$$

$$STC(q) = 8 \times L^*(q,\bar{K}) + 4 \times \bar{K}$$

SHORT-RUN TOTAL COST

Suppose that a firm's production function is $Q = 2L^{0.5}K$. Their output goal is **q units**.

The firm's capital is **fixed at $\bar{K}$**.

The price of a labor is $\boldsymbol{w = 8}$, and the price of capital is $\boldsymbol{r = 4}$.

1. What is the short run total cost curve for this production function when capital is fixed at a level $\bar{K}$?
2. Decompose total costs into variable and fixed costs.
3. Why are there fixed costs in this example?

$$STC(q) = \underbrace{2\left(\frac{q}{\bar{K}}\right)^2}_{\text{Variable costs}} + \underbrace{4\bar{K}}_{\text{Fixed costs}}$$

There are fixed costs because…

the firm has a fixed input (K)

RELATIONSHIP BETWEEN LONG-RUN AND SHORT-RUN TOTAL COST CURVES

Using the examples we have worked with all week…

A firm's production function is given by $\boldsymbol{Q = 2L^{0.5}K}$. Their output goal is $\boldsymbol{q}$ **units**.

The price of a labor is $\boldsymbol{w = 8}$, and the price of capital is $\boldsymbol{r = 4}$.

In the long-run when no inputs are fixed…

$$\boldsymbol{TC(q) = 6q^{\frac{2}{3}}}$$

In the short run if capital is fixed at $\boldsymbol{\overline{K} = 4}$…

$$STC(q) = 2\left(\frac{q}{\overline{K}}\right)^2 + 4\overline{K}$$

$$STC(q) = 2\left(\frac{q}{4}\right)^2 + 4 \times 4$$

$$\boldsymbol{STC(q) = \frac{q^2}{8} + 16}$$

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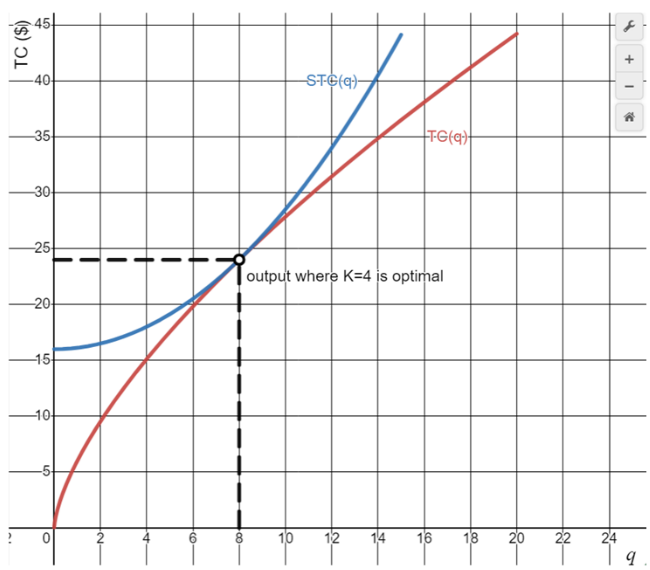
RELATIONSHIP BETWEEN LONG-RUN AND SHORT-RUN TOTAL COST CURVES

$$\boldsymbol{TC(q) = 6q^{\frac{2}{3}}}$$

$$\boldsymbol{STC(q) = \frac{q^2}{8} + 16}$$

$\boldsymbol{STC(q)}$ **is always greater than** $\boldsymbol{TC(q)}$ except at $(8, 24)$ – this is the output for which 4 units of capital is cost-minimizing!

So…at the point $(8, 24)$, both the long and short-run cost-minimization problems are being solved.



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SHORT-RUN AVERAGE AND MARGINAL COST CURVES

Short-run average cost – the firm's total cost per unit of output when it has one or more fixed inputs

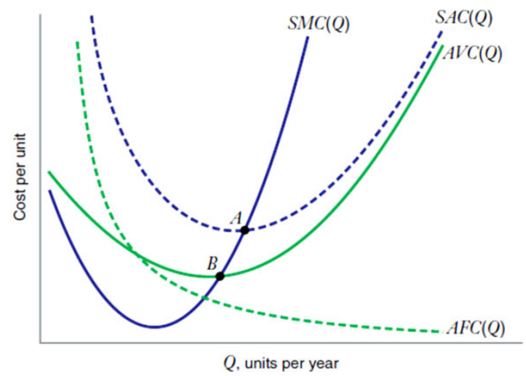
$$SAC(q) = \frac{STC(Q)}{Q}$$

$$SAC(q) = \frac{VC(Q)}{Q} + \frac{FC}{Q}$$

Average variable cost	**Average fixed cost**
Total variable cost per unit of output	Total fixed cost per unit of output

Short-run marginal cost – the slope of the short-run total cost curve.

$$SMC(Q) = \frac{dSTC(Q)}{dQ} = \frac{dVC(Q)}{dQ}$$



THE RELATIONSHIP BETWEEN SAC AND AC

The long-run average cost curve forms an **envelope** around the set of short-run average cost curves for different amounts of fixed inputs.

The short-run average cost is always greater than long-run average cost except for at the level of output for which the fixed capital is optimal.