

THE EFFECTS OF A CHANGE IN INCOME (Normal Goods)

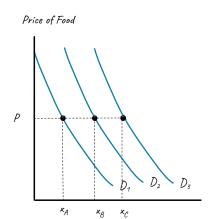
How does consumption of food and clothing change as income changes?

Income-Consumption Curve

Clothing, y Income-Consumption Curve

Food, x

Individual Demand Curve



Food, x

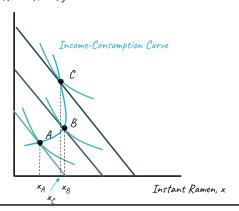
3

THE EFFECTS OF A CHANGE IN INCOME (Inferior Goods)

How does consumption of instant ramen and ramen from a ramen house change as income changes?

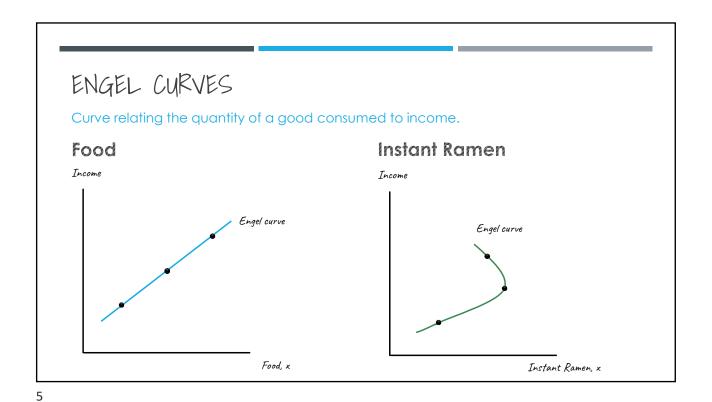
Income-Consumption Curve

Ramen House, y



For a low levels of income Instant Ramen is **normal**.

As income increases beyond that point, Instant Ramen becomes **inferior**.





DEMAND WITH AN INTERIOR SOLUTION

A consumer purchases two goods, food (x) and clothing (y) at prices P_x and P_y , respectively.

Their income is given by I, and their utility is given by

$$u(x, y) = xy$$

- a. Derive the demand for food and the demand for clothing.
- b. Use the demand curve to determine if food is a normal good.

Step 1: State the problem

Maximize: u(x,y) = xySubject to: $xP_x + yP_y = I$

Step 2: Form the Lagrangian

$$\phi(x, y, \lambda) = xy - \lambda(xP_x + yP_y - I)$$

7

DEMAND WITH AN INTERIOR SOLUTION

A consumer purchases two goods, food (x) and clothing (y) at prices P_x and P_y , respectively.

Their income is given by I, and their utility is given by

$$u(x, y) = xy$$

- a. Derive the demand for food and the demand for clothing.
- b. Use the demand curve to determine if food is a normal good.

$$\phi(x, y, \lambda) = xy - \lambda(xP_x + yP_y - I)$$

Step 3: Find the first order conditions

(1)
$$\frac{\partial \Phi}{\partial x} =$$

(2)
$$\frac{\partial \Phi}{\partial y} =$$

(3)
$$\frac{\partial \Phi}{\partial \lambda} =$$

DEMAND WITH AN INTERIOR SOLUTION

Step 4: Solve the resulting system of equations for x and y.

Plug (4) into (3) and solve for
$$x^*(I, P_x)$$
.

(1)
$$\frac{\partial \Phi}{\partial x} = y - \lambda P_x = 0 \implies \lambda = \frac{y}{P_x}$$

(2)
$$\frac{\partial \Phi}{\partial y} = x - \lambda P_y = 0 \quad \Rightarrow \lambda = \frac{x}{P_y}$$

(3)
$$\frac{\partial \Phi}{\partial \lambda} = -(xP_x + yP_y - I) = 0$$

Set (1) = (2) and solve for y.

Plug x^* into (4) and solve for $y^*(I, P_y)$.

9

DEMAND WITH AN INTERIOR SOLUTION

A consumer purchases two goods, food (x) and clothing (y) at prices P_x and P_y , respectively.

Their income is given by I, and their utility is given by

$$u(x, y) = xy$$

- a. Derive the demand for food and the demand for clothing.
- b. Use the demand curve to determine if food is a normal good.

Demand:
$$x^* = \frac{I}{2P_x}$$

Demand for a normal good will increase with income...

DEMAND W/ A POTENTIAL CORNER SOLUTION

David purchases two goods, food (x) and clothing (y). His income is \$100 and the price of food is \$1.

$$u(x,y) = xy + 10x$$

Goal: Derive David's demand for clothing, $y^*(P_y)$.

Step 1: Investigate the utility function.

Observe...

- 1. When y = 0 utility is still positive.
- 2. Potential corner point solution if the P_y is too high.

11

DEMAND W/ A POTENTIAL CORNER SOLUTION

David purchases two goods, food (x) and clothing (y). His income is \$100 and the price of food is \$1.

$$u(x,y)=xy+10x$$

Goal: Derive David's demand for clothing, $y^*(P_y)$.

Step 2: State the problem

Maximize: u(x, y) = xy + 10xSubject to: $x + yP_v = 100$

Step 3: Find MU_x , MU_y , and $MRS_{x,y}$

DEMAND W/ A POTENTIAL CORNER SOLUTION

David purchases two goods, food (x) and clothing (y). His income is \$100 and the price of food is \$1.

$$u(x,y) = xy + 10x$$

Goal: Derive David's demand for clothing, $y^*(P_v)$.

$$MRS_{x,y} = \frac{y+10}{x}$$

Step 4: Apply the tangency condition and solve for x.

$$MRS_{x,y} = \frac{P_x}{P_y}$$

13

DEMAND W/ A POTENTIAL CORNER SOLUTION

David purchases two goods, food (x) and clothing (y). His income is \$100 and the price of food is \$1.

$$u(x,y) = xy + 10x$$

Goal: Derive David's demand for clothing, $y^*(P)$.

Result of tangency: $x = yP_y + 10P_y$ Budget constraint: $x + yP_y = 100$ Step 5: Plug resulting equation into the budget constraint and solve for \boldsymbol{y}

DEMAND W/ A POTENTIAL CORNER SOLUTION

David purchases two goods, food (x) and clothing (y). His income is \$100 and the price of food is \$1.

$$u(x,y) = xy + 10x$$

Goal: Derive David's demand for clothing, $y^*(P_y)$.

$$y^*(P_y) = \begin{cases} \frac{50 - 5P_y}{P_y} & \text{if } P_y \le 10\\ 0 & \text{if } P_y > 10 \end{cases}$$

Step 6: Specify the prices for which this equation holds

$$y = \frac{50 - 5P_y}{P_y}$$



15

A CHANGE IN THE PRICE OF A GOOD

Demand curves are typically downward sloping...

A decrease in the price of the good typically results in an increase in the quantity demanded.

There are two separate effects at work

- 1. Substitution effect the change in consumption of a good associated with a change in its price, with the level of utility held constant.
- 2. Income effect the change in consumption of a good resulting from an increase in purchasing power, with relative prices held constant.

17

INCOME AND SUBSTITUTION EFFECTS

First consider only normal goods...

Suppose that a consumer buys two goods, food and clothing.

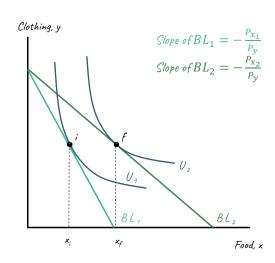
Both goods have a positive marginal utility.

Initially, the consumer chooses point i.

What happens when the price of food decreases?

Now the consumer chooses point f.

How much of the **change in x** comes from in the income effect? How much from the substitution effect?



INCOME AND SUBSTITUTION EFFECTS

First consider only normal goods...

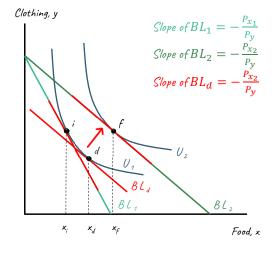
To answer this question, we need to find the **decomposition basket**.

Two features:

- 1. Reflects the price change (lies on a BL parallel to BL₂)
- 2. Achieves the same utility as basket i (BL is tangent to y)

Substitution effect = $X_d - X_i$

Income effect = $X_f - X_d$



19

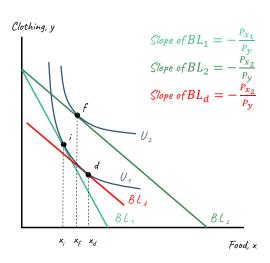
INCOME AND SUBSTITUTION EFFECTS

What happens when we have an inferior good?

Inferior goods have

- positive substitution effect (x_d-x_i) .
- negative income effect (x_f-x_d) .

Income effect is rarely large enough to outweigh the substitution effect.



INCOME AND SUBSTITUTION EFFECTS

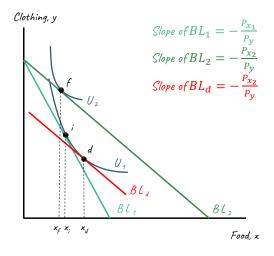
What is a Giffen good?

A good so inferior...

that the (negative) income effect is stronger than the (positive) substitution effect.

Substitution effect = $X_d - X_i > 0$ Income effect = $X_f - X_d < 0$

The result would be an upward sloping demand for x



21

DO GIFFEN GOODS EXIST?

Probably, but they are very rare...



The potato in Ireland during the Irish famine of the mid-1800s



Rice for very poor households in China's Hunan Province

COMPUTING THE INCOME AND SUBSTITUTION EFFECTS

Zuri has \$15 available to spend on cake and lemonade.

Her utility function is u(x, y) = xy + x, where x is the number of cakes and y is number of cups of lemonade.

The price of a cake is \$4 and the price of a cup of lemonade is \$1.

Goal: Calculate the income and substitution effects when the price of cake falls from \$4 to \$1.

Important initial work...

Marginal utilities

$$MU_x = y + 1$$

$$MU_{v} = x$$

Initial budget constraint

$$4x + y = 15$$

Final budget constraint

$$x + y = 15$$

23

COMPUTING THE INCOME AND SUBSTITUTION EFFECTS

Goal: Calculate the income and substitution effects when the price of cake falls from \$4 to \$1.

$$u(x, y) = xy + x$$

Marginal utilities

$$MU_x = y + 1$$

$$MU_{\nu} = x$$

Initial budget constraint $(P_x = 4, P_y = 1)$

$$4x + y = 15$$
, (BL_1)

Final budget constraint $(P_x = 1, P_y = 1)$

$$x + y = 15$$
, (BL_2)

Step 1: Initial basket (x_i, y_i)

Tangency Condition

$$\frac{y+1}{y} = \frac{4}{1}$$

Plug into BL_1

$$4x + (4x - 1) = 15$$

COMPUTING THE INCOME AND SUBSTITUTION EFFECTS

Goal: Calculate the income and substitution effects when the price of cake falls from \$4 to \$1.

$$u(x,y) = xy + x$$

Marginal utilities

$$MU_x = y + 1$$
$$MU_y = x$$

Initial budget constraint ($P_x = 4$, $P_y = 1$)

$$4x + y = 15$$
, (BL_1)

Final budget constraint ($P_x = 1$, $P_y = 1$)

$$x + y = 15$$
, (BL_2)

Step 2: Final basket (x_f, y_f)

Tangency Condition

$$\frac{y+1}{x} = \frac{1}{1}$$

Plug into BL_2

$$x + (x - 1) = 15$$

25

COMPUTING THE INCOME AND SUBSTITUTION EFFECTS

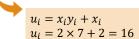
Step 3: Decomposition Basket (x_d, y_d)

i. Indifference curve is tangent to the decomposition budget line.



Slope of
$$BL_d = -\frac{P_{x_2}}{P_{y_1}} = -\frac{1}{1}$$

i. Achieves the same utility as initial basket $(x_i = 2, y_i = 7)$



$$\left(x_d y_d + x_d = 16\right)$$

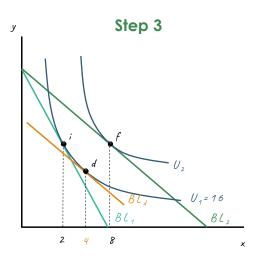
$$\frac{y_{d+1}}{x_d} = \frac{1}{1} \implies \left(y_d = x_d - 1 \right)$$

COMPUTING THE INCOME AND SUBSTITUTION EFFECTS

Step 4: Compute!

Substitution effect $= x_d - x_i$

Income effect = $x_f - x_d$



27



CONSUMER SURPLUS

Using a demand curve, we can calculate consumer surplus

Individual Consumer Surplus

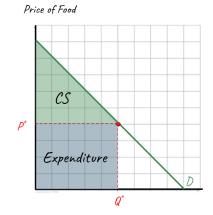
The difference between the maximum amount a consumer is willing to pay for a good and the amount that the consumer pays.

CS=WTP- Price

(Aggregate) Consumer Surplus

Measures how much better off individuals are in aggregate because they can buy goods in the market.

= total benefit - expenditure



Food, x

29

LET'S DO THE MATH ...

The demand function for widgets is given by $Q_D = 16 - 2P$.

Compute the change in consumer surplus and expenditure when price of a widget increases from \$1 to \$2.

P=\$1

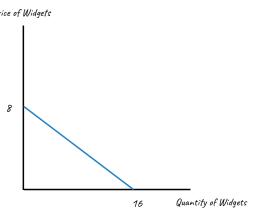
CS =

P=\$2 CS =

Exp. =

Exp. =

Price of Widgets



MARKET DEMAND CURVE

Curve relating the quantity of a good that all consumers in a market will buy to its price

Calculated by horizontally summing all individual demand curves in the market.

Calculate the Market Demand for OJ

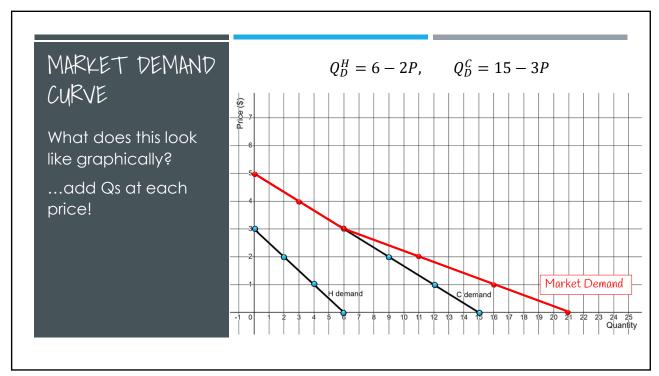
Example:

Suppose there are two consumers in the market for orange juice, a health-conscious consumer and a casual consumer.

$$Q_D^H = 6 - 2P$$

$$Q_D^C = 15 - 3P$$

31





POSITIVE NETWORK EXTERNALITIES

Occur if the quantity of a good demanded by a typical consumer increases in response to the growth in purchases of other consumers

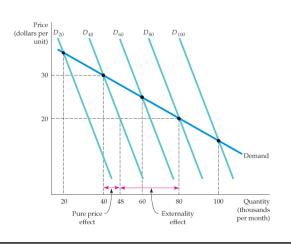
Basic Idea...

The consumer desires a good because others do.

Examples

- Word processing
- Social networking
- Multiplayer online games

This positive network externality effect is sometimes called the bandwagon effect.



NEGATIVE NETWORK EXTERNALITIES

Occur if the quantity of a good demanded by a typical consumer decreases in response to the growth in purchases of other consumers

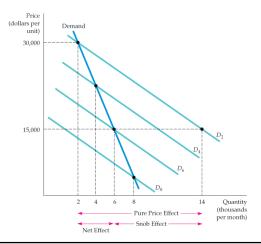
Basic Idea...

Consumers have a desire to own an exclusive or unique good.

Examples

- Congestion
- Rare items
- Exclusive club

This negative network externality effect is sometimes called the snob effect.



35



DEMAND FOR LEISURE

Informs us about the supply of labor...

Divide an individual's the day into two parts

- 1. Hours when she works.
- 2. Hours when she pursues leisure (all non-work activities).

(both consumption and leisure are normal goods)

Why does she work at all?

To earn income so she can buy goods

Basic model

24 hours in a day

- \circ L = hours of leisure
- H = hours of labor = 24 L

Each hour of labor yields wage = w

• Daily income = w(24 - L)

Income is used to buy a composite good

 $P_C = 1$

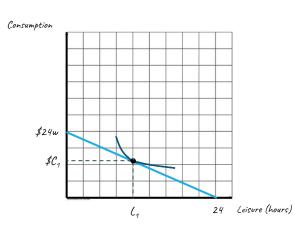
Utility depends on Leisure and consumption:

u(L,C)

38

DEMAND FOR LEISURE

Informs us about the supply of labor...



Problem to be solved:

Maximize u(L, C)

s.t.
$$C = w(24 - L)$$

Often rearranged to...

$$C + (w)L = 24w$$

 \dot{w} is the price of leisure

A QUICK DEMONSTRATION

Suppose that Marco consumes leisure (L) and a composite good (C).

To fund his purchases of the composite good, Marco must work.

He has 24 hours per day to divide between leisure activities and working.

His current wage is \$20 an hour and the price of the composite good is \$1.

Marco has utility over leisure and consumption (of the composite good) of u(L, C) = LC + 40L.

Determine Marco's optimal choice of L and C.

Tangency:
$$\frac{MU_L}{MU_C} = \frac{P_L}{P_C}$$

$$\frac{C + 40}{L} = 20$$

$$C = 20L - 40 \quad (1)$$

Budget Constraint:
$$C = w(24 - L)$$

 $C = 20(24 - L)$

$$C = 480 - 20L$$
 (2)

Set (1) = (2) and solve for L

$$20L - 40 = 480 - 20L$$

 $40L = 520$
 $L = 13$

Plug
$$L = 13$$
 into (2) to solve for C
 $C = 480 - 20(13) \Rightarrow C = 220$

40

SUBSTITUTION AND INCOME EFFECTS ON LABOR SUPPLY

An increase in wages has two effects:

- 1. The increase in the price of leisure induces a **substitution effect** toward less leisure and more work.
- 2. The rise in income has an **income effect**, so consumers buy more of all normal goods, including leisure.

Effects are opposite-signed, so the shape of labor supply is ambiguous.

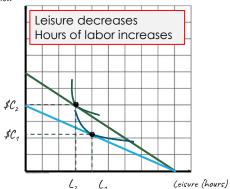
SUBSTITUTION AND INCOME EFFECTS ON LABOR SUPPLY

$$C = w(24 - L)$$

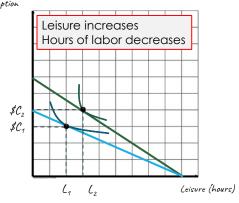
Substitution effect dominates

Income effect dominates

Consumption



Consumption



42

THE SHAPE OF LABOR SUPPLY CURVE

These two possibilities imply different shapes for the labor supply curve

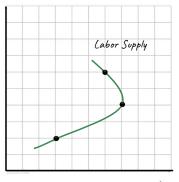
- If substitution effects dominate
 the labor supply curve is the typical upward -sloping shape
- 2. If income effects dominate

the labor supply curve will slope downward

Unlikely that labor supply curves are everywhere downward sloping...

Backward Bending Supply of Labor

Wage



(hours worked per day)

A SPECIAL CASE ...

When consumption and leisure are perfect complements

Suppose $u = \min(L, 2C)$

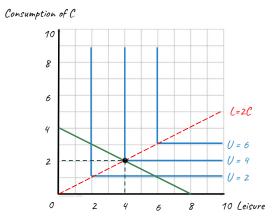
What do we know about this utility function?

Optimal choice

- 1. Must be on the budget line
- 2. Must have L = 2C

I call this the "no waste" condition

True at the corner of each indifference curve.



44

A SPECIAL CASE...

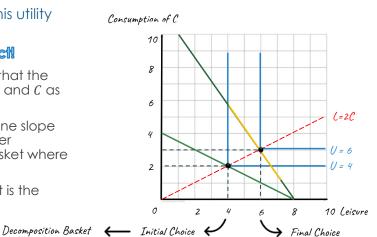
When consumption and leisure are perfect complements

Suppose $u = \min(L, 2C)$

What do we know about this utility function?

There is no substitution effect!

- the utility function implies that the consumer does not view L and C as substitutable.
- o If wage rises, the budget line slope changes, but the consumer continues to choose a basket where L = 2C
- The decomposition basket is the initial basket.



A SPECIAL CASE...

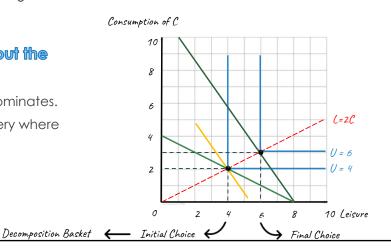
When consumption and leisure are perfect complements

Substitution Effect = 4 - 4 = 0

Income Effect = 6 - 4 = 2

What does this tell us about the labor supply curve?

- Income effect always dominates.
- Labor supply curve is every where downward sloping.



46

THE MARKET (LABOR) SUPPLY CURVE

The market supply curve shows the total amount of labor workers are willing to supply at every possible wage

Horizontal sum of every worker's labor supply curve

- To construct it, simply fix wage and add each worker's quantity of labor supplied at that wage.
- Once this is done for every possible wage, the market-level labor supply is the result.

Some individuals can have a backward-bending portion of their labor supply curve.

Research suggests that, at the market level, the substitution effect dominates the income effect.

The market labor supply curve is upward sloping!

A LABOR SUPPLY EXAMPLE ...

Consider Samira's optimal labor and leisure decision...

She earns a wage of \$w an hour and has 24 hours per day to split between labor and leisure.

Samira has a utility function over consumption spending and leisure equal to

$$U(L,C)=10C^{0.5}+5L$$

Where C is dollars of consumption and L is hours of leisure.

Goal: Derive Samira's supply of labor

Determine her budget line and identify the price of leisure.

48

A LABOR SUPPLY EXAMPLE ...

Consider Samira's optimal labor and leisure decision...

She earns a wage of w an hour and has 24 hours per day to split between labor and leisure.

Samira has a utility function over consumption spending and leisure equal to

$$U(L,C) = 10C^{0.5} + 5L$$

Where C is dollars of consumption and L is hours of leisure.

Goal: Derive Samira's supply of labor

Use the tangency condition $\left(MRS_{L,C} = \frac{P_L}{P_C}\right)$ to determine the **demand for** C.

A LABOR SUPPLY EXAMPLE ...

Consider Samira's optimal labor and leisure decision...

She earns a wage of \$w\$ an hour and has 24 hours per day to split between labor and leisure.

Samira has a utility function over consumption spending and leisure equal to

$$U(L,C)=10C^{0.5}+5L$$

Where C is dollars of consumption and L is hours of leisure.

Goal: Derive Samira's supply of labor

Plug $C^*(w) = w^2$ into the budget line to find **demand for** L (leisure)

50

A LABOR SUPPLY EXAMPLE ...

Consider Samira's optimal labor and leisure decision...

She earns a wage of w an hour and has 24 hours per day to split between labor and leisure.

Samira has a utility function over consumption spending and leisure equal to

$$U(L,C) = 10C^{0.5} + 5L$$

Where C is dollars of consumption and L is hours of leisure.

Goal: Derive Samira's supply of labor

Finally, we know that the **supply of labor** is $H^* = 24 - L^*$