COST CONCEPTS FOR DECISION-MAKING

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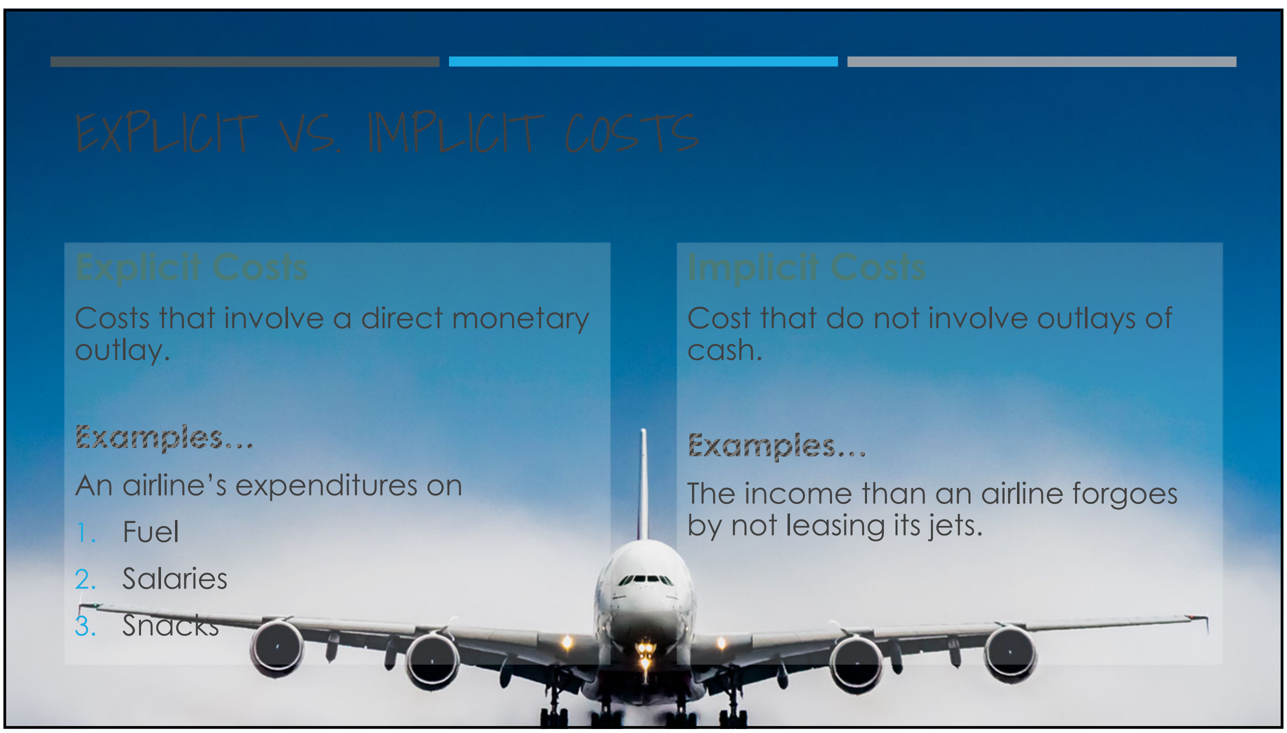

EXPLICIT VS. IMPLICIT COSTS

Explicit Costs

Costs that involve a direct monetary outlay.

Examples…

An airline's expenditures on

1. Fuel
2. Salaries
3. Snacks

Implicit Costs

Cost that do not involve outlays of cash.

Examples…

The income than an airline forgoes by not leasing its jets.

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OPPORTUNITY COST

The value of the next best alternative that is forgone when an alternative is chosen.

Includes all the implicit and explicit costs associated with the alternative → Forward looking → Depends on the decision being made

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ECONOMIC VS. ACCOUNTING COSTS

Accounting costs

The total of explicit costs that have been incurred in the past.

Economic costs

Synonymous with opportunity costs, the sum of the firm's explicit and implicit costs

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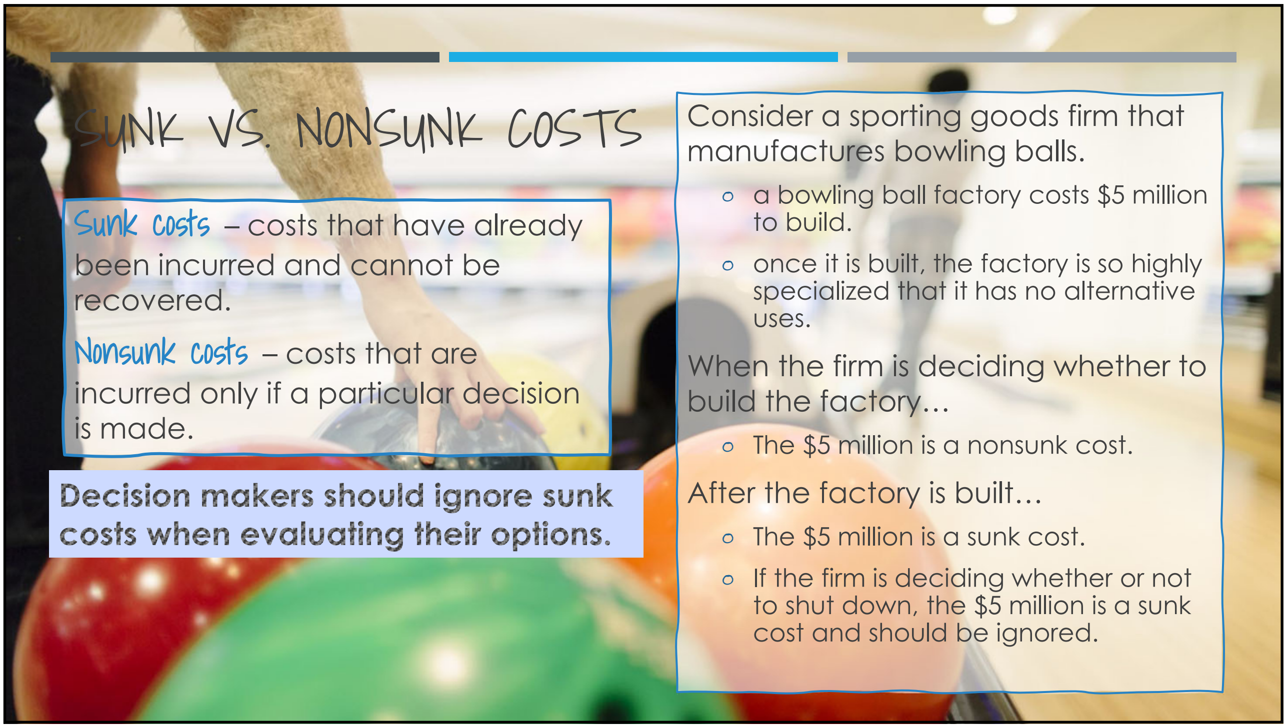

Sunk vs. Nonsunk Costs
Sunk costs – costs that have already been incurred and cannot be recovered.
Nonsunk costs – costs that are incurred only if a particular decision is made.
Decision makers should ignore sunk costs when evaluating their options.
Consider a sporting goods firm that manufactures bowling balls.
a bowling ball factory costs $5 million to build.
once it is built, the factory is so highly specialized that it has no alternative uses.
When the firm is deciding whether to build the factory…
The $5 million is a nonsunk cost.
After the factory is built…
The $5 million is a sunk cost.
If the firm is deciding whether or not to shut down, the $5 million is a sunk cost and should be ignored.

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The Cost-Minimization Problem

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WHAT IS THE COST MINIMIZATION PROBLEM?

The problem of finding the **input combination** that minimizes a firm's total cost of producing a particular level of output.

How does time impact this problem?

In the **long run**, all inputs are variable, and all costs are nonsunk.

In the **short run**, the amount of at least one input is fixed, the costs associated with those fixed inputs are fixed and potentially sunk.

THE LONG-RUN COST-MINIMIZATION PROBLEM…

…is a constrained minimization problem!

The firm only uses two inputs, L and K.

- The price of one unit of labor is the wage rate, w.
- The price of one unit of capital is the rental rate, r.

The firm wants to produce $\bar{Q}$ units of output (this is exogenously determined)

- Their production function is given by $Q = f(L, K)$

The problem they are solving is…

Minimize the function:

$$\boldsymbol{TC = wL + rK}$$

Subject to the constraint:

$$\boldsymbol{f(L, K) = \bar{Q}}$$

ISOCOST LINE

Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

Standard form:

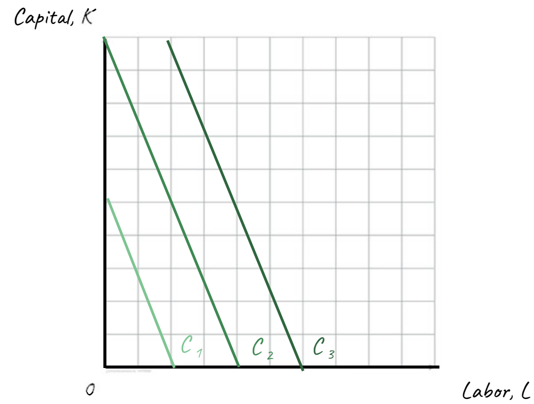
$$TC = wL + rK$$

Solve for $\boldsymbol{K}$:

$$K = \frac{TC}{r} - \frac{w}{r}L$$

K intercept

Slope



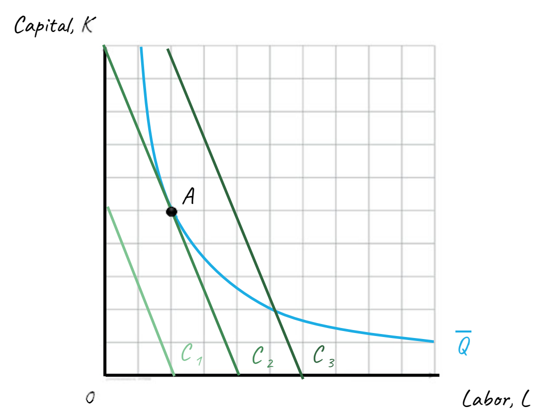
GRAPHICAL SOLUTION (Constrained Cost Minimization)

Given a production function and input prices…

a producer will choose inputs to achieve an output goal ($\overline{Q}$) at the lowest possible cost.

This will occur at the tangency between the isocost line and the isoquant.

Interior solution



BASIC MATHEMATICAL PROCESS

1. Isoquant is tangent to the Isocost line.

$$-\frac{MP_L}{MP_K} = -\frac{w}{r}$$

or

$$MRTS_{L,K} = \frac{w}{r}$$

2. Input choice is on the Isoquant.

$$f(L,K) = \bar{Q}$$

Rearrange to form …

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Costs are minimized when the additional output per dollar spent is equal across inputs.

DETERMINING THE COST MINIMIZING INPUTS

Consider a cost-minimizing firm

Suppose that the firm's production function is

$$Q = 2L^{0.5}K$$

The price of a **labor** is $10, and the price of **capital** is $5.

Determine the cost minimizing input combination if the firm wants to produce **1000** units per year.

Step 1: Determine the MP_L, MP_K, and $MRTS_{L,K}$.

$$\boldsymbol{MP_L = \frac{\partial Q}{\partial L} =}$$

$$\boldsymbol{MP_K = \frac{\partial Q}{\partial K} =}$$

$$\boldsymbol{MRTS_{L,K} = \frac{MP_L}{MP_K} =}$$

DETERMINING THE COST MINIMIZING INPUTS

Consider a cost-minimizing firm

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Step 2: Set $MRTS_{L,K} = \frac{w}{r}$ (tangency)

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DETERMINING THE COST MINIMIZING INPUTS

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The price of a **labor** is \$10, and the price of **capital** is \$5.

Determine the cost minimizing input combination if the firm wants to produce **1000** units per year.

Step 3: Plug resulting equation into the isoquant.

$\boxed{K = 4L}$

Isoquant: $\mathbf{2L^{0.5}K = 1000}$

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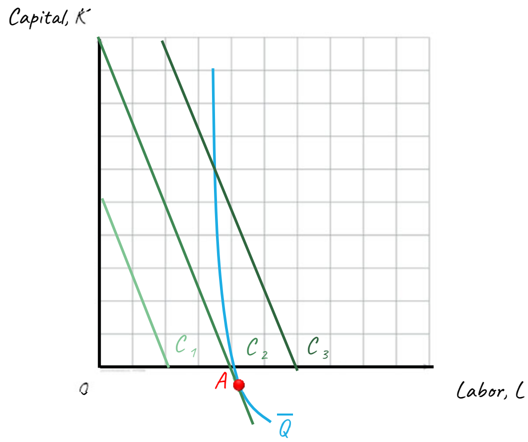
CORNER SOLUTIONS…

Convex isoquants that cross an axis



Examples…
$q = L^{0.5} + K, \qquad q = 0.1KL + L$

When you solve a problem like this using the standard methods, you will get a negative value for one of your inputs.

(In this case a negative K)

You can then safely conclude that…

- $K = 0$
- L is hired until the output goal of $\bar{Q}$ is reached.

CORNER SOLUTIONS…

To solve this question, compare…

$$\frac{MP_L}{w} \text{ to } \frac{MP_K}{r}$$

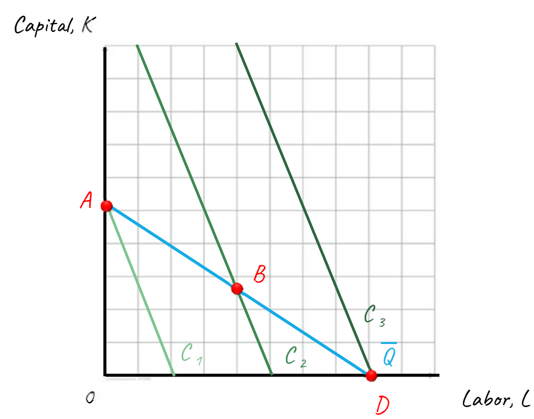
If $\frac{MP_L}{w} > \frac{MP_K}{r}$…

then $K = 0$ and L is hired until the output goal of $\bar{Q}$ is reached.

If $\frac{MP_K}{r} > \frac{MP_L}{w}$ …

then $L = 0$ and K is hired until the output goal of $\bar{Q}$ is reached.

Perfect Substitutes



A COST-MINIMIZING EXAMPLE (with a corner solution)

Suppose that a firm's production function is

$$Q = 2L + 6K$$

The price of a **labor** is $10, and the price of **capital** is $10.

Determine the cost minimizing input combination if the firm wants to produce **60** units.

Try the tangency condition…

Instead compare $\frac{MP_L}{w}$ to $\frac{MP_K}{r}$ …

A COST-MINIMIZING EXAMPLE (with a corner solution)

Suppose that a firm's production function is

$$Q = 2L + 6K$$

The price of a **labor** is $10, and the price of **capital** is $10.

Determine the cost minimizing input combination if the firm wants to produce **60** units.

We know the firm only hires K.

What is the cost-minimizing choice?

EXPANSION PATH

A line that connects the **cost-minimizing input** combinations as the quantity of **output varies**, holding input prices constant.

$\boldsymbol{Q = KL}$ (basic Cobb-Douglas); $\boldsymbol{w = 5, r = 5}$

If the firm wants to produce 100 units

$K = 10, L = 10$

$$TC_{100} = \$5 \times 10 + \$5 \times 10 = \$100$$

If the firm wants to produce 225 units

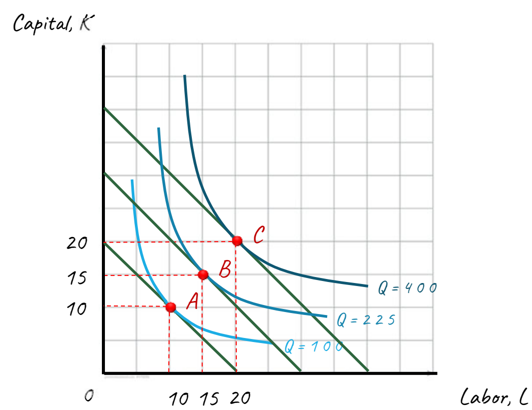
$K = 15, L = 15$

$$TC_{225} = \$5 \times 15 + \$5 \times 15 = \$150$$

If the firm wants to produce 400 units

$K = 20, L = 20$

$$TC_{400} = \$5 \times 20 + \$5 \times 20 = \$200$$



EXPANSION PATH

A line that connects the **cost-minimizing input** combinations as the quantity of **output varies**, holding input prices constant.

$\boldsymbol{Q = KL}$ (basic Cobb-Douglas); $\boldsymbol{w = 5, r = 5}$

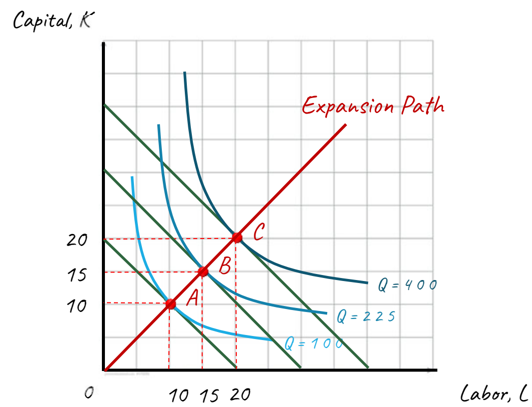
What must hold at points A, B, and C?

The tangency condition!

In this example…

$$\frac{K}{L} = \frac{5}{5}$$
$$\Rightarrow K = L$$

The **expansion** path is the upward sloping line where $\boldsymbol{K = L}$.



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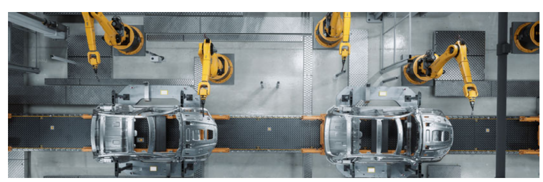
INPUT DEMAND CURVES

Labor Demand Curve

A curve that shows how the firm's cost-minimizing quantity of labor varies with the price of labor.

How do we find it?

Solve the cost minimization problem with an unspecified w (and potentially r & $\bar{Q}$).



Capital Demand Curve

A curve that shows how the firm's cost-minimizing quantity of capital varies with the price of capital.

How do we find it?

Solve the cost minimization problem with an unspecified r (and potentially w & $\bar{Q}$).

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INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

Determine the demand curves for labor and for capital.

Step 1: Write down the problem

Minimize $wL + rK$

Subject to $2L^{0.5}K = q$

Step 2: Form the Lagrangian

$$\mathcal{L} = \underbrace{wL + rK}_{\text{Objective Function}} - \lambda\underbrace{(2L^{0.5}K - q)}_{\text{Constraint, solved to be = 0}}$$

INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

Determine the demand curves for labor and for capital.

$$\mathcal{L} = wL + rK - \lambda(2L^{0.5}K - q)$$

Step 3: Find the first order conditions.

(1) $\frac{\partial \mathcal{L}}{\partial L} =$

(2) $\frac{\partial \mathcal{L}}{\partial K} =$

(3) $\frac{\partial \mathcal{L}}{\partial \lambda} =$

INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

Determine the demand curves for labor and for capital.

f.o.c.

(1) $\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \times L^{-0.5}K = 0$

(2) $\frac{\partial \mathcal{L}}{\partial K} = r - \lambda \times 2L^{0.5} = 0$

(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = -(2L^{0.5}K - q) = 0$

Step 4: Solve for $L^*(r, w, q)$

Solve (1) for λ

Solve (2) for λ

INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

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(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = -(2L^{0.5}K - q) = 0$

Step 4: Solve for $L^*(r, w, q)$

(4) $\lambda = \frac{wL^{0.5}}{K}$ (5) $\lambda = \frac{r}{2L^{0.5}}$

Set (4) = (5) and solve for $K = f(L)$

INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

Determine the demand curves for labor and for capital.

f.o.c.

(1) $\frac{\partial \mathcal{L}}{\partial L} = w - \lambda \times L^{-0.5}K = 0$

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(3) $\frac{\partial \mathcal{L}}{\partial \lambda} = -(2L^{0.5}K - q) = 0$

Step 4: Solve for $L^*(r, w, q)$

(6) $K = \frac{2w}{r}L$

Plug (6) into (3) and solve for L^*

INPUT DEMAND EXAMPLE

Suppose that the firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and the firm's output goal is q units.

Determine the demand curves for labor and for capital.

Relevant information…

(6) $K = \frac{2w}{r}L$

Notice…
The expansion path is a line with slope of $\frac{2w}{r}$

(7) $L^*(r, w, q) = \left(\frac{qr}{4w}\right)^{\frac{2}{3}}$

Process to follow…

plug (7) into (6) and solve for $K^*(r, w, q)$

WHAT HAVE WE LEARNED SO FAR?

Long run cost-minimization

Minimize $wL + rK$

s.t. $f(L, K) = q$

Where w, r, and q are provided exogenous constants.

How to Solve?

(1) $\frac{MP_L}{MP_K} = \frac{w}{r}$

(2) $f(K, L) = q$

Tangency

Isoquant

Or you can use the Lagrangian...

$$\mathcal{L} = wL + rK - (f(L, K) - q)$$

Long-run input demand

Minimize $wL + rK$

s.t. $f(L, K) = q$

Where at least one of w, r, and q is unspecified (a value is not provided).

How to Solve?

(1) $\frac{MP_L}{MP_K} = \frac{w}{r}$

(2) $f(K, L) = q$

Or you can use the Lagrangian...

$$\mathcal{L} = wL + rK - (f(L, K) - q)$$

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TOTAL, FIXED AND VARIABLE COSTS

Suppose that a firm cannot alter its quantity of capital $\bar{K}$…

$$Total\ Costs = wL + r\bar{K}$$

Total costs (TC or C) can be divided into two distinct categories:

$$TC = VC(Q) + FC$$

Total Variable Cost (VC)
Sum of expenditures on variable inputs at the short-run cost minimizing input combination.

Total Fixed Cost (FC)
The costs of fixed inputs; it does not vary with output.

ARE THESE COSTS SUNK?

Variable costs

If the firm produces no output…

Then they hire no labor and pay no wages.

These completely avoidable costs **are always nonsunk**

Fixed costs

If the firm produces no output…

Then they still must pay rent on their fixed capital, $\bar{K}$.

These costs are

- **Sunk if there are no alternative uses** for the fixed input.
- **Nonsunk if there are alternative uses** (and thus the costs can be recovered)

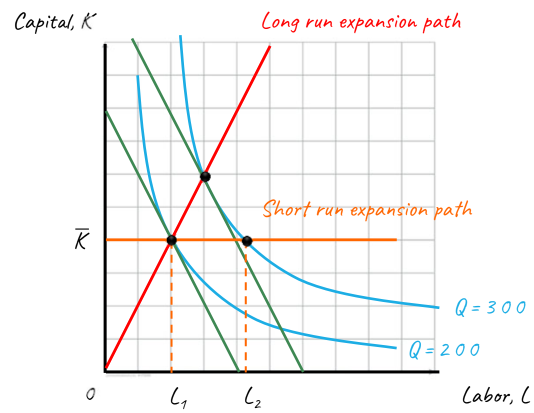
THE INFLEXIBILITY OF SHORT RUN PRODUCTION

In the short run, at least one input is fixed.

This fact will change the process of determining…

1. the cost minimizing choice of inputs, and
2. the expansion path.

Suppose capital is fixed at $\overline{K}$



SHORT-RUN COST MINIMIZATION EXAMPLE (one fixed input)

Suppose that a firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and their output goal is $\boldsymbol{q = 100}$ **units**.

The firm's capital is fixed at $\boldsymbol{\overline{K} = 10}$

How much labor should the firm hire to minimize their costs in the short run?

"Constraint" is all that matters here!

$$2L^{0.5}\overline{K} = 100$$

SHORT-RUN DEMAND EXAMPLE (For the variable input)

Suppose that a firm's production function is $Q = 2L^{0.5}K$. The wage is w, the rental rate is r and their output goal is **q units**.

The firm's capital is **fixed at $\overline{K}$**.

Solve for the firms short-run demand for labor.

"Constraint" is all that matters here!

$$2L^{0.5}\overline{K} = q$$