DESCRIBING RISK

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PROBABILITY

Likelihood that a given outcome will occur.

If a situation has multiple outcomes (called a lottery), then to describe risk we need to know the likelihood of each outcome…

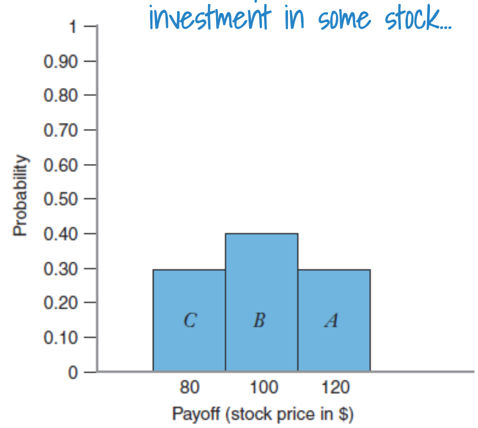
Probability – the likelihood that a particular outcome will occur.

- Probability of any particular outcome must be between 0 and 1.

Probability Distribution – the depiction of all possible payoffs in a lottery and their associated probabilities.

- The sum of the probabilities of all possible outcomes equals 1.

Probability distribution for a $100 investment in some stock…



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PROBABILITY

Likelihood that a given outcome will occur.

If a situation has multiple outcomes (called a lottery), then to describe risk we need to know the likelihood of each outcome…

Objective probability – relies on the frequency with which outcomes occur.

Subject probability – is the perception that an outcome will occur.

A simple example…

Suppose you are choosing between two part-time jobs

1. Based entirely on commission (C)
   - Outcome C_1: $I_1 = \$2000$, $P_1 = 0.5$
   - Outcome C_2: $I_2 = \$1000$, $P_2 = 0.5$
2. Salaried (S)
   - Outcome S_1: $I_1 = \$1510$, $P_1 = 0.99$
   - Outcome S_2: $I_2 = \$510$, $P_2 = 0.01$

EXPECTED VALUE

Probability-weighted average of payoffs associated with all possible outcomes

So, we are going to weight each outcome (X_i) by the probability that outcome occurs (P_i)…

Expected Value of X

$$E(X) = \sum_{i=1}^{n} P_i X_i$$

Revisiting our example…

1. Based entirely on commission (C)
   - Outcome C_1: $I_1 = \$2000$, $P_1 = 0.5$
   - Outcome C_2: $I_2 = \$1000$, $P_2 = 0.5$

$E(C) =$

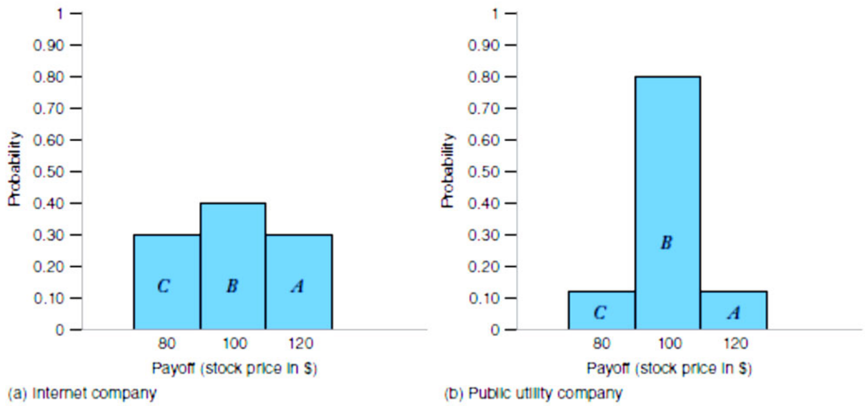
2. Salaried (S)
   - Outcome S_1: $I_1 = \$1510$, $P_1 = 0.99$
   - Outcome S_2: $I_2 = \$510$, $P_2 = 0.01$

$E(S) =$

VARIABILITY

Extent to which possible outcomes of an uncertain event differ.

We can visualize variability by looking at probability distributions



VARIABILITY

Extent to which possible outcomes of an uncertain event differ.

We can measure variability by calculating the variance and standard deviation…

This is a 3-step process:

1. Determine the **deviation** for each outcome
   - difference between expected payoff and actual payoff.
2. Determine the **variance**
   - the probability-weighted average of the squared deviations.
3. Determine the **standard deviation** by taking the square root of the variance.

$$\sigma = \sqrt{P_1\left[\left(X_1 - E(X)\right)^2\right] + P_2\left[\left(X_2 - E(X)\right)^2\right]}$$

VARIABILITY

$$\sigma = \sqrt{P_1\left[\left(X_1 - E(X)\right)^2\right] + P_2\left[\left(X_2 - E(X)\right)^2\right]}$$

Extent to which possible outcomes of an uncertain event differ.

Revisiting our example…

1. Based entirely on commission (C)
   - Outcome C_1: $I_1 = \$2000,\ P_1 = 0.5$
   - Outcome C_2: $I_2 = \$1000,\ P_2 = 0.5$

$E(C) = 0.5 \times 2000 + 0.5 \times 1000 = 1500$

2. Salaried (S)
   - Outcome S_1: $I_1 = \$1510,\ P_1 = 0.99$
   - Outcome S_2: $I_2 = \$510,\ P_2 = 0.01$

$E(S) = 0.99 \times 1510 + 0.01 \times 510 = 1500$

Calculate the standard deviation of each job offer.

DECISION MAKING

Which job would you take?

If you don't like risk, then you choose the salaried job.

1. Same expected income
2. Lower variability

The decision is much more difficult if the two jobs have different expected incomes and the one with the higher expected income has higher variability.

How do we decide then?

It depends on preferences toward risk – which we will discuss on Wednesday.

DETERRING CRIME

We can use this idea of risk avoidance to design punishments that effectively deter crime

Its costly to catch lawbreakers…

Rather than trying to catch them all, just make the punishment for those who are caught very high.

This makes the expected cost of law breaking high.

As a result, most risk averse-individuals avoid committing crimes.

Less time is spent policing, so costs are lower.

Theory in Action

High fines for speeding, double parking, tax evasion and air polluting.

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UTILITY FROM INCOME (or consumption)

For example…

$$u(I) = \sqrt{I}$$

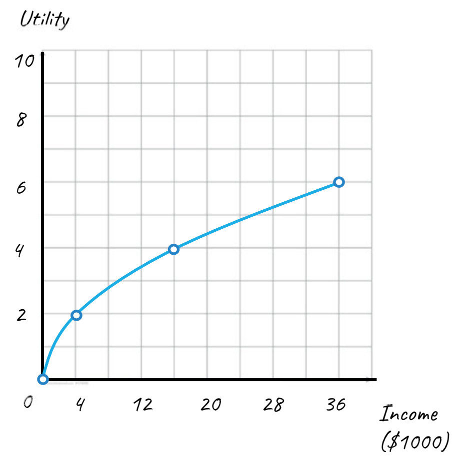
Where I represents income in thousands.

Notice the shape of the utility function

$$u'(I) = MU = 0.5I^{-0.5} \geq 0$$

$$u''(I) = MU' = -0.25I^{-1.5} \leq 0$$

Its concave!



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EXPECTED UTILITY

Sum of the utilities associated with all possible outcomes, weighted by the probability that each outcome will occur.

If there are two outcomes, then: $E[u(X)] = P_1 \times u(X_1) + P_2 \times u(X_2)$

Revisiting our example…with $u(I) = \sqrt{I}$

1. Based entirely on commission (C)

   Outcome C_1: $I_1 = \$2000$, $P_1 = 0.5$

   Outcome C_2: $I_2 = \$1000$, $P_2 = 0.5$

$E[u(C)] =$

2. Salaried (S)

   Outcome S_1: $I_1 = \$1510$, $P_1 = 0.99$

   Outcome S_2: $I_2 = \$510$, $P_2 = 0.01$

$E[u(S)] =$

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PREFERENCES TOWARDS RISK

Let's consider a second scenario…

At Samantha's current job (c), she has an income of $15,000

Potential new job (n)

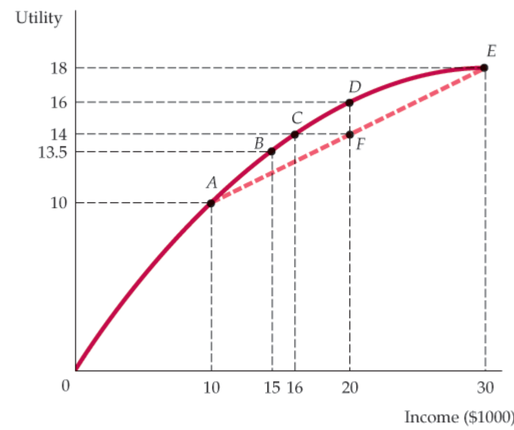
- $I_1 = \$30{,}000$ with probability $P_1 = 0.5$
- $I_2 = \$10{,}000$ with probability $P_2 = 0.5$
- $E[I_n] = 0.5 \times 30{,}000 + 0.5 \times 10{,}000 = \$20{,}000$

$u(c) = u(15{,}000) = 13.5$

$E[u(n)] = 0.5 \times u(10{,}000) + 0.5 \times u(30{,}000)$

$= 0.5 \times 10 + 0.5 \times 18 = 14$

She chooses the new job…14 > 13.5



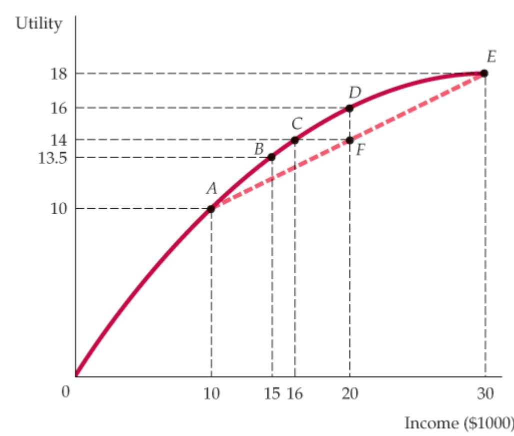
PREFERENCES TOWARDS RISK

Risk Averse – Condition of preferring a certain income to a risky income with the same expected value.

Utility function exhibits diminishing marginal utility of income (concave)

Example:

$$u(I) = \sqrt{I}$$



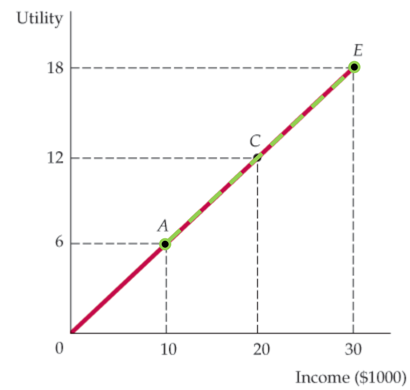
PREFERENCES TOWARDS RISK

Risk Neutral – Condition of being indifferent between a certain income and an uncertain income with the same expected value.

Utility function exhibits constant marginal utility of income (linear)

Example:

$$u(I) = 2I$$



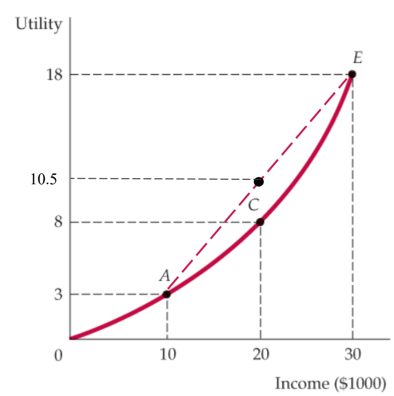
PREFERENCES TOWARDS RISK

Risk Loving – Condition of preferring a risky income to a certain income with the same expected value.

Utility function exhibits increasing marginal utility of income (convex)

Example:

$$u(I) = I^2$$



BEARING AND ELIMINATING RISK

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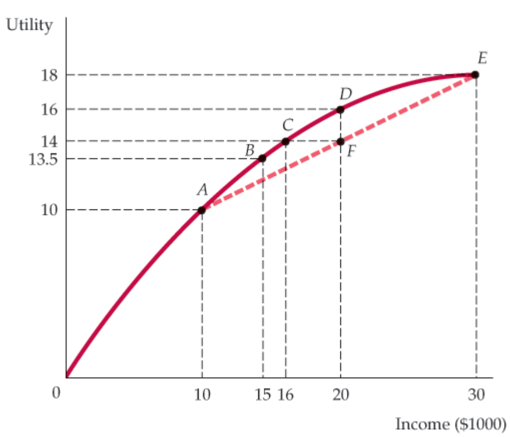
CERTAINTY EQUIVALENT AND RISK PREMIUM

Samantha's preferences



Certainty equivalent (ce) – certain payment that makes an individual indifferent between taking the risk and taking the certain payment.

$$E(u) = u(ce)$$

Samantha's $ce = 16$

Risk premium (rp)- Maximum amount of money that a risk averse person will pay to avoid taking risk.

$$rp = E(I) - ce$$

Samantha's $rp = E[n] - ce = 20 - 16 = 4$

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MORE DIRECT WAY TO FIND THE RISK PREMIUM

Set…

$$E(U) = U(E[Income] - rp)$$

…then solve for rp

THE DEMAND FOR INSURANCE

Risk-averse individuals are willing to pay to avoid risk…

How does insurance work?

1. Policyholders pay a monthly (or yearly) premium
2. In the event of an adverse event, some or all "damages" are paid for by the insurer.
   - Full Insurance – covers all damages
   - Partial Insurance – covers only some damages

Due to the law of large numbers, insurers are always willing to sell insurance at the **actuarially fair premium**

↓

Equal to the expected value of the promised insurance payout

THE DEMAND FOR INSURANCE

Suppose you are **risk averse** and have just purchased a new car.

There are two possible outcomes

1. No accident

   $I_1 = \$50{,}000,\ P_1 = 0.95$

2. Accident

   $I_2 = \$40{,}000,\ P_2 = 0.05$

$$E[I] = 0.95 \times 50{,}000 + 0.05 \times 40{,}000 = 49{,}500$$

You have the opportunity to buy insurance coverage

- Coverage: up to \$10,000 in damages (full insurance).
- Cost: \$500 per year (actuarially fair)

If you purchase the insurance, income is the same whether you have an accident or not

- $I_1 = 50{,}000 - 500 = 49{,}500$
- $I_2 = 40{,}000 - 500 + 10{,}000 = 49{,}500$

…risk premium tells us how much a risk averse individual will pay to avoid risk beyond the actuarially fair insurance premium.

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A WORKED-OUT EXAMPLE…

You have a utility function given by $\boldsymbol{u(I) = I^{0.25}}$, where I is income.

You are considering two job opportunities.

1. The <u>first</u> pays a salary of $\mathbf{\$58,000}$ for sure.
2. The <u>second</u> pays a base salary of $\mathbf{\$20,000}$. However, it also offers the possibility of a $\mathbf{\$50,000}$ **bonus** on top of your base salary (for a total salary of $70,000).

   You believe that you will earn the bonus with a probability of $\mathbf{0.8}$.

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- $20,000 with probability 0.2
- $70,000 with probability 0.8

Use your utility function to prove that you are risk averse.

$u'(I) =$

$u''(I) =$

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- $20,000 with probability 0.2
- $70,000 with probability 0.8

Calculate the expected income of the second job offer.

$$E[I_{Job\ 2}] = 0.2 \times 20{,}000 + 0.8 \times 70{,}000$$

$$= 4{,}000 + 56{,}000$$

$$= \$60{,}000$$

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- $20,000 with probability 0.2
- $70,000 with probability 0.8

$E[I_{Job\ 2}] = \$60{,}000$

Calculate the utility from job 1.

$u(I_{Job\ 1}) = 58{,}000^{0.25} = \mathbf{15.519}$

Calculate the expected utility from job 2.

$$E[u(I_{Job\ 2})]$$

$$= 0.2 \times 20{,}000^{0.25} + 0.8 \times 70{,}000^{0.25}$$

$$= 0.2 \times 11.8921 + 0.8 \times 16.2658$$

$$= \mathbf{15.391}$$

Which job opportunity should you choose?

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- \$20,000 with probability 0.2
- \$70,000 with probability 0.8

$E[I_{Job\ 2}] = \$60{,}000$

$E[u(I_{Job\ 2})] = 15.391$

Calculate the certainty equivalent of job 2.

$$u(ce) = E[u(I_{Job\ 2})]$$

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- \$20,000 with probability 0.2
- \$70,000 with probability 0.8

$E[I_{Job\ 2}] = \$60{,}000$

$E[u(I_{Job\ 2})] = 15.391$

$ce = \$56{,}113.89$

Calculate the risk premium of job 2.

$$rp = E[I_{Job\ 2}] - ce$$

Do these results confirm your risk aversion?

YES!
Risk premium is positive

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- \$20,000 with probability 0.2
- \$70,000 with probability 0.8

$E[I_{Job\ 2}] = \$60{,}000$

$E[u(I_{Job\ 2})] = 15.391$

$ce = \$56{,}113.89$

$rp = \$3{,}886.11$

Suppose that there is insurance available that will protect you against the possibility you do not receive the bonus.

They pay out \$50,000 with probability 0.2.

How much would you be willing to pay for this insurance?

$$WTP = actuarially\ fair + rp$$

Reminder…

actuarially fair ins. = expected payout

A WORKED-OUT EXAMPLE…

$u(I) = I^{0.25}$

$I_{Job\ 1} = \$58{,}000$

$I_{Job\ 2}$ is either…

- \$20,000 with probability 0.2
- \$70,000 with probability 0.8

$E[I_{Job\ 2}] = \$60{,}000$

$E[u(I_{Job\ 2})] = 15.391$

$ce = \$56{,}113.89$

$rp = \$3{,}886.11$

Suppose that there is insurance available that will protect you against the possibility you do not receive the bonus.

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How much would you be willing to pay for this insurance?

Alternative method…

$$u(70{,}000 - max\ payment) = E[u(I_{Job\ 2})]$$