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## UTILITY MAXIMIZATION (A Calculus Approach)

#### Let's start by restating our consumer choice problem

Suppose George buys two goods, x and y, where prices per unit are  $P_x$  and  $P_y$ .

George has a utility function over x and y of u(x, y).

His income is *I*.

George's goal is to

Maximize u(x, y)

Subject to:  $P_x x + P_y y = I$ 

#### How should we solve this problem?

- One option: find optimal ratio using tangency  $(\frac{MU_X}{MU_Y} = \frac{P_X}{P_Y})$ , then plug into the constraint.
- o A second (potentially better) option: use the Lagrange multiplier method.

### LAGRANGE MULTIPLIER METHOD

Technique to maximize or minimize a function subject to one or more constraints.

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Subject to:  $P_x x + P_y y = I$ 

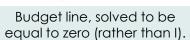
### Step 1: Form the Lagrangian

 $L = objective function - \lambda(constraint solve for zero)$ 



Function we are seeking to maximize (or minimize).

u(x, y)



$$P_x x + P_y y - I$$

$$\mathcal{L} = u(x, y) - \lambda (P_x x + P_y y - I)$$

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### LAGRANGE MULTIPLIER METHOD

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#### George's goal is to

### Step 2: Differentiate the Lagrangian

(first order conditions for a maximum)

Maximize u(x, y)

Subject to:  $P_x x + P_y y = I$ 

 $\frac{\partial \mathcal{L}}{\partial x} =$ 

1. Lagrangian:

 $\mathcal{L} = u(x, y) - \lambda (P_x x + P_y y - I)$ 

 $\frac{\partial \mathcal{L}}{\partial u} =$ 

 $\frac{\partial \mathcal{L}}{\partial \lambda} =$ 

### LAGRANGE MULTIPLIER METHOD

Technique to maximize or minimize a function subject to one or more constraints.

George's goal is to

Step 3: Solve the resulting equations

Maximize u(x, y)

Subject to:  $P_x x + P_y y = I$ 

1. Lagrangian

$$\mathcal{L} = u(x, y) - \lambda (P_x x + P_y y - I)$$

2. Derivatives

(1) 
$$\frac{\partial \mathcal{L}}{\partial x} = MU_x - \lambda P_x = 0$$

(2) 
$$\frac{\partial \mathcal{L}}{\partial y} = MU_y - \lambda P_y = 0$$

(3) 
$$\frac{\partial \mathcal{L}}{\partial \lambda} = -(P_x x + P_y y - I) = 0$$

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# WHAT DOES 2 TELL US?

Represents the extra utility generated when the budget constraint is relaxed... ...the marginal utility of income, also referred to as the "shadow price".

From previous work we know that...

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

So, we could say that...

$$MU_I = \lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

A derivation of this result can be found on Canvas

I will never ask you about this derivation.

### AN EXAMPLE USING THE LAGRANGE

Suppose that a consumer's utility function for two goods (x and y) is

$$u(x,y) = 10x^{0.5} + 2y$$

$$P_{x} = $5$$

$$P_{v} = $10$$

The consumer has total income of \$175

Goal: Determine the optimal basket

1. Write the statement of the constrained optimization problem.

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# AN EXAMPLE USING THE LAGRANGE

Maximize  $10x^{0.5} + 2y$ s.t. 5x + 10y = 175

2. Use the Lagrangian to solve the utility maximization problem.

$$\mathcal{L} = 10x^{0.5} + 2y - \lambda(5x + 10y - 175)$$

$$\frac{\partial \mathcal{L}}{\partial x} =$$

$$\frac{\partial \mathcal{L}}{\partial v} =$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} =$$

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$$P_{x} = $5$$

$$P_y = $10$$

The consumer has total income of \$175

$$\lambda = \frac{1}{5}, \ x = 25, \ y = 5$$

More ways to find  $MU_I$ 

$$MU_x = \frac{5}{\sqrt{x}} = \frac{5}{\sqrt{25}} = 1$$
  $\frac{MU_x}{P_x} = \frac{1}{5}$   $MU_y = 2$   $\frac{MU_y}{P_y} = \frac{2}{10} = \frac{1}{5}$ 

3. Interpret the Lagrange multiplier.

4. What type of utility function is this?