Applied Statistics

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Fill me in later

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Introduction

`R` is both a programming language and software environment for statistical computing, which is *free* and *open-source*. To get started, you will need to install two pieces of software:

- `R`, the actual programming language, which can be installed from http://cran.r-project.org/
  - Chose your operating system, and select the most recent version. (As of writing, 3.3.0.)
- RStudio, an excellent IDE for working with `R`, which can be obtained from http://www.rstudio.com/ (Note, you must have `R` installed to use RStudio. RStudio is simply a way to interact with `R`.)

`R`'s popularity is on the rise, and everyday it becomes a better tool for statistical analysis. It even generated this document! (A skill you will learn in this coruse.) To get started, we'll use `R` like a simple calculator. Note, in `R` the `#` symbol is used for comments. Lines which begin with two, `##` will indicate output.

There are many good resoruces for learning `R`. They are not necessary for this course, but you may find them useful if you would like a deeper understandinf of `R`:

- Try R from Code School.
  - An interactive introduction to the basics of `R`. Could be very useful for getting up to speed on `R`'s syntax.
- The Art of `R` Programming by Norman Matloff
  - Gentle introduction to the programming side of `R`. (Whereas we will focus more on the data anlaysis side.) Free electronic version availible through Illinois library.
- Advanced `R` by Hadley Wickham
  - From the author of several extremely popular `R` packages. Good follow-up to The Art of `R` Programming. (And more up-to-date material.)
- The R Inferno by Patrick Burns
  - Likens learning the tricks of `R` to descending through the levels of hell. Very advanced material, but may be important if `R` becomes a part of your everyday toolkit.

RStudio has a large number of useful keyboard shortcuts. A list of these can be found using a keyboard shortcut, the keyboard shortcut to rule them all:

- On Windows: `Option` + `Shift` + `K`
- On Mac: `Alt` + `Shift` + `K`

The RStudio team has developed a number of "cheatsheets" for working with both `R` and RStudio which can be found here or from the help menu inside of RStudio. This one for Base `R` in particular will summarize many of the concepts in this document.

Chapter 4

Basic Calculations

R's most most basic funtion is that of a simple calculator.

- Addition, Subtraction, Multiplication and Division

```
3 + 2
```

```
## [1] 5
```

```
3 - 2
```

```
## [1] 1
```

```
3 * 2
```

```
## [1] 6
```

```
3 / 2
```

```
## [1] 1.5
```

- Exponents

```
3 ^ 2
```

```
## [1] 9
```

```
2 ^ (-3)
```

```
## [1] 0.125
```

```
100 ^ (1 / 2)
```

```
## [1] 10
```

```r
sqrt(1 / 2)
```

```
## [1] 0.7071068
```

```r
exp(1)
```

```
## [1] 2.718282
```

- Mathematical Constants

```r
pi
```

```
## [1] 3.141593
```

```r
exp(1)
```

```
## [1] 2.718282
```

- Logarithms

```r
log(10) # natural log
```

```
## [1] 2.302585
```

```r
log10(1000) # base 10 log
```

```
## [1] 3
```

```r
log2(8) # base 2 log
```

```
## [1] 3
```

```r
log(16, base = 4) # base 4 log
```

```
## [1] 2
```

- Trigonometry

```r
sin(pi / 2)
```

```
## [1] 1
```

```r
cos(0)
```

```
## [1] 1
```

Chapter 5

Getting Help

In using `R` as a calculator, we have seen a number of functions. `sqrt()`, `exp()`, `log()` and `sin()` are all `R` functions. To get documentation about a function in `R`, simply put a question mark in front of the function name and RStudio will display the documentation, for example:

```
?log
?sin
?paste
?lm
```

TODO: how to ask for help

Chapter 6

Vectors

Many operations in `R` make heavy use of vectors. Note that vectors in `R` are indexed starting at 1.

TODO: Note about [1] in output. Ouput a big vector?

TODO: make a vector TODO: assignment

```
x <- c(1, 3, 5, 7, 8, 9)
x
```

```
## [1] 1 3 5 7 8 9
```

TODO: vector sequence

```
y <- 1:20
y
```

```
##  [1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
```

TODO: fine control sequence TODO: directly output

```
(z <- seq(1, 2, 0.1))
```

```
##  [1] 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
```

TODO: add `rep`

TODO: Accessing elements

```
x
```

```
## [1] 1 3 5 7 8 9
```

```
x[3]
```

```
## [1] 5
```

```
x[1:3]
```

```
## [1] 1 3 5
```

```
x[-2]
```

```
## [1] 1 5 7 8 9
```

One of the biggest strengths of `R` is its use of vectorized operations. (Frequently the lack of understanding of this concept leads of a belief that `R` is *slow*. `R` isn't the fastest language, but it has a reputation for being slower than it really is.)

```
x <- 1:10
x + 1
```

```
##  [1]  2  3  4  5  6  7  8  9 10 11
```

```
2 * x
```

```
##  [1]  2  4  6  8 10 12 14 16 18 20
```

```
2 ^ x
```

```
##  [1]    2    4    8   16   32   64  128  256  512 1024
```

```
sqrt(x)
```

```
##  [1] 1.000000 1.414214 1.732051 2.000000 2.236068 2.449490
##  [7] 2.645751 2.828427 3.000000 3.162278
```

```
log(x)
```

```
##  [1] 0.0000000 0.6931472 1.0986123 1.3862944 1.6094379 1.7917595
##  [7] 1.9459101 2.0794415 2.1972246 2.3025851
```

We see that when a function like `log()` is called on a vector `x`, a vector is returned which has applied the function to each element of the vector `x`.

Chapter 7

Matrix Calculations

`R` can also be used for matrix calculations. Matrices can be created using the `matrix` function.

TODO: matrix all same "data"" type. "order matters". has rows and columns

By default the `matrix` function reorders a vector into columns, but we can also tell `R` to use rows instead.

```
x <- 1:9
x
```

```
## [1] 1 2 3 4 5 6 7 8 9
```

```
X <- matrix(x, nrow = 3, ncol = 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```
Y <- matrix(x, nrow = 3, ncol = 3, byrow = TRUE)
Y
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
```

```
Z <- matrix(0, 2, 4)
Z
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

```
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```
X[1, 2]
```

```
## [1] 4
```

```
X[1, ]
```

```
## [1] 1 4 7
```

```
X[, 2]
```

```
## [1] 4 5 6
```

```
X[2, c(1, 3)]
```

```
## [1] 2 8
```

Matrices can also be created by combining vectors as columns, using `cbind` or combining vectors as rows using `rbind`.

```
x <- 1:9
rev(x)
```

```
## [1] 9 8 7 6 5 4 3 2 1
```

```
rep(1, 9)
```

```
## [1] 1 1 1 1 1 1 1 1 1
```

```
cbind(x, rev(x), rep(1, 9))
```

```
##       x
##  [1,] 1 9 1
##  [2,] 2 8 1
##  [3,] 3 7 1
##  [4,] 4 6 1
##  [5,] 5 5 1
##  [6,] 6 4 1
##  [7,] 7 3 1
##  [8,] 8 2 1
##  [9,] 9 1 1
```

```r
rbind(x, rev(x), rep(1, 9))
```

```
##   [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
## x    1    2    3    4    5    6    7    8    9
##      9    8    7    6    5    4    3    2    1
##      1    1    1    1    1    1    1    1    1
```

R can then be used to perform matrix calculations.

```r
x <- 1:9
y <- 9:1
X <- matrix(x, 3, 3)
Y <- matrix(y, 3, 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```r
Y
```

```
##      [,1] [,2] [,3]
## [1,]    9    6    3
## [2,]    8    5    2
## [3,]    7    4    1
```

```r
X + Y
```

```
##      [,1] [,2] [,3]
## [1,]   10   10   10
## [2,]   10   10   10
## [3,]   10   10   10
```

```r
X - Y
```

```
##      [,1] [,2] [,3]
## [1,]   -8   -2    4
## [2,]   -6    0    6
## [3,]   -4    2    8
```

```r
X * Y
```

```
##      [,1] [,2] [,3]
## [1,]    9   24   21
## [2,]   16   25   16
## [3,]   21   24    9
```

```
X / Y
```

```
##           [,1]      [,2]     [,3]
## [1,] 0.1111111 0.6666667 2.333333
## [2,] 0.2500000 1.0000000 4.000000
## [3,] 0.4285714 1.5000000 9.000000
```

Note that `X * Y` is not matrix multiplication. It is element by element multiplication. (Same for `X / Y`). Instead, matrix multiplication uses `%*%`. `t()` gives the transpose of a matrix, and `solve()` returns the inverse of a matrix.

```
X %*% Y
```

```
##      [,1] [,2] [,3]
## [1,]   90   54   18
## [2,]  114   69   24
## [3,]  138   84   30
```

```
t(X)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    4    5    6
## [3,]    7    8    9
```

```
Z <- matrix(c(9, 2, -3, 2, 4, -2, -3, -2, 16), 3, byrow = T)
Z
```

```
##      [,1] [,2] [,3]
## [1,]    9    2   -3
## [2,]    2    4   -2
## [3,]   -3   -2   16
```

```
solve(Z)
```

```
##             [,1]        [,2]       [,3]
## [1,]  0.12931034 -0.05603448 0.01724138
## [2,] -0.05603448  0.29094828 0.02586207
## [3,]  0.01724138  0.02586207 0.06896552
```

```
X <- matrix(1:6, 2, 3)
X
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    5
## [2,]    2    4    6
```

```r
dim(X)
```

```
## [1] 2 3
```

```r
rowSums(X)
```

```
## [1]  9 12
```

```r
colSums(X)
```

```
## [1]  3  7 11
```

```r
rowMeans(X)
```

```
## [1] 3 4
```

```r
colMeans(X)
```

```
## [1] 1.5 3.5 5.5
```

```r
diag(Z)
```

```
## [1]  9  4 16
```

```r
diag(1:5)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    0    0
## [2,]    0    2    0    0    0
## [3,]    0    0    3    0    0
## [4,]    0    0    0    4    0
## [5,]    0    0    0    0    5
```

```r
diag(5)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    0    0
## [2,]    0    1    0    0    0
## [3,]    0    0    1    0    0
## [4,]    0    0    0    1    0
## [5,]    0    0    0    0    1
```

Chapter 8

Distributions

When working with different statistical distributions, we often want to make probabilistic statements based on the distribution.

We typically want to know one of four things:

- The density (pdf) value at a particular value of `x`.
- The distribution (cdf) value at a particular value of `x`.
- The quantile `x` value corresponding to a particular probability.
- A random value from a particular distribution.

This used to be done with statistical tables printed in the back of textbooks. Now, R has functions for obtaining density, distribution, quantile and random values.

The general naming structure of the relevant R functions is:

- `dname` calculates density (pdf) value at input `x`.
- `pname` calculates distribution (cdf) value at input `x`.
- `qname` calculates quantile `x` value at input probability.
- `rname` generates a random value from a particular distribution.

Note that `name` represents the name of the given distribution.

For example, to calculate the value of the pdf for a $N(2, 25)$ for `x = 3`, use:

```
dnorm(3, mean = 2, sd = 5)
```

```
## [1] 0.07820854
```

Or, to calculate the value of the cdf for a $N(2, 25)$ for `x = 3`, use:

```
pnorm(3, mean = 2, sd = 5)
```

```
## [1] 0.5792597
```

Or, to calculate the quantile for probability 0.975, use:

```r
qnorm(0.975, mean = 2, sd = 5)
```

```
## [1] 11.79982
```

Lastly, to generate a random sample of size `n = 10`, use:

```r
rnorm(10, mean = 2, sd = 5)
```

```
##  [1]  7.382739 -3.506444  6.119234  6.036602 11.937247  5.376396
##  [7]  3.299657  7.078087  6.055764  2.017482
```

These functions exist for many other distributions, including but not limited to:

Command	Distribution
`*binom`	Binomial
`*t`	t
`*pois`	Poisson
`*f`	F
`*chisq`	Chi-Squared

Where `*` can be `d`, `p`, `q`, and `r`.

Chapter 9

Programming Basics

9.1 Logical Operators

Operator	Summary
`<`	Less than
`>`	Greater than
`<=`	Less than or equal to
`>=`	Greater than or equal to
`==`	Equal to
`!=`	Not equal to
`!x`	NOT x
`x | y`	x OR y
`x & y`	x AND y

In R, logical operators are vectorized.

```
heights <- c(110, 120, 115, 136, 205, 156, 175)
weights <- c(64, 67, 62, 60, 77, 70, 66)
heights < 121 | heights == 156
```

```
## [1]  TRUE  TRUE  TRUE FALSE FALSE  TRUE FALSE
```

```
weights[heights > 150]
```

```
## [1] 77 70 66
```

In R, the if/else syntax is:

```
if (...) {
  some R code
} else {
  more R code
}
```

For example,

```
x <- 1
y <- 3
if (x > y) {
  z <- x * y
  print("x is larger than y")
} else {
  z <- x + 5 * y
  print("x is less than or equal to y")
}
```

```
## [1] "x is less than or equal to y"
```

```
z
```

```
## [1] 16
```

TODO: ifelse

Now a `for` loop example,

```
x <- 11:15
for (i in 1:5) {
  x[i] <- x[i] + 1
}

x
```

```
## [1] 12 13 14 15 16
```

Note that this `for` loop is very normal in many programming languages, but not in `R`. In `R` we would not use a loop, instead we would simply use a vectorized operation:

```
x <- 11:15
x <- x + 1
x
```

```
## [1] 12 13 14 15 16
```

Lastly, we can write our own functions in `R`. For example,

```
standardize <- function(x) {
  m <- mean(x)
  std <- sd(x)
  result <- (x - m) / std
  result
}

x <- rnorm(10, 2, 25)
standardize(x)
```

```
##  [1] -1.32701248  0.47991303  1.82844465  0.07662533 -1.45444869
##  [6] -0.87337561  0.59854062 -0.00797926  0.64980787  0.02948454
```

TODO: function with arguments, control flow, if based return, how return works

```
get_sd <- function(y, biased = FALSE) {
  n <- length(y)
  if (biased) {
    std <- sqrt((1 / n) * sum((y - mean(y)) ^ 2))
  } else {
    std <- sqrt((1 / (n - 1)) * sum((y - mean(y)) ^ 2))
  }
  std
}
```

TODO: Potentially save this for the SLR document, since it is already somewhat there.

Chapter 10

Data Frames

TODO: data frames. have observations and variables

Chapter 11

Importing Data

TODO: read() or RStudio

Chapter 12

Scatter Plots

Chapter 13

Hypothesis Tests in R

13.1 One Sample t Test: Review

Suppose $x_i \sim \mathrm{N}(\mu, \sigma^2)$ and we want to test $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$.

Assuming σ is unknown, use the one-sample Student's t test statistic:

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}$$

where $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ and $s = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

A $100(1-\alpha)\%$ CI for μ is given by

$$\bar{x} \pm t_{n-1}^{(\alpha/2)} \frac{s}{\sqrt{n}}$$

where $t_{n-1}^{(\alpha/2)}$ is the critical value such that $P\left(T > t_{n-1}^{(\alpha/2)}\right) = \alpha/2$ for $n-1$ degrees of freedom.

13.2 One Sample t Test: Example

A store sells "16-ounce" boxes of *Captain Crisp* cereal. A random sample of 9 boxes was taken and weighed. The results were

$$15.5 \quad 16.2 \quad 16.1 \quad 15.8 \quad 15.6 \quad 16.0 \quad 15.8 \quad 15.9 \quad 16.2$$

ounces. Assume the weight of cereal in a box is normally distributed.

a) Compute the sample mean $\bar{x}$ and the sample standard deviation s.

$$
\begin{aligned}
\bar{x} &= \frac{1}{n}\sum_{i=1}^{n} x_i = (1/9)(15.5 + \cdots + 16.2) = (1/9)(143.1) = \mathbf{15.9} \\
s^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right] \\
&= (1/8)\left[2275.79 - 9(15.9^2)\right] = (1/8)(0.5) = 0.0625 \\
s &= \sqrt{0.0625} = \mathbf{0.25}
\end{aligned}
$$

```
x <- c(15.5, 16.2, 16.1, 15.8, 15.6, 16.0, 15.8, 15.9, 16.2)
mean(x)
```

```
## [1] 15.9
```

```
sd(x)
```

```
## [1] 0.25
```

b) Construct a 95% confidence interval for the overall average weight of boxes of *Captain Crisp* cereal.

$t_{n-1}^{(\alpha/2)} = t_8^{(0.025)} = 2.306$, so the 95% CI for the average weight of a cereal box is:

$$15.9 \pm 2.306\sqrt{\frac{0.0625}{9}} = [15.708, 16.092]$$

Or, in R:

```
t.test(x, alternative = c("two.sided"), conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  x
## t = 190.8, df = 8, p-value = 6.372e-16
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  15.70783 16.09217
## sample estimates:
## mean of x
##      15.9
```

Or if we only wanted to display the interval:

```
tt <- t.test(x, alternative = c("two.sided"), conf.level = 0.95)
tt$conf.int
```

```
## [1] 15.70783 16.09217
## attr(,"conf.level")
## [1] 0.95
```

Or, we could calculate it "by hand" in R.

```
qt(0.975, 8)
```

```
## [1] 2.306004
```

```
c(mean(x) - qt(0.975, 8) * sd(x) / sqrt(9),
  mean(x) + qt(0.975, 8) * sd(x) / sqrt(9))
```

```
## [1] 15.70783 16.09217
```

c) The company that makes *Captain Crisp* cereal claims that the average weight of its box is at least 16 ounces. Use a 0.05 level of significance to test the company's claim. What is the p-value of this test?

To test $H_0 : \mu \geq 16$ versus $H_1 : \mu < 16$, the test statistic is

$$T = \frac{15.9 - 16}{\sqrt{0.0625/9}} = -1.2$$

We know that $T \sim t_8$, so the rejection reject is $T < -t_{n-1}^{(\alpha)} = -t_8^{(0.05)} = -1.860$.

Therefore, we **do NOT reject the null hypothesis** at the $\alpha = .05$ level. We could have also bounded the p-value of the test using the t table.

```
t.test(x, mu = 16, alternative = c("less"), conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  x
## t = -1.2, df = 8, p-value = 0.1322
## alternative hypothesis: true mean is less than 16
## 95 percent confidence interval:
##      -Inf 16.05496
## sample estimates:
## mean of x
##      15.9
```

13.3 Two Sample t Test: Review

Suppose $x_i \sim \mathrm{N}(\mu_x, \sigma^2)$ and $y_i \sim \mathrm{N}(\mu_y, \sigma^2)$.

Want to test $H_0 : \mu_x - \mu_y = \mu_0$ versus $H_1 : \mu_x - \mu_y \neq \mu_0$.

Assuming σ is unknown, use the two-sample Student's t test statistic:

$$T = \frac{(\bar{x} - \bar{y}) - \mu_0}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}} \sim t_{n+m-2}$$

where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $\bar{y} = \frac{\sum_{i=1}^{m} y_i}{m}$, and $s_p^2 = \frac{(n-1)s_1^2 + (m-1)s_2^2}{n+m-2}$

A $100(1-\alpha)\%$ CI for $\mu_x - \mu_y$ is given by

$$(\bar{x} - \bar{y}) \pm t_{n+m-2}^{(\alpha/2)} \left(s_p \sqrt{\tfrac{1}{n} + \tfrac{1}{m}} \right)$$

where $t_{n+m-2}^{(\alpha/2)}$ is critical t_{n+m-2} value such that $P\left(T > t_{n+m-2}^{(\alpha/2)}\right) = \alpha/2$.

13.4 Two Sample t Test: Example

Assume that the distributions of X and Y are $\mathrm{N}(\mu_1, \sigma^2)$ and $\mathrm{N}(\mu_2, \sigma^2)$, respectively. Given the $n = 6$ observations of X,

$$70, \quad 82, \quad 78, \quad 74, \quad 94, \quad 82$$

and the $m = 8$ observations of Y,

$$64, \quad 72, \quad 60, \quad 76, \quad 72, \quad 80, \quad 84, \quad 68$$

find the p-value for the test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 > \mu_2$.

First, note that the sample means and variances are given by

$$\begin{aligned}
\bar{x} &= (1/6) \textstyle\sum_{i=1}^{6} x_i = (1/6)480 = 80 \\
\bar{y} &= (1/8) \textstyle\sum_{i=1}^{8} y_i = (1/8)576 = 72 \\
s_x^2 &= (1/5) \textstyle\sum_{i=1}^{6} (x_i - \bar{x})^2 = (1/5)344 = 68.8 \\
s_y^2 &= (1/7) \textstyle\sum_{i=1}^{8} (y_i - \bar{y})^2 = (1/7)448 = 64
\end{aligned}$$

which implies that the pooled variance estimate is given by

$$\begin{aligned}
s_p^2 &= \frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2} \\
&= \frac{344 + 448}{12} \\
&= 66
\end{aligned}$$

Thus, the relevant t test statistic is given by

$$\begin{aligned}
T &= \frac{(\bar{x} - \bar{y}) - \mu_0}{s_p \sqrt{\frac{1}{n} + \frac{1}{m}}} \\
&= \frac{(80 - 72) - 0}{\sqrt{66}\sqrt{\frac{1}{6} + \frac{1}{8}}} \\
&= 1.82337
\end{aligned}$$

Note that $T \sim t_{12}$, so

$$0.025 < p - value < 0.05$$

since

$$t_{12}^{(0.025)} = 1.782 < 1.82337 < t_{12}^{(0.05)} = 2.179.$$

```
x <- c(70, 82, 78, 74, 94, 82)
y <- c(64, 72, 60, 76, 72, 80, 84, 68)
t.test(x, y, alternative = c("greater"), var.equal = TRUE)
```

```
##
##  Two Sample t-test
##
## data:  x and y
## t = 1.8234, df = 12, p-value = 0.04662
## alternative hypothesis: true difference in means is greater than 0
## 95 percent confidence interval:
##  0.1802451       Inf
## sample estimates:
## mean of x mean of y
##        80        72
```

Or, performing the calculations `by hand' inR`:

```
sPooled2 <- ((6 - 1) * var(x) + (8 - 1) * var(y)) / (6 + 8 - 2)
sPooled2
```

```
## [1] 66
```

```
test_stat <- (mean(x) - mean(y)) / sqrt(sPooled2 * (1 / 6 + 1 / 8))
test_stat
```

```
## [1] 1.823369
```

```
1 - pt(test_stat, 6 + 8 - 2)
```

```
## [1] 0.04661961
```

Chapter 14

Simulation in R

14.1 Paired Differences

Consider the model:

$$\begin{aligned} X_{11}, X_{12}, \ldots, X_{1n} &\sim N(\mu_1, \sigma^2) \\ X_{21}, X_{22}, \ldots, X_{2n} &\sim N(\mu_2, \sigma^2) \end{aligned}$$

Assume that $\mu_1 = 6$, $\mu_2 = 5$, $\sigma^2 = 4$ and $n = 25$.

Let $\bar{X}_1 = \frac{1}{n}\sum_{i=1}^{n} X_{1i}$, $\bar{X}_2 = \frac{1}{n}\sum_{i=1}^{n} X_{2i}$ and $D = \bar{X}_1 - \bar{X}_2$.

Find $P(0 < D < 2)$**.**

$$D = \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma^2}{n} + \frac{\sigma^2}{n}\right) = N\left(6 - 5, \frac{4}{25} + \frac{4}{25}\right)$$

So,

$$D \sim N(1, 0.32)$$

Thus,

$$P(0 < D < 2) = P(1.77 < Z < 1.77) = 0.9616 - 0.0384 = 0.9232.$$

```
z <- 1 / sqrt(0.32)
pnorm(z) - pnorm(-z)
```

```
## [1] 0.9229001
```

Empirical distribution of D

Generate $S = 1000$ datasets for each of group 1 and group 2. For each of the $s = 1 : 1000$ datasets, compute $d_s = \bar{x}_{1s} - \bar{x}_{2s}$. Make a histogram for the 1000 values of d. What is the proportion of values of d (among the 1000 values of d generated) that are between 0 and 2?

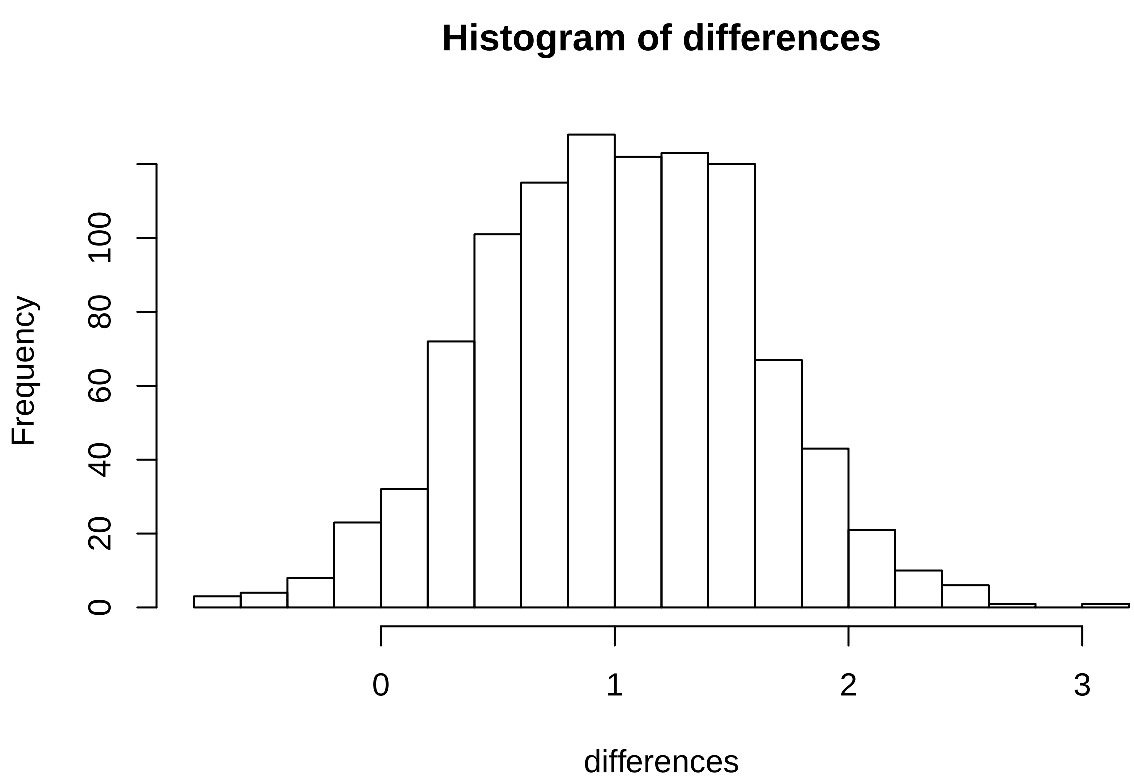
```
set.seed(42)
sampleSize <- 25
mu1 <- 6
mu2 <- 5
std <- 2

samples <- 1000
count <- 0
differences <- c(1:samples)

for (i in 1:samples) {
  x1 <- rnorm(sampleSize, mu1, std)
  x2 <- rnorm(sampleSize, mu2, std)
  differences[i] <- mean(x1) - mean(x2)
  if ((differences[i] > 0) & (differences[i] < 2)) {
  count <- count + 1
  }
}

count / samples
```

```
## [1] 0.923
```

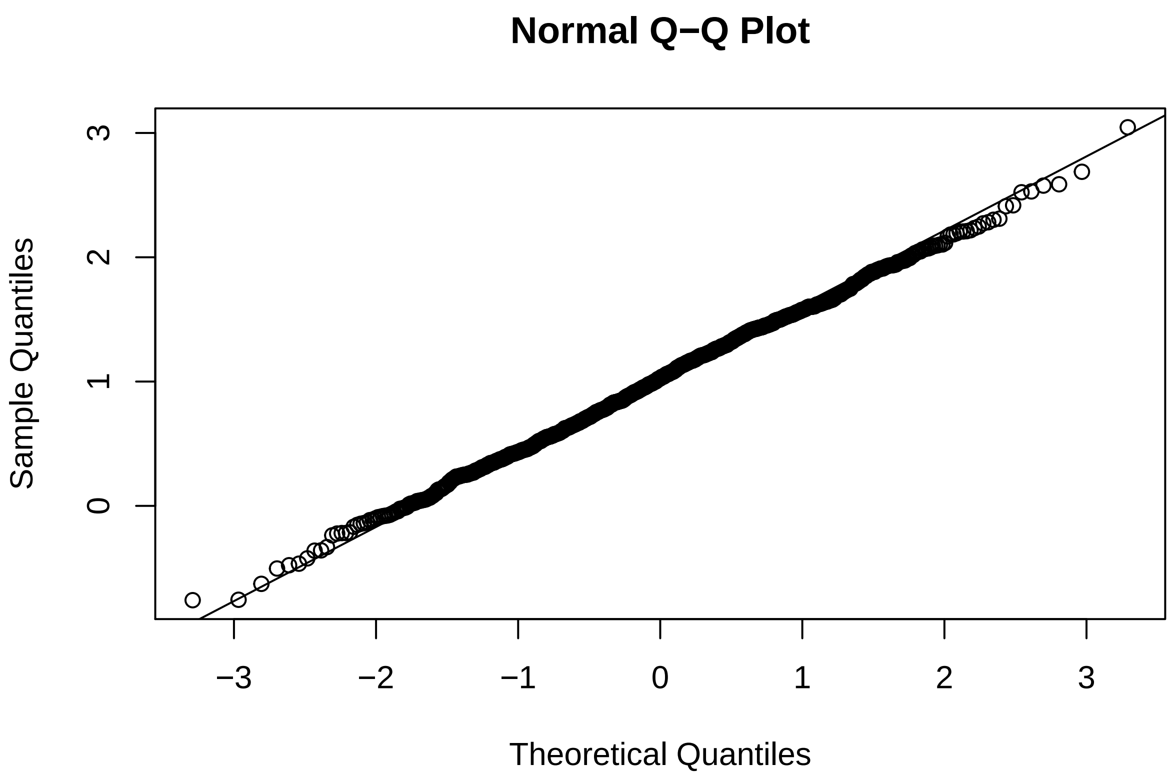
```
mean(0 < differences & differences < 2)
```

```
## [1] 0.923
```

```
hist(differences, breaks = 20)
```

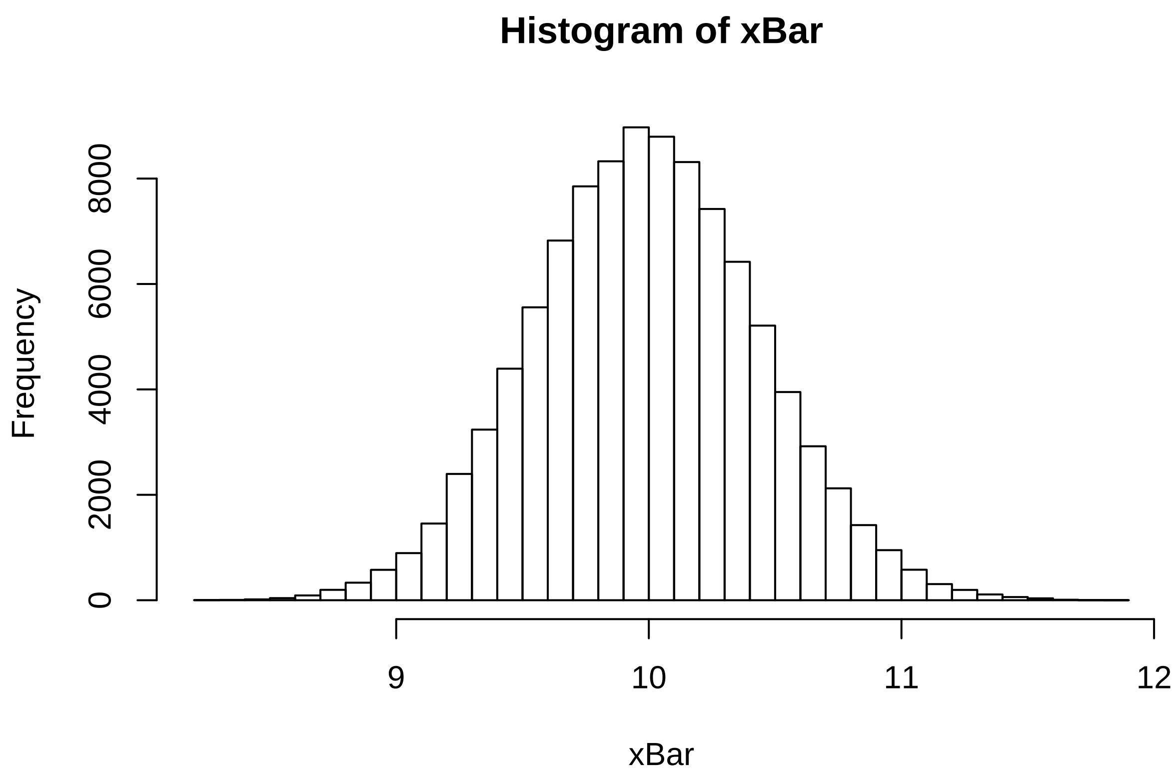


```
qqnorm(differences)
qqline(differences)
```



14.2 Distribution of $\bar{x}$

```
sampleSize <- 50
mu <- 10
samples <- 100000
xBar <- rep(0, samples)

for(i in 1:samples){
  xBar[i] <- mean(rpois(sampleSize, lambda = mu))
}

hist(xBar, breaks = 50)
```




```
c(mean(xBar), mu)
```

```
## [1] 10.00015 10.00000
```

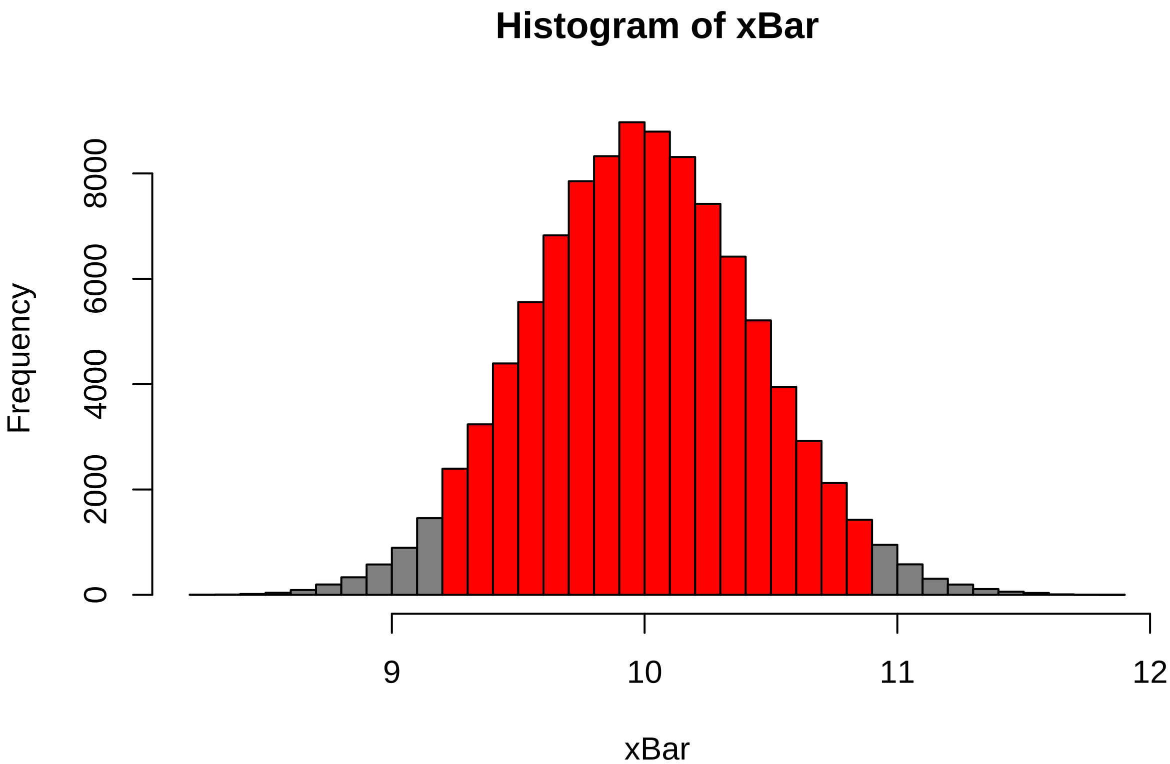
```
c(sd(xBar), sqrt(mu) / sqrt(sampleSize))
```

```
## [1] 0.4470015 0.4472136
```

```
mean(xBar > mu - 2 * sqrt(mu) / sqrt(sampleSize) &
       xBar < mu + 2 * sqrt(mu) / sqrt(sampleSize))
```

```
## [1] 0.95389
```

```
histgm <- hist(xBar, breaks = 50, plot = FALSE)
plot(
  histgm,
  col = ifelse(
    histgm$breaks > mu - 2 * sqrt(mu) / sqrt(sampleSize) &
      histgm$breaks < mu + 2 * sqrt(mu) / sqrt(sampleSize),
    "red",
    "gray50"
  )
)
```



"All models are wrong, but some are useful."
- George E. P. Box

Chapter 15

Motivating Example

Suppose you are the owner of the *Momma Leona's Pizza*, a restaurant chain located near several college campuses. You currently own 10 stores and have data on the size of the student population as well as quarterly sales for each. Your data is summarized in the table below.

Restaurant, i	Student Population, x_i, in 1000s	Quarterly Sales, y_i, in \$1000s
1	2	58
2	6	105
3	8	88
4	8	118
5	12	117
6	16	137
7	20	157
8	20	169
9	22	149
10	26	202

How can you use this data to

- Explain relationship
- Predict

One tool will do both, LINEAR REGRESSION

Chapter 16

WH_____ ?s

Chapter 17

Model

Chapter 18

What is the Best Line?

historical, easy math

Chapter 19

Least Squares Approach

Chapter 20

MLE Approach

Chapter 21

Example

Chapter 22

R

Chapter 23

Decomposition

Chapter 24

R^2

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 + \sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2$$

Bibliography