Unit I - INTRODUCTION TO STATICS & FORCE SYSTEMS

Chapter 1 - INTRODUCTION TO STATICS

1. Explain the terms: a) Space b) Time c) Mass d) Force e) Particle f) Rigid Body

a) Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

b) Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

c) Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

d) Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity.

e) *Particle* is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

f) Rigid body A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, which normally follows statics in the curriculum.

2. Explain the difference between Scalars and Vectors.

Scalar quantities are those with which only a magnitude is associated.
Examples of scalar quantities are time, volume, density, speed, energy, and mass.

Vector quantities, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition.
Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Vectors representing physical quantities can be classified as free, sliding, or fixed.

3. Explain the terms a) Free Vector b) Sliding Vector c) Fixed Vector

a) ***Free Vector*** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

b) ***Sliding Vector*** has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole *(principle of transmissibility)*, and thus it is a sliding vector.

c) ***Fixed Vector*** is one for which a unique point of application is specified. The action of a force on a deformable or non rigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

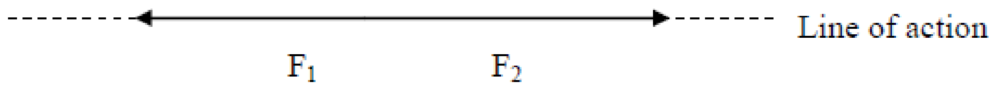
Chapter 2 - FORCE SYSTEMS

Topic - Resolution of force

4. Explain the terms (with sketches) a. Collinear Forces b. Coplanar parallel Forces c. Coplanar Concurrent Forces d. Coplanar Non-Concurrent Forces e. Non-Coplanar parallel Forces f. Non-Coplanar Concurrent Forces g. Non-Coplanar Non-Concurrent Forces.

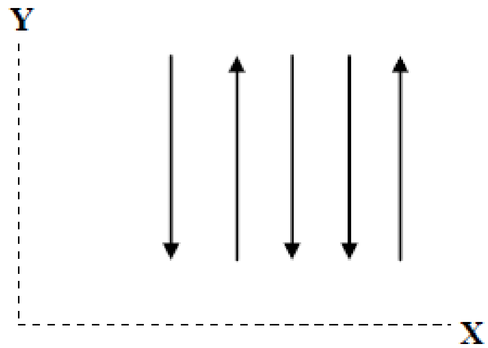
Classification of force systems: Depending upon their relative positions, points of applications and lines of actions, the different force systems can be classified as follows.

1) **Collinear forces**: It is a force system, in which all the forces have the same line of action.



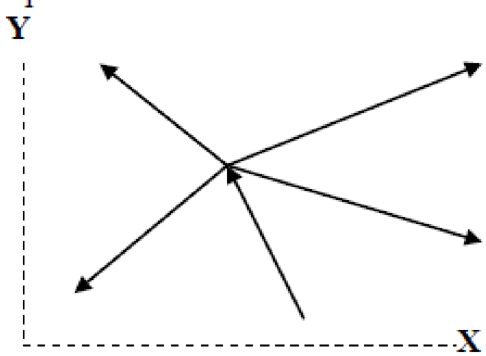
Ex.: Forces in a rope in a tug of war.

2) **Coplanar parallel forces**: It is a force system, in which all the forces are lying in the same plane and have parallel lines of action.



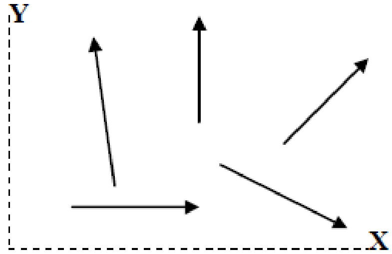
Ex.: The forces or loads and the support reactions in case of beams.

3) Coplanar Concurrent forces: It is a force system, in which all the forces are lying in the same plane and lines of action meet a single point.



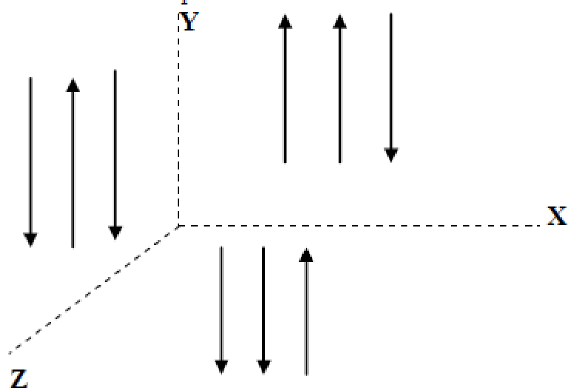
Ex.: The forces in the rope and pulley arrangement.

4) **Coplanar non-concurrent forces:** It is a force system, in which all the forces are lying in the same plane but lines of action do not meet a single point.



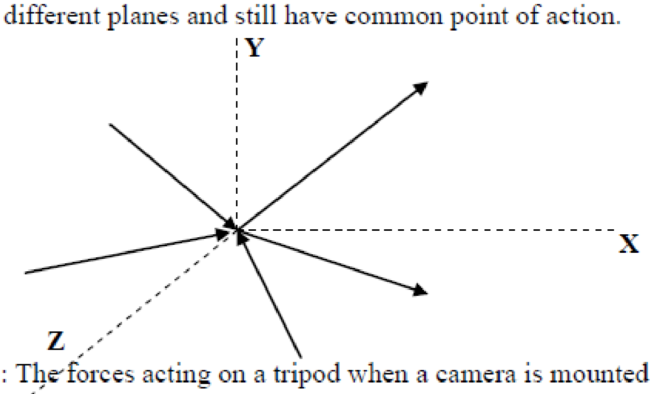
Ex.: Forces on a ladder and reactions from floor and wall, when a ladder rests on a floor and leans against a wall.

5) **Non- coplanar parallel forces**: It is a force system, in which all the forces are lying in the different planes and still have parallel lines of action.



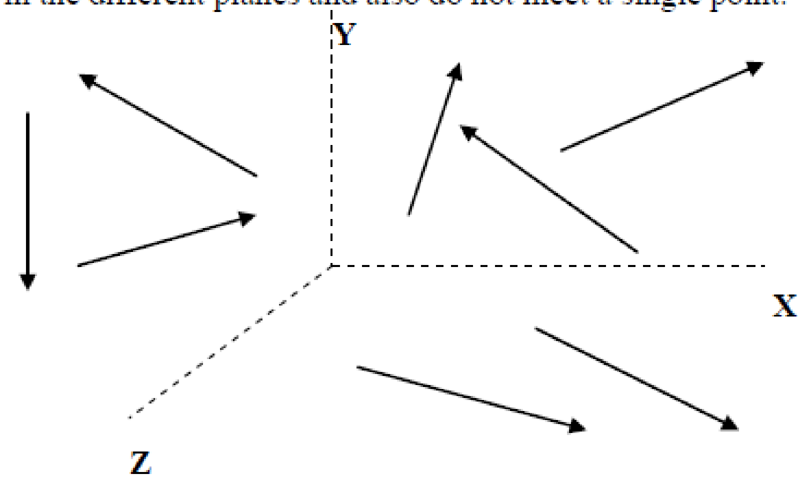
Ex: The forces acting and the reactions at the points of contact of bench with floor in a classroom.

6) **Non- coplanar concurrent forces** It is a force system, in which all the forces are lying in the different planes and still have common point of action.



Ex.: The forces acting on a tripod when a camera is mounted on a tripod.

7) **Non- coplanar non-concurrent forces**: It is a force system, in which all the forces are lying in the different planes and also do not meet a single point.



Ex.: Forces acting on a building frame.

5. What are (Explain with Sketches) a) Rectangular Components of a Force b) Components of a Force c) Projections of a Force?

a) Rectangular Components of a force

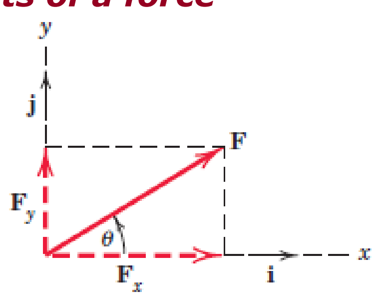


Figure. 5 a.

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of Fig.6 a) may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of **F** in the *x*- and *y*-directions.

Each of the two vector components may be written as a scalar times the appropriate unit vector. In terms of the unit vectors are **i** and **j**, $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\,\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 5 a, the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$F_x = F\cos\theta \qquad F = \sqrt{{F_x}^2 + {F_y}^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

b) Components of a Force

We often need to replace a force by its vector components in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force R in figure 5 b may be replaced by or resolved into, two vector components F_1 and F_2 with the specified directions by completing the parallelogram as shown to obtain the magnitudes of F_1 and F_2.

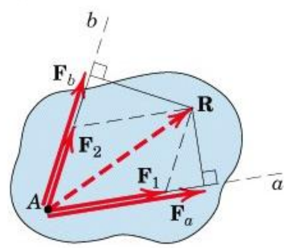


Figure. 5 b

c) Projections of a Force

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular projections onto the same axes. Figure 6 b) shows the perpendicular projection P_a and P_b of the given force R onto axes a and b, which are parallel to the vector components F1 and F2

6. Explain the Transmissibility of a force with a neat sketch.

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Figure 6 may be applied at *A* or at *B* or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at *O* and the force exerted on the plate by the roller support at *C*.

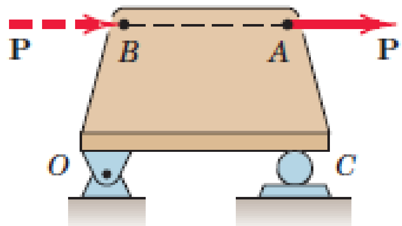


Figure. 6

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*.

7. Define Force and State its Characteristics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action.

The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*.

Topic - Moment

8. Explain the term Moment of a force with neat sketch

The rotation effect caused by force about a point is called as moment of force about that point.

Moment of force= Force X Perpendicular distance

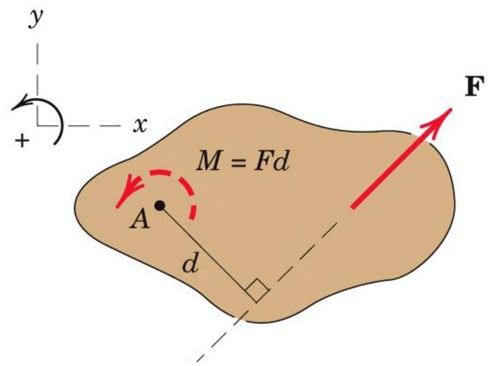


Figure 8

As a familiar example of the concept of moment, consider the Figure 8 One effect of the force applied perpendicular, is the tendency to rotate about point A. The magnitude of this tendency depends on both the magnitude *F* of the force and the perpendicular length *d*.

9. State and Prove the Varignon's theorem.

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

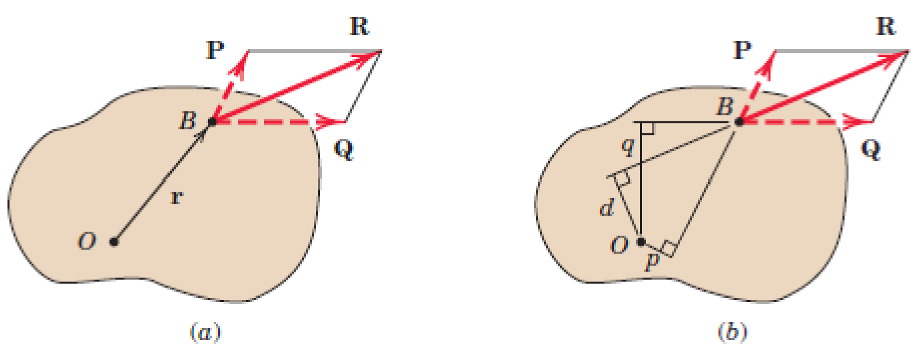


Figure 9

To prove this theorem, consider the force **R** acting in the plane of the body shown in Figure 9 (a). The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point *O* is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$, we may write

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

Using the distributive law for cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

Which says that the moment of **R** about *O* equals the sum of the moments about *O* of its components **P** and **Q**. This proves the theorem.
Figure 9 (*b)* illustrates the usefulness of Varignon's theorem. The moment of **R** about point *O* is *Rd*. However, if *d* is more difficult to determine than *p* and *q*, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

Topic - Couple

10. What is Couple?

Two equal and unlike parallel forces constitute a couple. The moment produced by two equal, opposite, and non-collinear forces is called a *couple*. The couple can produce clockwise rotation or anticlockwise rotation.

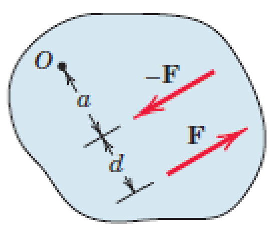


Figure 10

Consider the action of two equal and opposite forces **F** and **-F** a distance *d* apart, as shown in Figure 10. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as *O* in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

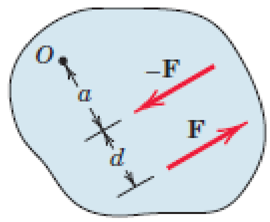
$$M = Fd$$

11) Define the term Couple and State its Characteristics.

The moment produced by two equal, opposite, and non-collinear forces is called a *couple*.

Characteristics:

- The effect of couple on body is independent of its position
- The magnitude of both forces should be same
- They act in opposite direction separated by a small distance
- The algebraic sum of forces, constituting the couple is zero i.e F-F=0
- The algebraic sum of the moments of the forces, constituting the couple, about any point is equal to the moment of the couple itself



Taking moment of forces about the point O

$$M = F(a + d) - Fa$$

$$M = Fd \quad \text{..............(1)}$$

Moment of the Couple $$M = Fd \quad \text{..............(2)}$$

From (1) and (2) it is proved

- A couple can be balanced by an equal and opposite couple in the same plane.
- Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.
- Any two couples whose moments are equal and of same sign are equivalent

12. Explain the term Force –Couple System with the help of neat sketch.

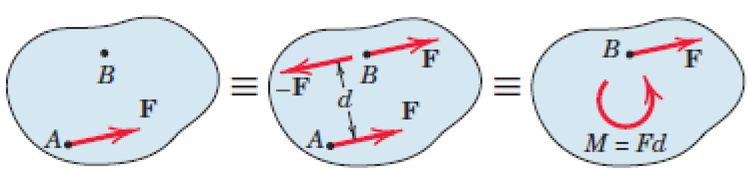


Figure 12

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig.12, where the given force **F** acting at point *A* is replaced by an equal force **F** at some point *B* and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces **F** and **-F** are added at point *B* without introducing any net external effects on the body. We now see that the original force at *A* and the equal and opposite one at *B* constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at *A* by the same force acting at a different point *B* and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Figure 12 is referred to as a *force–couple system*.

Topic - Resultant

13. State and Prove the Principle of moments.

Principle of Moments
This process is summarized in equation form by

$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma(Fd)$$

$$Rd = M_O$$

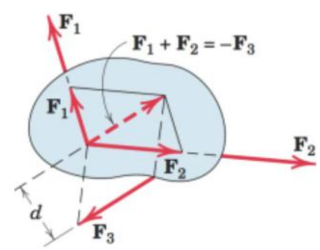


Figure 13

The first two of the Eqs. above reduce a given system of forces to a force-couple system at an arbitrarily chosen but convenient point 0. The last equation specifies the distance d from point 0 to the line of action of R, and states that the moment of the resultant force about any point 0 equals the sum of the moments of the original forces of the system about the same point.
This extends Varignon's theorem to the case of non-concurrent force systems; we call this extension the principle of moments.

For a concurrent system of forces where the lines of action of all forces pass through a common point 0, the moment sum $\sum M_o$ about that point is zero. Thus, the line of action of the resultant $R = \sum F$, determined by the first of Eqs. above, passes through point 0. For a parallel force system, select a coordinate axis in the direction of the forces.
If the resultant force R for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple.
The three forces in Figure 13 above, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3d$.