Project Euler Notes

Avid David

July 1, 2025

1. **Multiples of 3 or 5**

   If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3,5,6, and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000.

   Solution: We can calculate the sum directly. The multiples of k below a number n is given by

   $$S(k,n) = k \sum_{i=1}^{\lfloor \frac{n-1}{k} \rfloor} i = k \frac{\lfloor \frac{n-1}{k} \rfloor \left(\lfloor \frac{n-1}{k} \rfloor + 1 \right)}{2} \tag{1}$$

   Then we just need to calculate $S(3,1000) + S(5,1000) - S(15,1000)$ (subtracting 15 because PIE)

   Answer: 233168

2. **Even Fibonacci numbers**

   Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

   $$1, 2, 3, 5, 8, 13, 21, 34, 55, 89$$

   By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

   Solution: We implement 2 variables and a result variable. The key is to calculate the Fibonacci sequence by only taking into account the two current terms. Then, use an if statement to only add even terms to the sum.

   Answer: 4613732

3. **Largest Prime Factor**

   The prime factors of 13195 are 5, 17, 13 and 29. What is the largest prime factor of the number 600851475143?

   Solution: We implement a sieve of sorts to gradually filter out every prime factor. Firstly, we filter out powers of 2 from the number. Then, incrementing by 2 at a time

until the square root of that number, we filter out prime factors. Finally we should be left with the largest prime factor.

Answer: 6857

4. **Largest Palindrome Product**

   A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \times 99$. Find the largest palindrome made from the product of two 3-digit numbers.

   Solution: We can brute force this, but at least let's do so smartly. Start from the top down: 999×999, then 999×998, and so on. The first palindrome we see will be the largest.

   Answer: 906609

5. **Smallest Multiple**

   2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

   Solution: It is easy to see that $2520 = \text{lcm}(1, 2, \ldots, 10)$. Then we just need to find a way to efficiently compute $\text{lcm}(1, 2, \ldots, 20)$. We use the formula:

$$\text{lcm}(1, 2, \ldots, m) = \prod_{p \leq m} p^{\lfloor \log_p m \rfloor}$$

   Use sieve of Eratosthenes to generate all primes up to m and implement a loop for the logarithmic product.

   Answer: 232792560

6. **Sum Square Difference**

   The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \ldots + 10^2 = 385$$

   The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \ldots + 10)^2 = 55^2 = 3025$$

   Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$. Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

   Solution: This can be done very easily through the following two formulae:

$$1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(1 + 2 + \ldots + n)^2 = \left[\frac{n(n+1)}{2}\right]^2$$

   Answer: 25164150