

Number Theory

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Part I

Elementary Number Theory

1 Preliminaries

1.1 Mathematical Induction

This hinges on the Well Ordering Principle:

Concept

Well Ordering Principle: Every nonempty set S of nonnegative integers contains a least element that is, there is some integer a in S such that $a \leq b$ for all b 's belonging to S .

With a clever set construction, we can prove the **Archimedean property**: If a and b are any positive integers, then there exists a positive integer n such that $na \geq b$.

Now to the main character. We present the First Principle of Finite Induction:

Concept

First Principle of Finite Induction: Let $S \subseteq \mathbb{Z}_{>0}$ with the following properties:

- $1 \in S$
- If $k \in S$ then $k + 1 \in S$.

Then $S = \mathbb{Z}_{>0}$

This is also called weak induction. Sometimes we need strong induction, which is the Second Principle of Finite Induction. The problem statement stays the same, but the second condition changes to "If $1, 2, 3, \dots, k \in S$ then $k + 1 \in S$."

These two forms can be generalised to start with any starting positive integer n_0 . Then, in a problem, we'll have to verify the base case, assume it's true with n_0 , and prove the inductive step.