Exoplanet Measures

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Goals

Obtain estimates for

- Mass
- Radius
- Density

Explore data archives and computational tools

Modeling of detection data

Motivation

Observables are limited

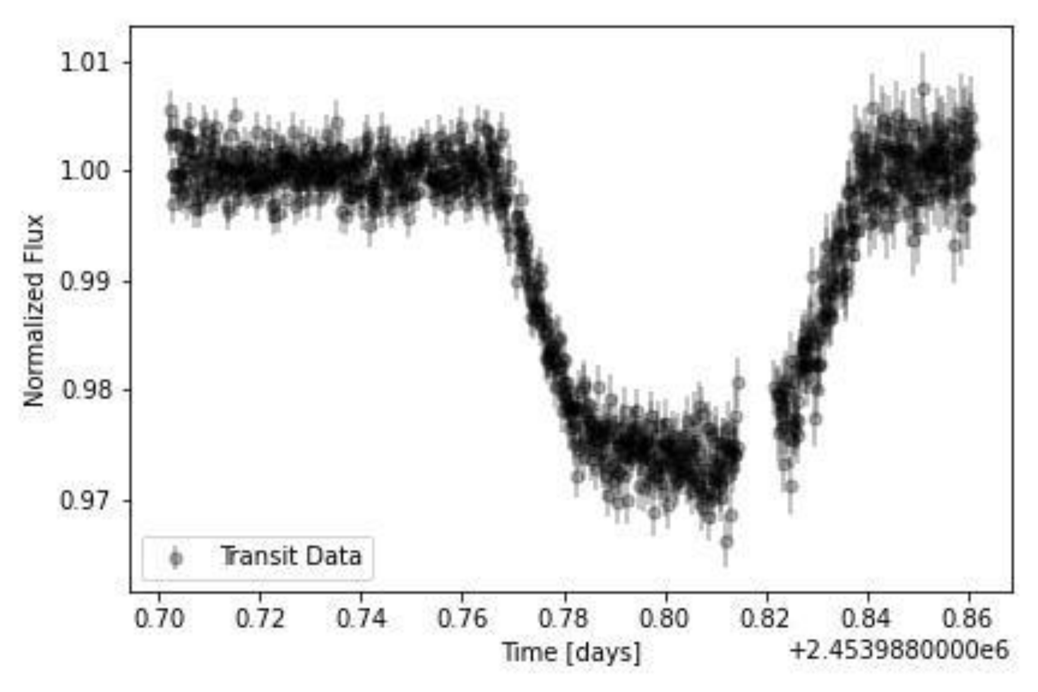
- Wealth of information inferred from analysis

Mass-Radius Relation

- Improved modeling
- Formation tendencies and limitations

Uncertainties

- Propagation thru to derived values



$$\delta = \left(\frac{R_p}{R_*}\right)^2$$

$$R_p = \sqrt{\delta} \cdot R_*$$

Radius

Rely on transit light curve data

Observable:

- Changes in stellar luminosity

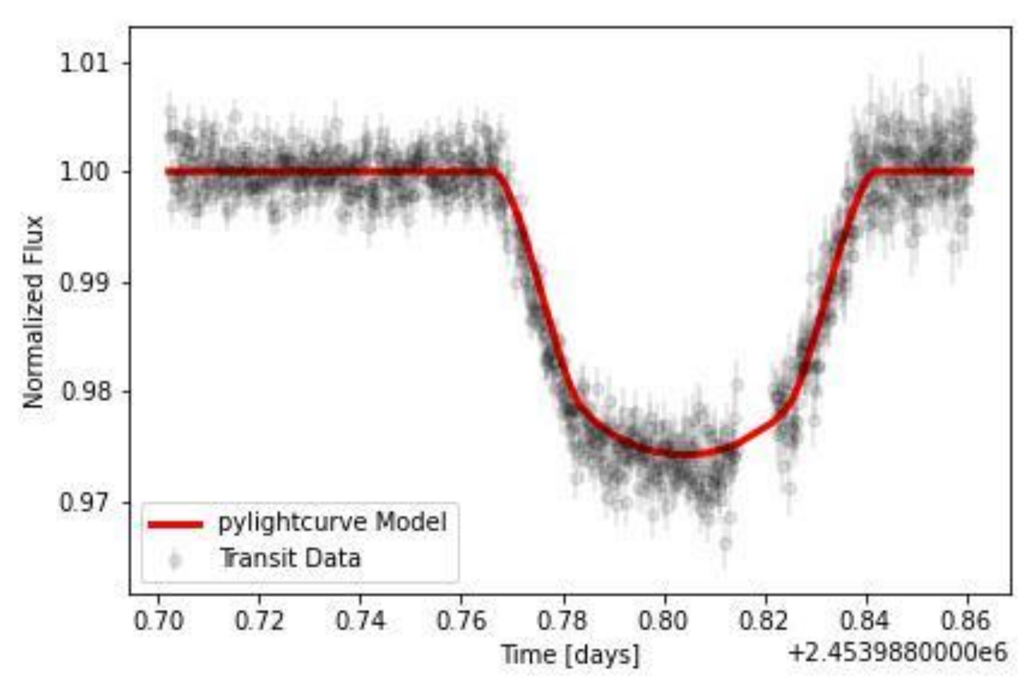
Parameters:

- δ, R_*

Derived:

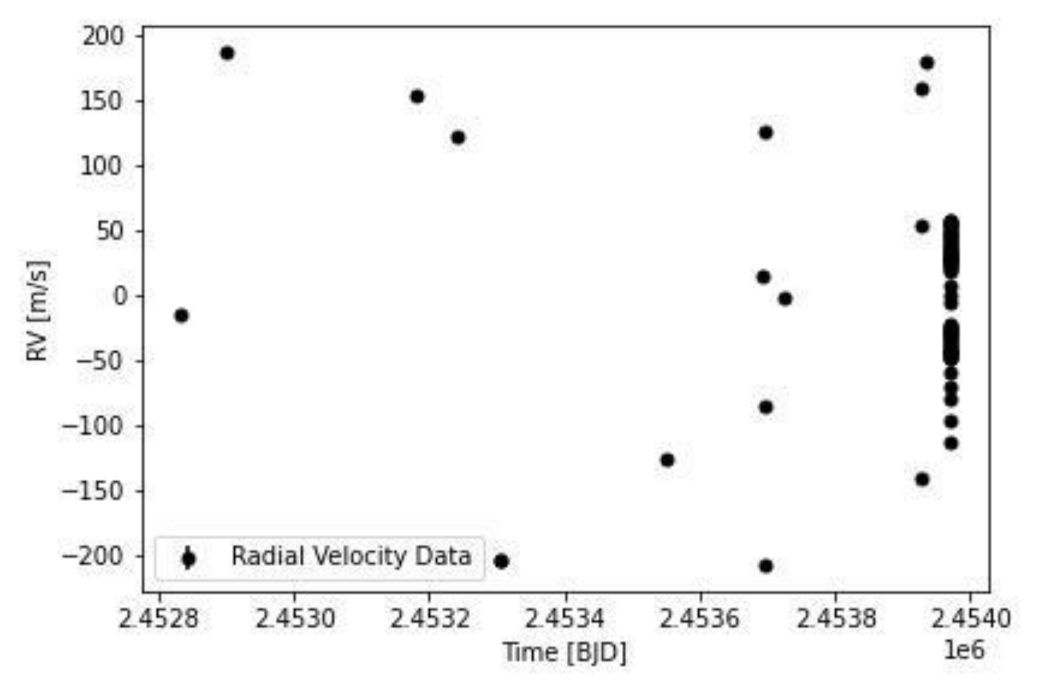
- Radius Estimate

Light Curve Model



Light curve model using *pylightcurve*

- *pylightcurve* imports own data



$$K = \frac{m_p}{m_*} \cdot \sqrt{\frac{Gm_*}{a}} \cdot \sin i \; ; \; i \approx 90^\circ$$

$$m_p = K \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

Mass

Rely on radial velocity data

Observable:

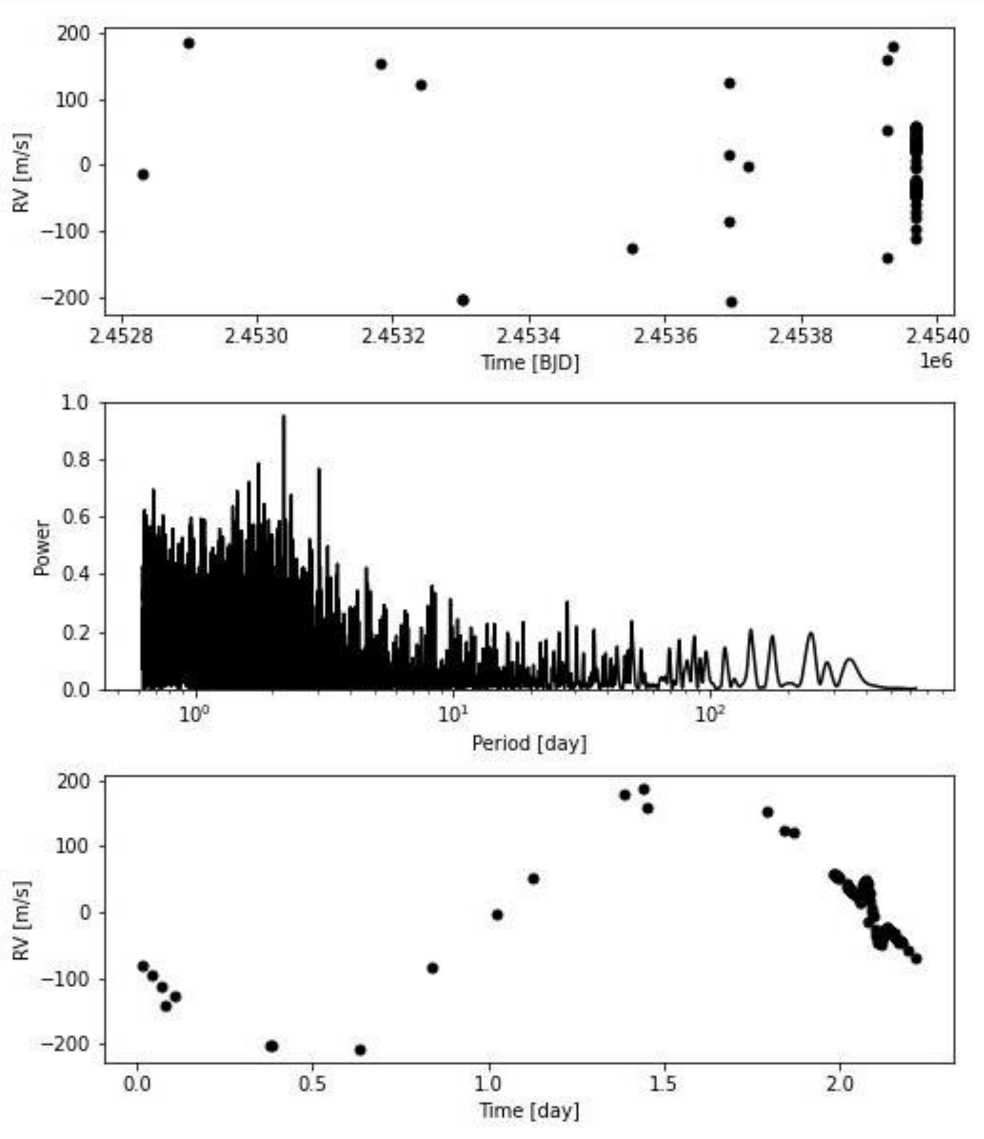
- Red/Blue shift in spectra

Parameters:

- G, m_*, a, K, i

Derived:

- Mass Estimate

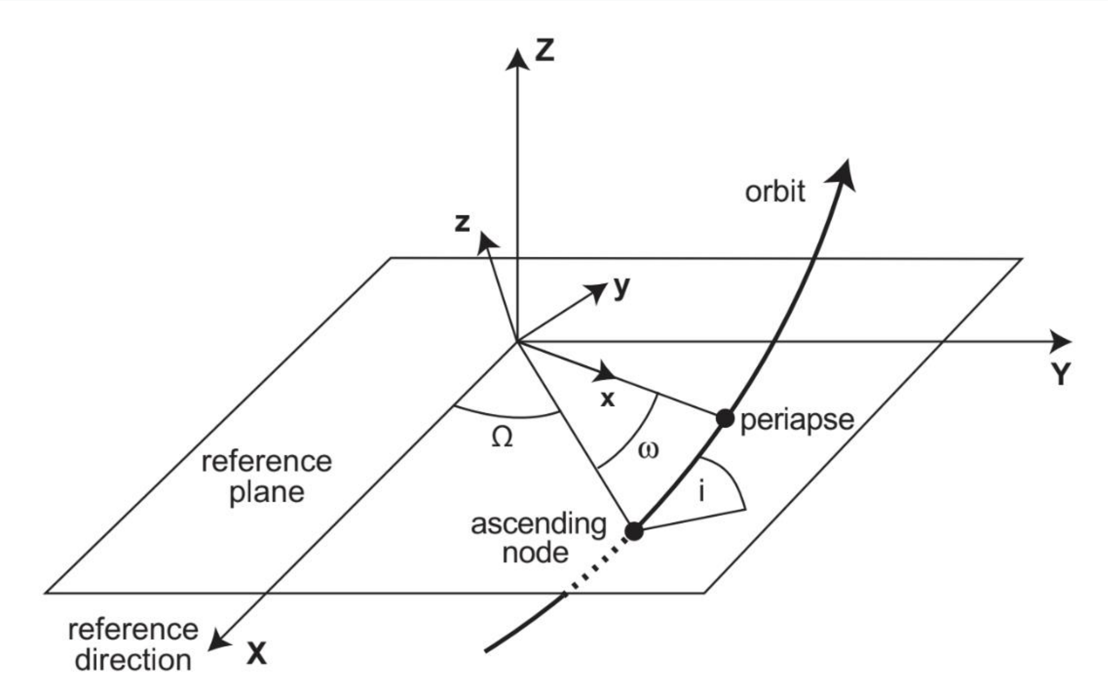


Lomb-Scargle Periodogram

Period identification

Radial Velocity Phase

Radial Velocity Curve Model



Orbital solution using EXOFAST

Input:

- Light Curve data
- Radial Velocity Data

Parameters Output:

- P, e, ω, t_0, K

Model radial velocity curve using *radvel*

Image Source: Professor Ji Wang

Density and Uncertainties

Radius Uncertainty

$$\sigma_{R_p} = R_p \left(\frac{\sigma_{\bar{\delta}}}{\bar{\delta}} \cdot \frac{1}{2} - \frac{\sigma_{R_*}}{R_*} \right)$$

$$\sigma_{\bar{\delta}} = \frac{\sigma_\delta}{\sqrt{N}}$$

Mass Uncertainty

$$\sigma_{m_p} = \frac{1}{2\sqrt{G}} \sqrt{\frac{K^2 m_*}{a} \cdot \sigma_a^2 + \frac{K^2 a}{m_*} \cdot \sigma_{m_*}^2 + 4 m_* a \cdot \sigma_K^2}$$

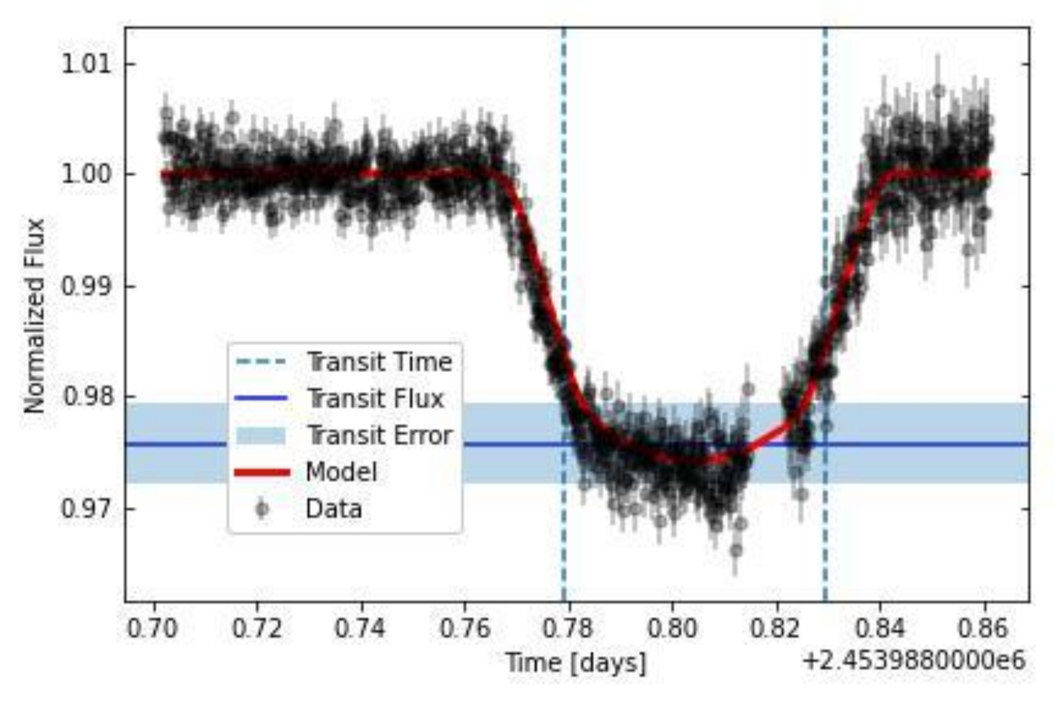
$$\sigma_K = \frac{\sqrt{\sigma_{\max}^2 + \sigma_{\min}^2}}{2}$$

Density and Uncertainty

$$\rho_p = \frac{m_p}{V_p} = \frac{m_p}{\frac{4}{3}\pi R_p^3}$$

$$\sigma_{\rho_p} = \rho_p \left(\frac{\sigma_{m_p}}{m_p} + 3\frac{\sigma_{R_p}}{R_p} \right)$$

Modeling/Parameters



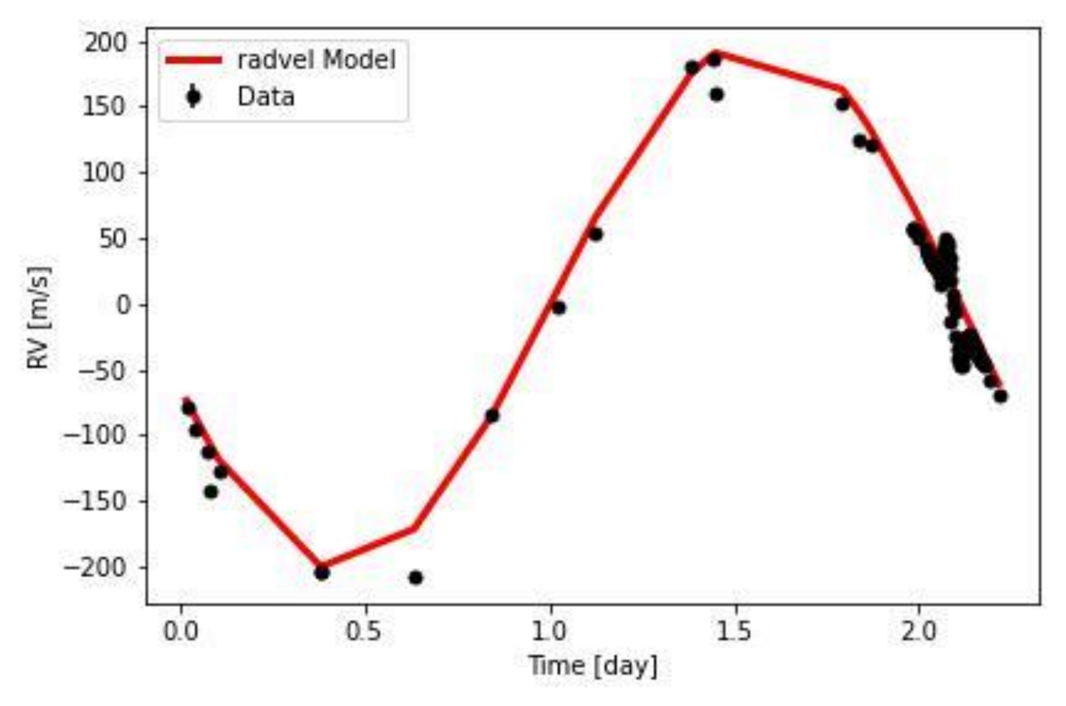
$$R_p = \sqrt{\delta} \cdot R_*$$

δ - 0.024 ± 0.004

R_* - 0.783 ± 0.013 R_{Sun}

R_star: Rosenthal et al. 2021
R_star uncertainty: Rosenthal et al. 2021

Modeling/Parameters



$$m_p = K \cdot m_* \cdot \sqrt{\frac{a}{Gm_*}}$$

K - 197.0 ± 1.0 m s^{-1}

a - 0.0314 ± 0.0004 AU

m_* - 0.84 ± 0.02 M_{Sun}

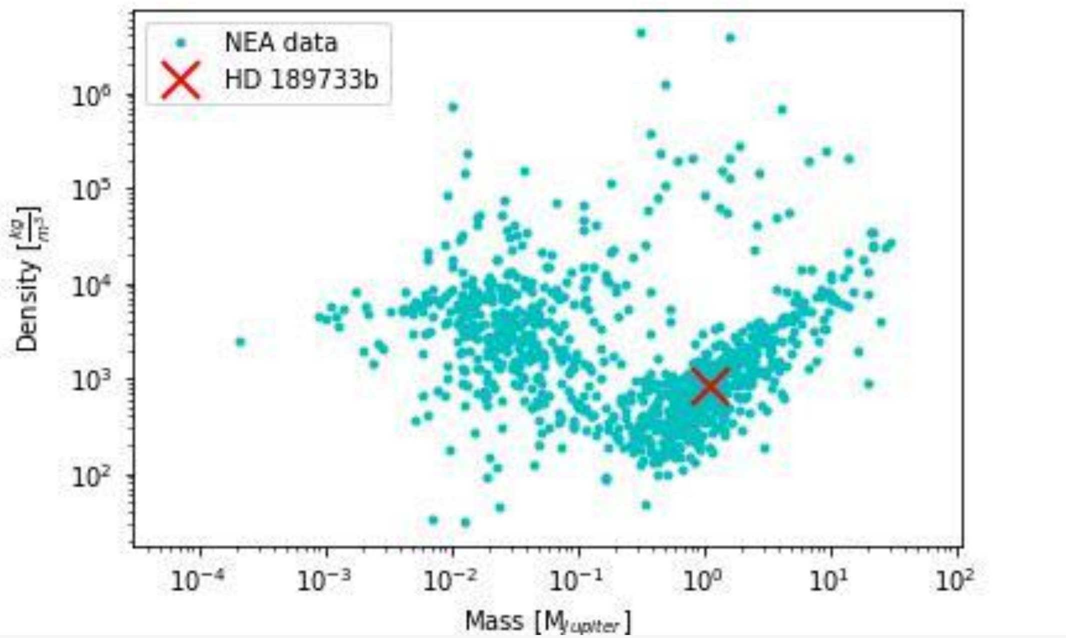
G - 6.67408 × 10^{-11} m^3 kg^{-1} s^{-2}

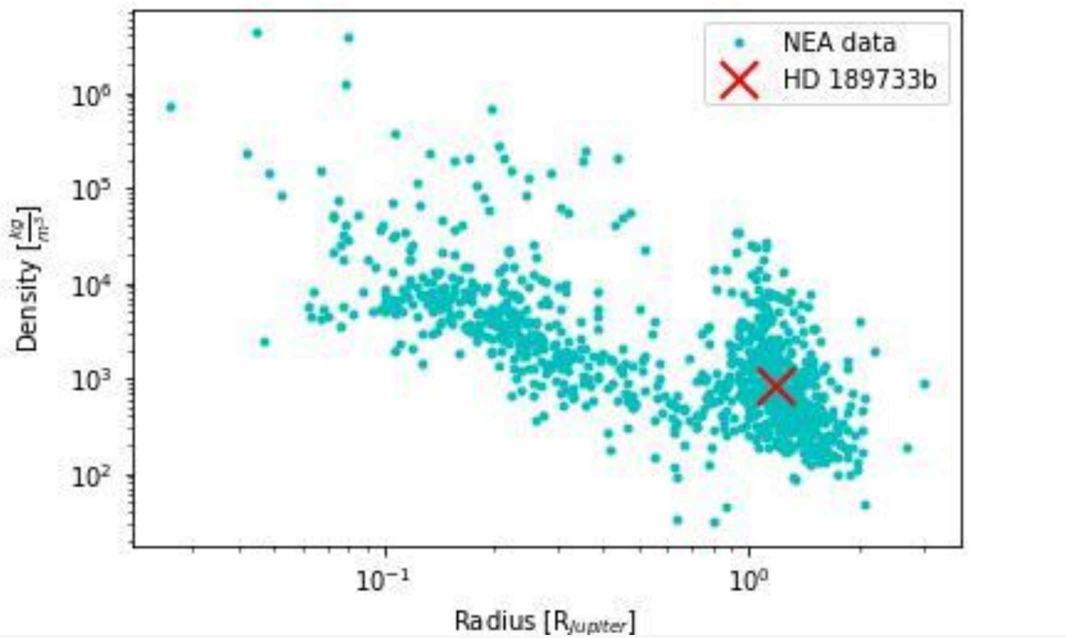
a uncertainty: (Rosenthal et al. 2021)
M_star uncertainty: NASA Exoplanet Archive

Estimates vs Accepted

	Radius (R_Jup)	Mass (M_Jup)	Density (Kg m^{-3})
Estimate	1.19 ± 0.07	1.124 ± 0.016	832.09 ± 156.17
Accepted	1.13 ± 0.01	1.13 ± 0.08	943 ± 77
Source	Stassun et al. 2017	Stassun et al. 2017	Bonomo et al. 2017

Archive Comparison





Chen and Kipping

R ~ M^(-0.04)

Given calculated mass
of 1.124 ± 0.016 M_jup

0.9953 ± 0.0006 R_jup

Good 1st order approximation

Accepted Value
1.13 ± 0.01 R_jup
Stassun et al. 2017

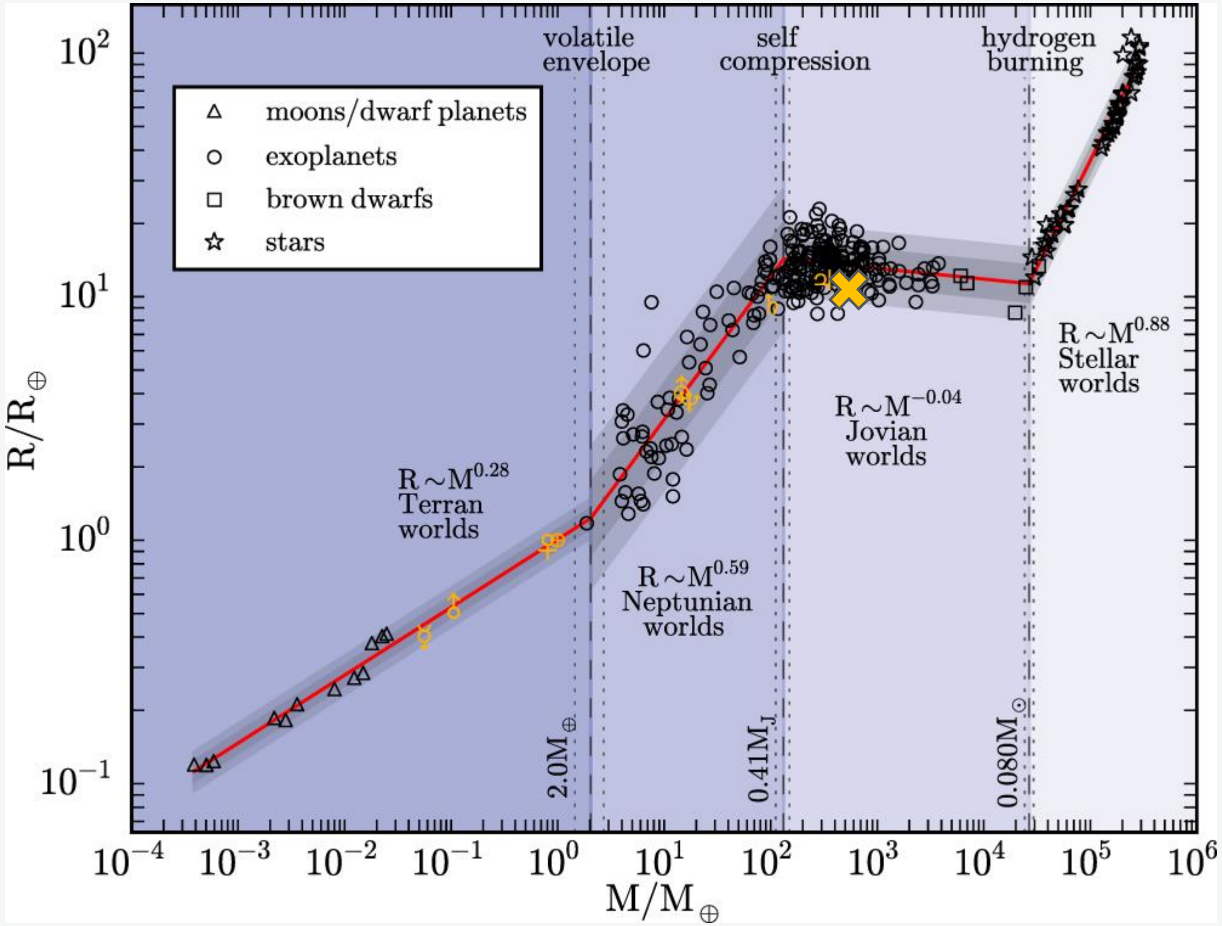
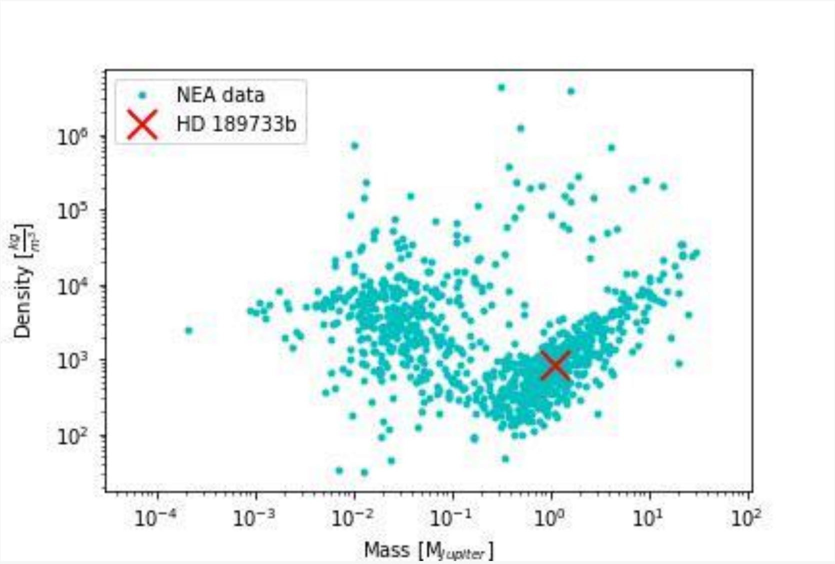
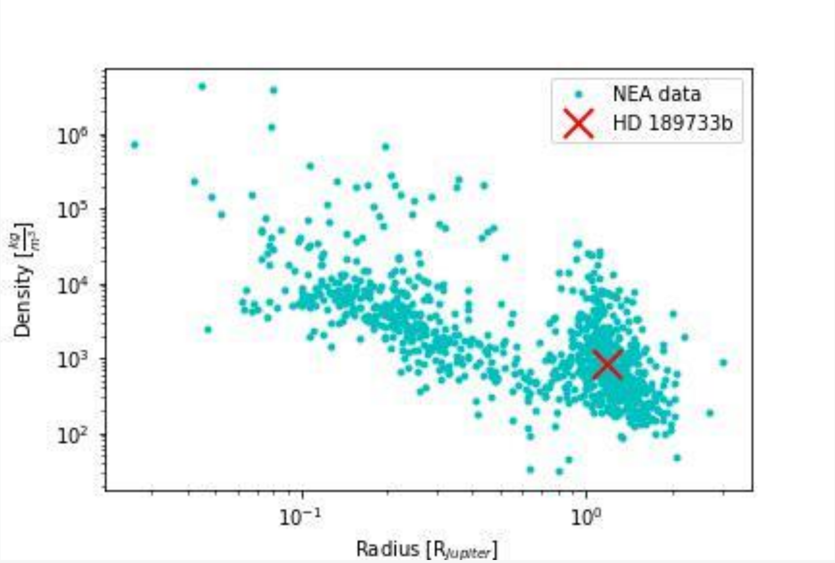


Image Source: Jingjing Chen and David Kipping 2017 *ApJ* **834** 17
https://iopscience.iop.org/article/10.3847/1538-4357/834/1/17/



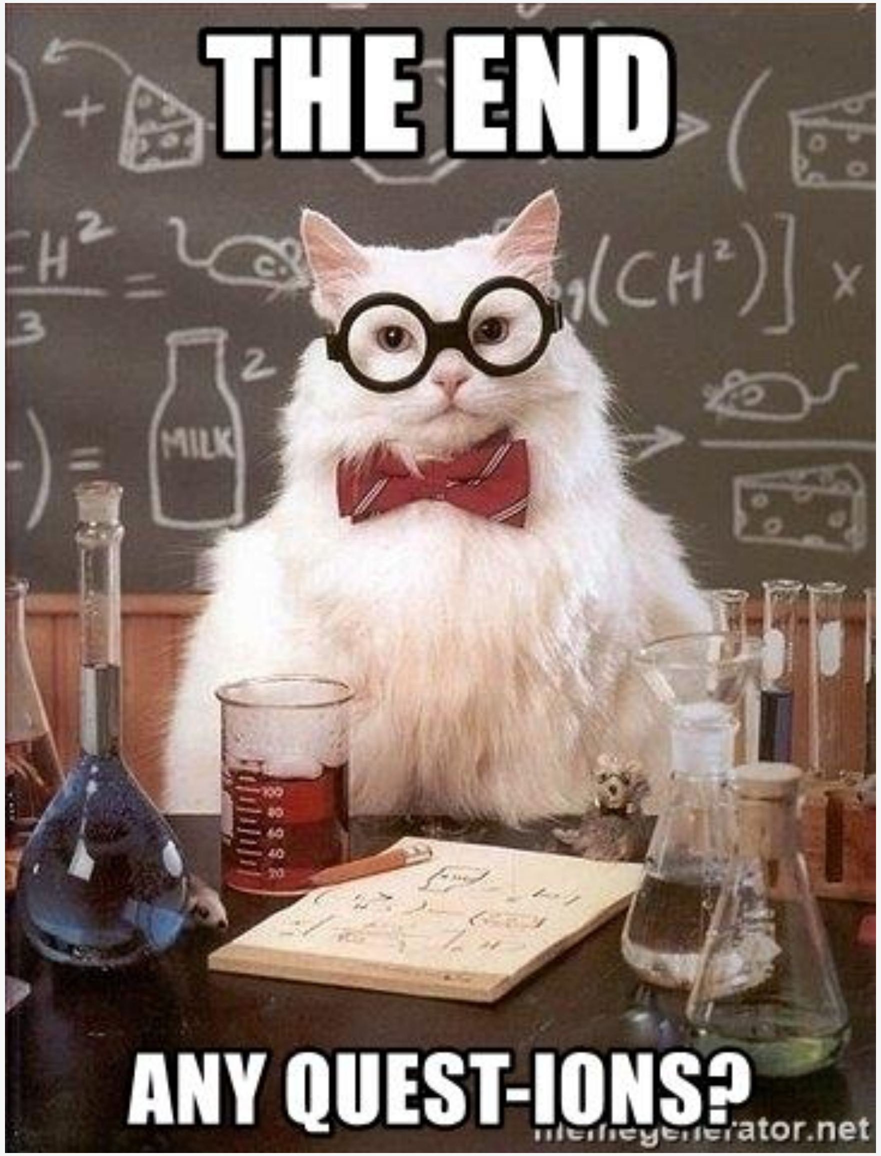


Conclusion

Our derived measures are consistent

- with values obtained from tools
- with accepted values

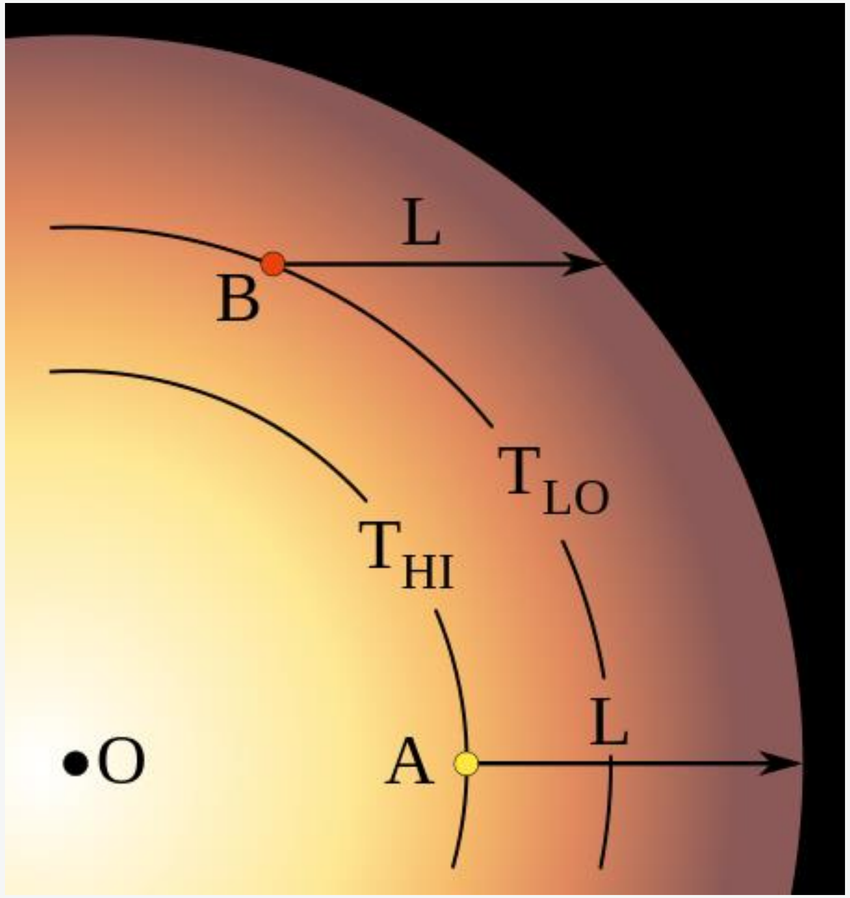
HD 189733 b is a prime example of mass/radius/density relation



Quest-ions.

Questions?

"L" is Optical Depth



Limb Darkening

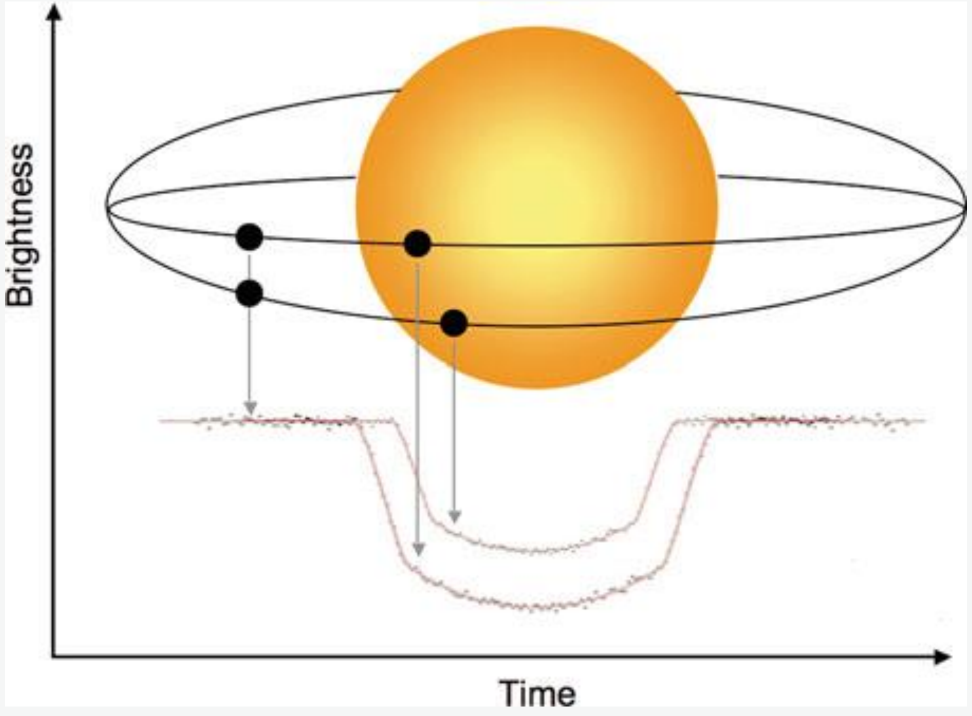


Image Source: https://en.wikipedia.org/wiki/Limb_darkening

radvel orbital solution params

K - 202.283082 m s^{-1}

P - 2.21863 days

e - 0.019434

ω - 83.30°

t_0 - 2453935.531264

Image Source:
Rouan D. (2014) Limb Darkening. In: Amils R. et al. (eds) Encyclopedia of Astrobiology. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-27833-4_885-2

Plotting Orbital Phase

$$M(t_{obs}) = \frac{(t_{obs} - T_0) \bmod P}{P}$$

Startig with the parenthesis in the numerator; for every time observed, we want it's difference from the start of a period to the end. The T_0 in the equation offsets the phase of the sinusoid, and would be set to the time when the potential planet is at the point in its orbit between the star and Earth (when the star would be moving radially towards Earth). We then want to take the modulos of the resulting time to get its point in time within a single period. Finally, we normalize the value by dividing by the period. The result is a value between 0 and 1 for each radial velocity that when plotted should show a single noisy sinusoid if the "best frequency" was accurate.

Image Source:
Rouan D. (2014) Limb Darkening. In: Amils R. et al. (eds) Encyclopedia of Astrobiology. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-27833-4_885-2