Differentiating Stellar Dynamos: Studying Fully and Partially Convective M-dwarfs in TESS Sectors 1 and 2

Undergraduate Research Thesis

Presented in Partial Fulfillment of the Requirements for graduation *with research distinction* in Astronomy in the undergraduate colleges of The Ohio State University

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ABSTRACT

Finding the mass-boundary that divides Fully and Partially convective stars can prove to be very useful in understanding the properties exhibited by these different types of stars, most importantly the role of the *Tachocline*, which is the boundary in *Low Mass Stars* that divides the convective and radiative regions of a star. Here, we analyzed data from NASA's Transiting Exoplanet Survey Satellite (TESS) survey, focusing primarily on M-dwarfs present in TESS Sectors 1 and 2, to determine this mass-boundary. The two distinguishing observables for fully and partially convective stars that we used in our analysis are the Relative Flare Energy and the Spot Filling Factor. Furthermore, this analysis can be expanded to the other sectors as well to get more extensive results regarding the position of the mass-boundary.

ACKNOWLEDGEMENTS

I would like to thank Dr. Ji Wang for his ever encouraging and supportive mentorship throughout this project, beginning from my selection into the Summer Undergraduate Research Program and continuing into the Astronomy 5205 class. I would also like to thank Dr. Max Guenther who wrote the initial paper which contained the TESS data we used in our analysis and his help in using the *Allesfitter* python package.

Additionally, I would like to thank my parents for their everlasting support and encouragement especially in these difficult past few years. I certainly owe my good mental health and ability to achieve all I did in my life to them. Also, a final thanks to Dr. Donald Terndrup and Dr. Wayne Schlingman for being great motivators during my undergraduate years. I would not be where I am now without their help.

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1. INTRODUCTION

The origin of stellar magnetism is still unknown to a large extent. Since magnetic fields are responsible for determining important properties of stars such as stellar evolution and mass loss (Guerrero et al., 2016), figuring out their origin would prove to be extremely useful in learning more about stars and in turn help in determining other stellar properties that can affect habitable planets around a star. In *Low Mass Stars,* the surface magnetic fields are believed to be produced due to a dynamo action in the convective region where a part of the kinetic energy is converted into magnetic energy. The 'rotational shear' boundary between the convective and radiative region of the star, known as the *Tachocline,* plays a role in creating stellar magnetism in these low mass stars. Since it doesn't exist in Fully Convective stars, separating them from Partially Convective stars will make it easier to understand the effects of the Tachocline.

For this project, we specifically selected M-dwarfs as our sample of low mass stars as it is easier to compare properties of stars that have the same spectral type. The data used for this analysis was obtained from Guenther et al. (2020) from which we specifically selected the M-dwarf candidates (455 stars) which had complete datasets for apparent TESS Magnitude (T_{mag}), Radius (R), Effective Temperature (T_{eff}), Bolometric Energy (E_{bol}) and Period of Rotation (P_{rot}). These values along with the stellar mass are what we used in our analysis in this project.

Additionally, we want to select M-dwarfs have a mass of less than 0.6 $M_\odot$ and a Rossby Number (R_o) of less than 0.1. Rossby Number is the ratio of a star's period of rotation and the convective turnover time. It gives a measure of how efficient the dynamo of the star is. These two quantities were not listed in the data set, so we had to derive them using the listed quantities. Our primary technique for analyzing the data and determining the mass-boundary was the Kolmogorov-Smirnov (KS) test which is used for statistical analysis of two cumulative distribution functions (CDFs) and determines how similar or different they are.

2. METHODS

2.1 *Selecting the Stars*

The first criteria for filtering out M-dwarfs from the dataset was the stellar mass. Unfortunately, the data did not contain measurements of the mass (Guenther et al., 2020), so we derived it instead using the stellar luminosity (L_*) which in turn we derived from the temperature (T_*) and radius (R_*) of the stars, data for which we have. Equation (1) highlights the relationship between said Luminosity, Temperature and Radius.

$$L_* = 4\pi\sigma R_*^2 T_*^4 \text{ --- (1)}$$

Here, σ is the Stefan-Boltzmann constant and the '$4\pi R^2$' refers to the surface area of the star, assuming a full spherical surface. The derived values for the Luminosity can be found in Table 1.

Another way to extract the Luminosity is through the TESS magnitude (T_{mag}) which is a relative magnitude scale used by TESS. Using the distance to the stars (d), Equations (2) and (3) relate it to T_{mag}.

$$M = T_{mag} - 5 + 5 \times log_{10} d \text{ --- (2)}$$

$$L_* = L_{\odot} \times 10^{\frac{-M}{2.5}} \text{ --- (3)}$$

Here, M is the absolute magnitude converted from the apparent T_{mag} and has a direct scaling relation with the Luminosity. It is of note that the two different methods of luminosity calculation yield almost similar measurements. When the two datasets were compared using the KS test, we got a p-value of 0.00014. The lower the p-value, the different the samples are. The low p-value shows that both the samples were mostly indistinguishable and proves to verify that our methods of deriving the luminosity were legitimate. For the rest of the analysis, data obtained using Equation (1) was used.

We then used the Luminosity-Mass relation found in *Foundations of Astrophysics by Ryden and Peterson* (2011) to find the mass of these stars. It is a scaling relation that works for two different mass ranges. Equations (4) and (5) depict said relation:

$$\frac{L_*}{L_\odot} = 0.35 \times \left(\frac{M_*}{M_\odot}\right)^{2.62} \text{ for } M_* < 0.7\ M_\odot \text{ --- (4)}$$

$$\frac{L_*}{L_\odot} = 1.02 \times \left(\frac{M_*}{M_\odot}\right)^{3.92} \text{ for } M_* > 0.7\ M_\odot \text{ --- (4)}$$

Here the '$\odot$' symbol denotes the quantities measured for the Sun. Since M-dwarfs have a mass of less than 0.6 $M_\odot$, Equation (2) is the one we used here. This narrowed down the sample size from 1228 stars to 455 stars. The mass values for these 455 stars can be found in Table 1.

Now that the mass values were obtained, we calculated the Rossby Numbers for these stars, which was the second criteria that the stars needed to pass to be considered as for our analysis. The Rossby Number (R_o) is a relation between the Period of Rotation (P_{rot}) and the Convective Turnover Timescale (τ_{conv}) of the stars and is a measure of how efficient the dynamo system of the star is (Brandenburg & Subramanian, 2005). If the R_o is less than 1, the dynamo system is considered to be efficient and if it is less than 0.1, then it is saturated. A saturated dynamo system implies that the magnetic field no longer depends on R_o, hence making it easier to compare the magnetic fields of Fully and Partially Convective stars. Equation (6) showcases the relation between R_o, P_{rot} and τ_{conv}:

$$R_o = \frac{P_{rot}}{\tau_{conv}} \text{ --- (6)}$$

The data for P_{rot} was available in Guenther et al. (2020), however τ_{conv} was not. Fortunately, a relation between τ_{conv} and the logarithm of the effective temperature of a star (*log* T_{eff}) was highlighted in Kim & Demarque (1996) which we then used to find τ_{conv}. For our sample of 455 stars, the highest *log* T_{eff} value was found to be 3.63 and the lowest was 3.45, which when compared to Figure 5 of Kim & Demarque (1996) showed that τ_{conv} for our sample ranged from 100 days to over 150 days. We modelled the relevant section of the graph as an approximate straight line using Equation (7) and calculated τ_{conv} and plugged it into (6) to get the R_o for all the stars in our sample. Coincidentally, all 455 stars had a Rossby Number of less than or equal to 0.1, which meant that all of them made it to our final sample.

$$\tau_{conv} = -800 \times log(T_{eff}) + 2980 \text{ days --- (7)}$$

2.2 *Analysis*

Once we had our sample, we began the analysis part of the project by first assuming a mass-boundary value, 0.4 $M_\odot$ in this case, and separated the ‘supposed’ Fully and Partially convective stars. The two sets now contain 372 Fully Convective stars and 82 Partially Convective stars. This was done to iron out the various steps involved in calculating the Relative Flare Energy and Spot Filling Factor, the two key distinguishing features for fully and partially convective M-dwarfs as they give a good account of the magnetic flux and topology of the surface of a star.

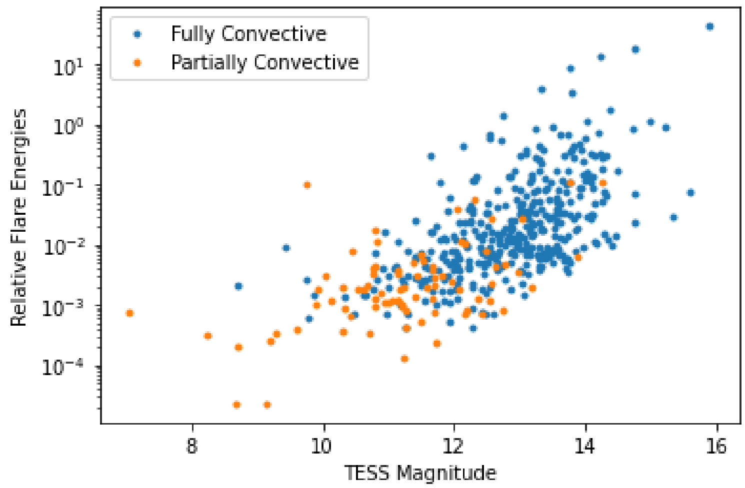
i) *Relative Flare Energy*

The Relative Flare Energy (E_{rel}) is a quantity that scales with the square of the Bolometric Energy (E_{bol}) and is given by the relation in Equation (8).

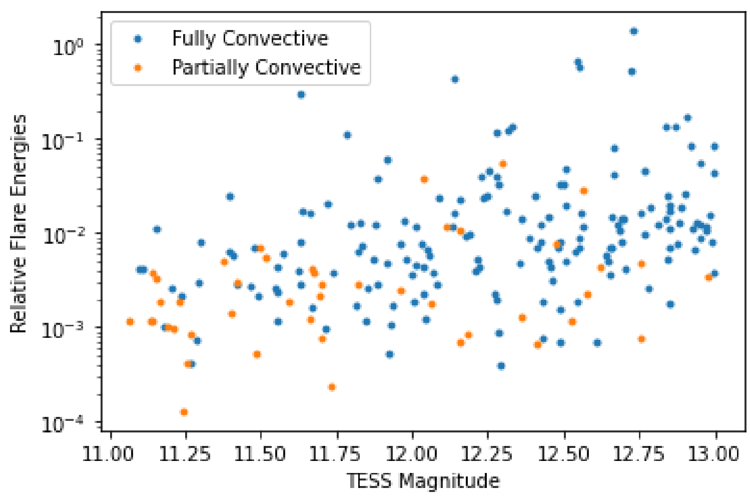
$$E_{rel} \sim E_{bol}^2 \frac{L^3}{8\pi} \text{ --- (8)}$$

Here, L is the typical scale of magnetic field producing region and for M-dwarfs it is usually on the scale of $\sim 10^8$ cm and we can treat it as an approximately constant quantity for all the stars in our sample. This narrows down the dependence of E_{rel} to just the square of E_{bol} and since the E_{bol} values in Guenther et al. (2020) are relative, we can ignore the constants in (8) as well to get a measure of E_{rel} in relative units. The values for both E_{rel} and E_{bol} are listed in Table 1.

Figure 1a shows the E_{rel} v/s T_{mag} graph with the fully and partially convective stars, where the mass-boundary was assumed to be 0.4 $M_\odot$, highlighted in blue and orange respectively. The outlying stars in the graph that have a T_{mag} of less than 9 and greater than 15 pose a risk of selection bias as they are few and far between, separated from the majority of the other stars in the sample. Additionally, there are significantly more fully convective stars than there are partially convective, which could possibly be a selection bias from TESS, so we decided to select only the stars that lie between T_{mag} of 11 and 13, as it includes the majority of partially convective stars, which are a minority in this particular sample of stars based on the decided mass-boundary. Figure 1b shows this sub-sample, which is used to compare the E_{rel} for the two types of stars.



a)



b)

Figure 1: a) All 455 (Fully and Partially convective) stars plotted on a Relative Flare Energy v/s TESS Magnitude graph. b) A selection of datapoints from Figure 1a to minimize selection bias.

The next step in the analysis was to make a final comparison between the two samples to see how different or similar they were. For this, we used the aforementioned Kolmogorov-Smirnov (KS) test to compare the cumulative distribution functions (CDF) for the two samples of stars. The python package *scipy* has an in-built '2-value KS test' which we used to find the ‘statistic’ and ‘p’-values. For the assumed mass-boundary of 0.4 $M_{\odot}$, the statistic value was 0.526 and the p-value was 1.568 x 10^{-9}. Table 2 contains these KS test results for all the mass-boundary calculations done in this project. The very low p-value shows that the samples were significantly distinguishable and could quite possibly be the correct mass-boundary, which is why we tested several other mass-boundary values asl well. Figure 2a shows the E_{rel} CDF v/s E_{rel} graph that gives insight on how well the two CDFs align with each other.

ii) *Spot Filling Factor*

The Spot Filling Factor (SFF) is a measure of the amount of surface of a star that is covered with starspots. A reliable way to measure the SFF for a star is to observe it’s lightcurve, where the variations in flux help in determining the density of starspots present on the star’s surface. For example, there would be a characteristic ‘dimming’ in the normalized flux when a starspot passes through and the shape of this ‘dip’ is distinct from the one caused due to a transiting planet.

We used the *Lightkurve* python package to extract the lightcurves for all 455 stars in our sample and the standard deviation and median values were calculated for each light curve. An important step to minimize any skewing of our results was to eliminate any possible flares in the lightcurves. Flares are large spikes in the flux of a star that last for a very short amount of time. Since they are the possible ‘outliers’ in the dataset, we eliminated any datapoints that are beyond the 3σ range, where is the σ standard deviation, thus only keeping the 99.7% of points in the data.

The important quantity in this analysis is the Fractional Variation (δf) which is defined as the ratio of the standard deviation and median of the flux of a star, as shown in Equation (9), and is a measure of how much the flux changes.

$$\delta f = \frac{\sigma}{\text{median}} \text{ --- (9)}$$

Plotting δf v/s T_{mag}, as shown in Figure 2, is the best way to extract the SFF values for each star in our sample. The green line is an exponential fit that barely grazes the bottommost points of the plot. This was done to determine the theoretical fractional variation for that sample of stars, but the observed δf is different, as evident in the graph, because of the spots occupying the surface. Equation (10) shows the relation used to derive SFF from the graph.

$$\text{SFF} = \sqrt{\delta f^2 - Y^2} \text{ --- (10)}$$

Where Y is the y-coordinate value of the fitted green curve corresponding to the particular T_{mag} value for each star.

Once we obtained the SFF values, we repeated the KS test procedure used for E_{rel} and obtained the statistic value as 0.105 and p-value as 3.33×10^{-16}. Figure 3b shows the CDFs of the SFF to give a pictorial representation of how similar the two datasets were for the selected mass-boundary. Equation (11) contains the equation of the exponential green curve, where X are the T_{mag} values or the x-coordinate.

$$\text{Y} = 1.3743 \times 10^{-7} \times 2.23149X \text{ --- (11)}$$

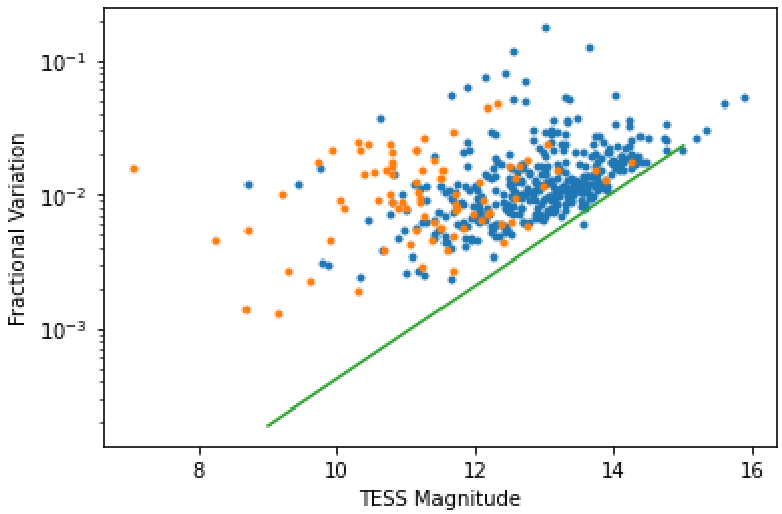
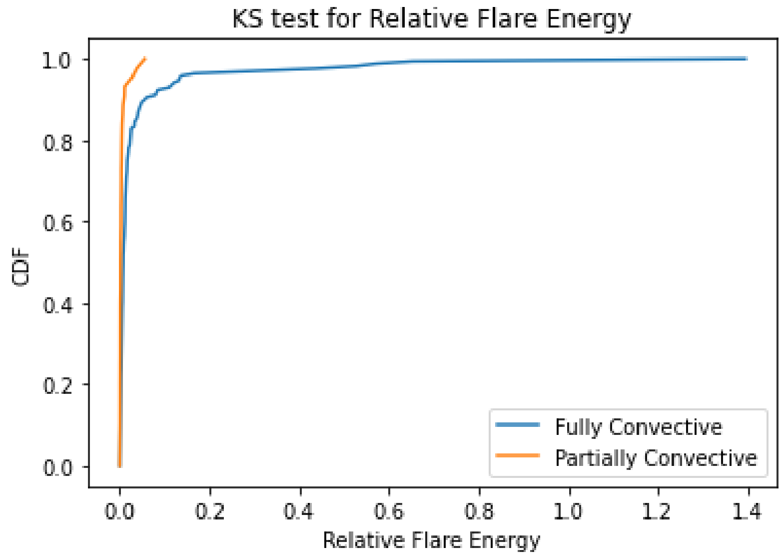
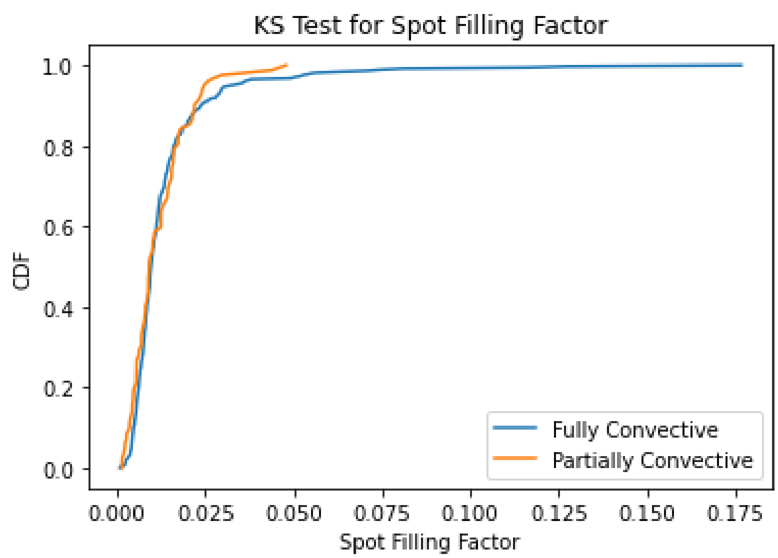


Figure 2: Fractional Variation v/s TESS Magnitude plot used to calculate Spot Filling Factor



a)



b)

Figure 3: a) A comparison of the CDFs of Relative Flare Energy for Fully and Partially convective stars used for the KS test. b) A comparison of the CDFs of Spot Filling Factor for Fully and Partially convective stars used for the KS test.

iii) *Changing the Assumed Mass-Boundary*

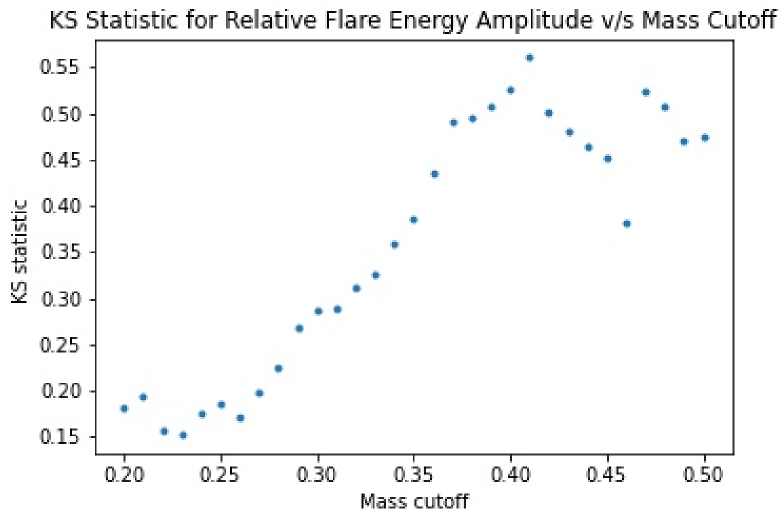
Once the analytical process was completely laid out, we changed the mass-boundary value and repeated the process for each of those new values. This process was automated through a loop such that a mass-slider was set up for the boundary value that ranged from 0.2 $M_\odot$ to 0.5 $M_\odot$, in intervals of 0.01 $M_\odot$. We then plotted the resultant p-values from the E_{rel} KS test results along with their respective mass-boundaries, with the mass-boundary giving the lowest p-value as the most probably point when fully convective M-dwarfs transition into partially convective M-dwarfs.

An important thing to take into consideration while automating the process was sorting the E_{rel} sample for each mass-boundary value such that there was no selection bias, similar to how we only selected stars that lay between 11 and 13 T_{mag} for the mass-boundary of 0.4 $M_\odot$. To do this, we selected all the stars that lay within 0.05 $M_\odot$ of the selected mass-boundary and calculated the median T_{mag} for just those stars, which helped us in identifying what T_{mag} value that particular set of stars was centered around. We then filtered out the stars that had a T_{mag} that lay outside of 1.5 on either side of the calculated median. As mentioned before, Table 2 contains all the measured p-values.

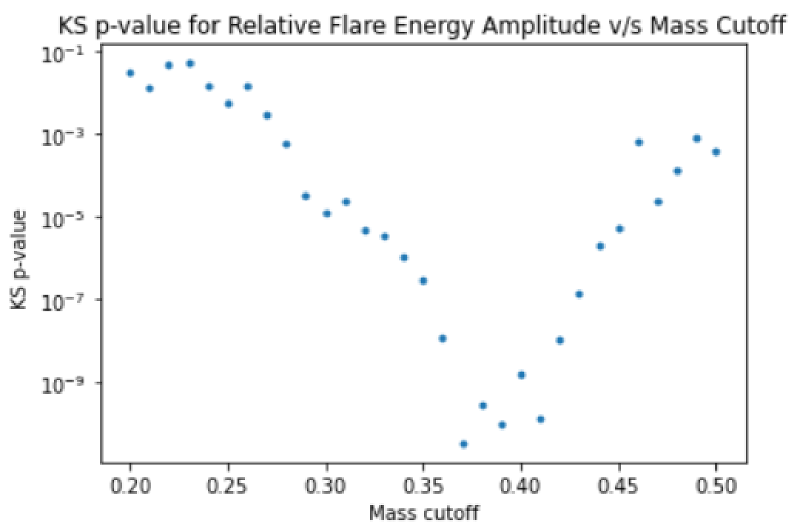
3. RESULTS

Figure 4 features the aforementioned statistic and p-values along with the respective mass-boundaries ranging from 0.2 $M_\odot$ to 0.5 $M_\odot$. It is of note that the datapoints towards the ends, closer to 0.2 $M_\odot$ and 0.5 $M_\odot$, stray away from the general pattern. This can be explained by a possible selection bias coming into play when the median T_{mag} was selected for the mass-cutoff (mass-boundary) and only the stars that fall within 1.5 magnitude of that value were selected for the final KS test analysis. Figure 5 shows the same contents of Figure 4b with the addition of the sample size of fully and partially convective stars that were selected for each mass-boundary. The decrease in the number of stars in each sample observed towards the end mass-boundaries confirms out assumption about the selection bias skewing those results.

However, the sharp dip in the p-value graph (Figure 4b) towards the middle shows that the transition mass does not lie on the ends of the selected mass-boundary range. The datapoints located between 0.37 $M_\odot$ and 0.4 $M_\odot$ communicate the fact that the two samples of fully and partially convective M-dwarfs are the most distinct in that range, since their p-values are the lowest. Hence, the most likely transition mass-cutoff lies between 0.37 $M_\odot$ and 0.4 $M_\odot$.



a)



b)

Figure 4: a) Shows that KS Test statistic results for the sample with the assumed mass-boundaries. b) Shows that KS Test p-value results for the sample with the assumed mass-boundaries.

4. CONCLUSION

We selected a sample of 455 M-dwarfs to determine the mass- boundary when Fully Convective M-dwarfs transition into Partially Convective stars. The luminosity of said stars in the sample was calculated using the stellar radius and effective temperature and the luminosity-mass relation was then used to obtain the stellar mass and filter out the particular M-dwarfs of interest ($M_* < 0.6\ M_\odot$). Additionally, stars that exhibit a saturated dynamo system ($R_o < 1$) were selected to create a more even sample size and make it easier to compare the properties of fully and partially convective stars.

Relative Flare Energy and Spot Filling Factor were the two properties that we used to distinguish fully and partially convective stars based on the results obtained from a 2-value Kolmogorov-Smirnov test. The resultant p-value of the KS test was found to be the lowest for mass-boundaries that lie within 0.37 $M_\odot$ and 0.4 $M_\odot$, which meant that Fully Convective M-dwarfs transitioned into Partially Convective M-dwarfs at some mass value within that range. The next step would be to repeat this analysis for M-dwarfs in other TESS Sectors to see if the mass-boundary results are consistent.

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APPENDIX A

TESS Input Catalog identifier	Tess magnitude	Luminosity (in L_sun)	Effective Temp. (in K)	Radius (in R_sun)	Period of Rotation (in days)	Rossby Number	Bolometric Energy (in erg units)	Relative flare amplitude	Mass (in M_sun)
2761472	12.551	0.0035751	3041	0.216026	0.958129	0.009581	1.47E+33	0.75583	0.173843
5656273	9.2107	0.0654345	3815	0.587235	0.431193	0.004312	1.06E+33	0.01613	0.527276
5725904	13.8295	0.0046825	3124	0.234268	0.330389	0.003304	1.19E+33	0.66936	0.192703
5796048	10.8081	0.0481887	3621	0.559388	0.555649	0.005556	7.54E+33	0.133	0.469167
7151484	9.87974	0.0231222	3403	0.438721	1.15218	0.011522	4.13E+32	0.03822	0.35449
12421477	11.701	0.0435049	3461	0.581786	0.358996	0.00359	4.08E+33	0.05334	0.451209
12423835	12.9949	0.010283	3190	0.332948	2.844444	0.028444	3.13E+33	0.28766	0.260188
12471629	11.7126	0.0188095	3213	0.443879	2.2294	0.022294	4.64E+32	0.03139	0.327631
12509218	11.4937	0.0172881	3192	0.431167	0.83507	0.008351	6.93E+32	0.04613	0.317252
12931952	12.4529	0.0044876	3171	0.222593	0.350149	0.003501	2.33E+32	0.12177	0.189601
12996375	15.3331	0.0043225	3064	0.233985	1.1033	0.011033	8.97E+32	0.17302	0.186908
20892891	11.5143	0.0328911	3516	0.490161	4.790643	0.047906	1.76E+33	0.07501	0.405526
25081629	14.0971	0.0070849	3153	0.282889	0.274577	0.002746	1.02E+33	0.36275	0.225702
25118964	12.0301	0.0282283	3323	0.508369	1.017008	0.01017	1.19E+33	0.08724	0.382541
25132999	10.8124	0.079914	3945	0.606897	4.92012	0.049201	1.05E+34	0.10711	0.56908
25153167	13.5675	0.002539	3036	0.182652	0.373484	0.003735	3.04E+32	0.2462	0.152557
25200252	10.955	0.0067812	3223	0.264868	1.205592	0.012056	3.64E+32	0.12833	0.221959
25243422	11.9169	0.0067327	3138	0.27841	0.958129	0.009581	1.76E+32	0.06889	0.221352
25374751	11.5526	0.0058896	3286	0.237468	4.551111	0.045511	9.20E+31	0.03458	0.210333
27955268	13.2609	0.0040624	2928	0.248396	0.616577	0.006166	1.93E+32	0.08117	0.182532
29257288	11.8478	0.006173	3258	0.247312	0.446461	0.004465	8.86E+31	0.03391	0.214141

Table 1: A table containing all the relevant, derived and observed, data about the selected 455 stars. Only the first 30 entries are listed here, the full table can be found on my Github: *https://github.com/PrimusAddy1910/Astronomy-Summer-Research--2021-*

Mass cutoff	KS statistic	KS p-value
0.2	0.182125	0.031603
0.21	0.19411	0.012864
0.22	0.157402	0.049517
0.23	0.15217	0.052548
0.24	0.175346	0.014684
0.25	0.186418	0.005856
0.26	0.170363	0.015322
0.27	0.198406	0.003025
0.28	0.224133	0.000612
0.29	0.26802	3.42E-05
0.3	0.286819	1.30E-05
0.31	0.289073	2.44E-05
0.32	0.311805	4.98E-06
0.33	0.326848	3.59E-06
0.34	0.359786	1.08E-06
0.35	0.385792	3.00E-07
0.36	0.436216	1.14E-08
0.37	0.491715	3.20E-11
0.38	0.494406	2.58E-10
0.39	0.506873	9.42E-11
0.4	0.526027	1.57E-09
0.41	0.560259	1.25E-10
0.42	0.501456	1.03E-08
0.43	0.481592	1.37E-07
0.44	0.46345	1.95E-06
0.45	0.452632	5.11E-06
0.46	0.382418	0.000674
0.47	0.524281	2.27E-05
0.48	0.506865	0.000134
0.49	0.47096	0.000787
0.5	0.473469	0.000398

Table 2: A table containing all the p-values from Relative Flare Energy KS test result

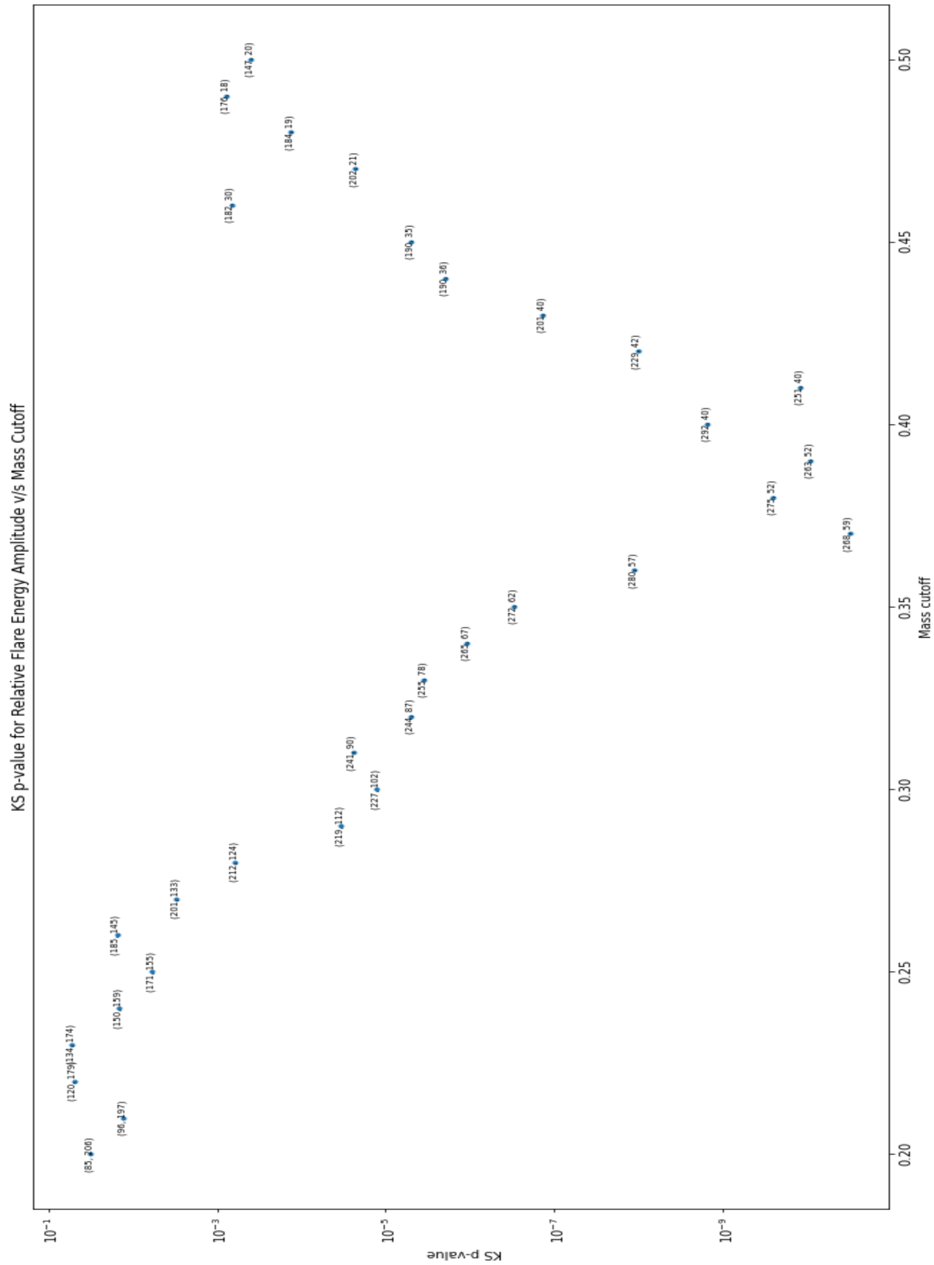

Figure 5: A figure containing the same KS p-values as Figure 4b, but with the addition of the number of stars in each sample for each mass-cutoff (mass-boundary). Both Figure 4 a) and b) graphs with the sample size can also be found on my Github page: *https://github.com/PrimusAddy1910/Astronomy-Summer-Research--2021-*