

# Risk-averse Heteroscedastic Bayesian Optimization

Anastasia Makarova, Ilnura Usmanova, Ilija Bogunovic, Andreas Krause

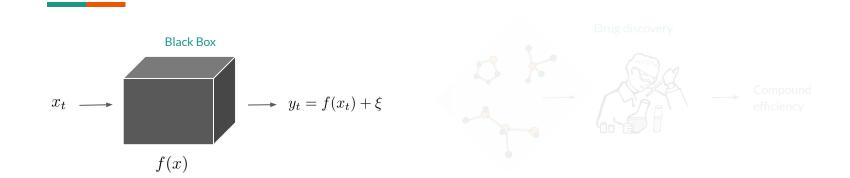






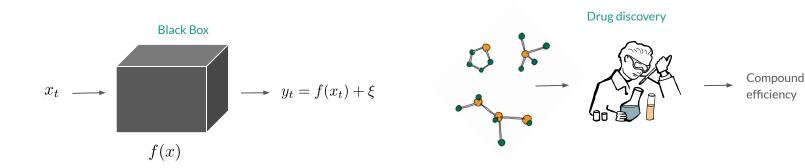






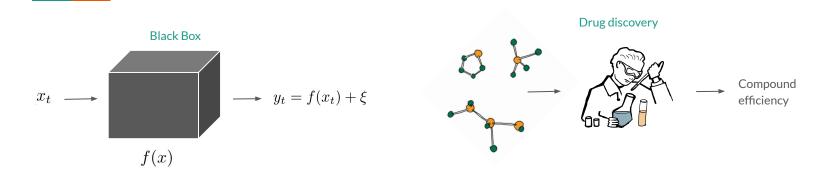


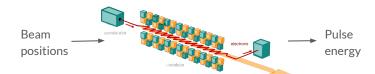
where one needs to trade off attaining high utility vs minimizing risk

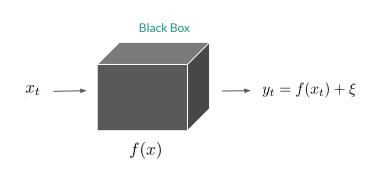




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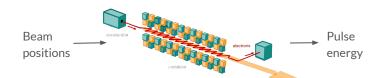








### Tuning accelerators



In these applications, one needs to trade off attaining high utility vs minimizing risk

trading off exploration & exploitation

objective f(x)  $x^* \in \arg\max_{x \in \mathcal{X}} f(x)$ 

regret 
$$R_T = \sum_{t=1}^T \Big[ f(x^*) - f(x_t) \Big]$$

Noise-perturbed evaluations

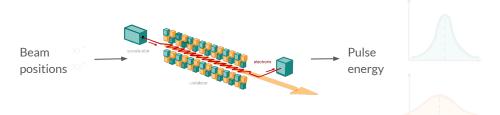
$$y_t = f(x_t) + \xi(x_t)$$

 $\rho$  -sub-Gaussian noise

Optimism under uncertainty via GP Upper Confidence bound

$$x_t \in \argmax_{x \in \mathcal{X}} \underbrace{\mu_{t-1}(x) + \beta_t \sigma_{t-1}(x)}_{=: \mathrm{ucb}_t^f(x)}$$

**Tuning accelerators** 



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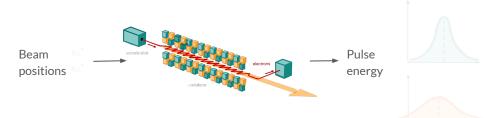
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$$= \text{e:ucb}_t^f(x)$$
GP posterior mean and variance

#### **Tuning accelerators**



trading off exploration & exploitation

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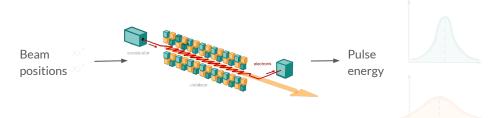
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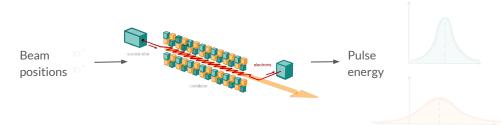
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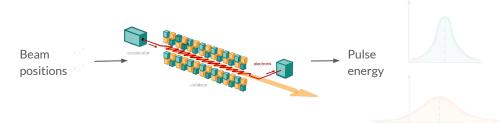
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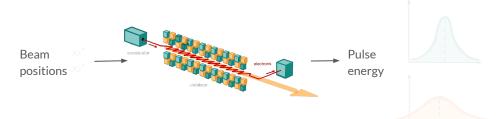
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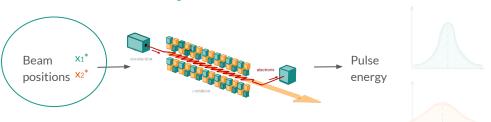
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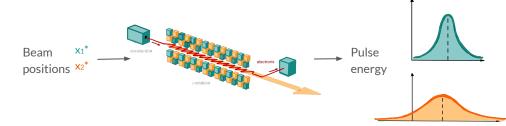
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Optimism under uncertainty via GP Upper Confidence bound

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$$GP \operatorname{posterior}_{\text{mean and variance}}$$

Tuning accelerators



f(x)objective  $\in \arg \max f(x)$ 

regret

 $R_T = \sum_{t=1}^{T} \left[ f(x^*) - f(x_t) \right]$ 

Noise-perturbed evaluations

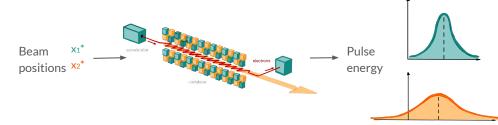
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**Tuning accelerators** 



Risk-averse regret  $R_T = \sum_{t=1}^T \left[ ext{MV}(x^*) - ext{MV}(x_t) 
ight]$ 

Noise-perturbed evaluations

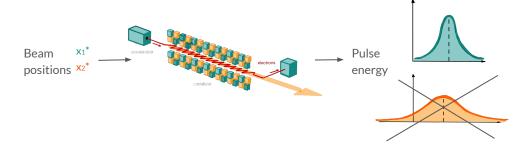
$$y_t = f(x_t) + \xi(x_t)$$

ho(x)-sub-Gaussian noise

Optimism under uncertainty via RAHBO

$$x_t \in \underset{x \in \mathcal{X}}{\operatorname{arg\,max}} \operatorname{ucb}_{t-1}^f(x) - \alpha \operatorname{lcb}_{t-1}^{\rho^2}(x)$$

#### **Tuning accelerators**



Mean-Variance objective  $\mathrm{MV}(x) = f(x) - \alpha \rho^2(x)$   $x^* \in \arg\max_{x \in \mathcal{X}} \mathrm{MV}(x)$ 

Risk-averse regret  $R_T = \sum_{t=1}^T \Big[ ext{MV}(x^*) - ext{MV}(x_t) \Big]$ 

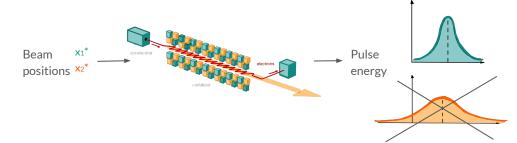
Noise-perturbed evaluations

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Optimism under uncertainty via RAHBO:

$$x_t \in \operatorname*{arg\,max}_{x \in \mathcal{X}} \ \operatorname{ucb}_{t-1}^f(x) - \alpha \ \operatorname{lcb}_{t-1}^{\rho^2}(x)$$

#### **Tuning accelerators**



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Risk-averse regret  $R_{\ell}$ 

$$R_T = \sum_{t=1}^{T} \left[ \text{MV}(x^*) - \text{MV}(x_t) \right]$$

Noise-perturbed evaluations

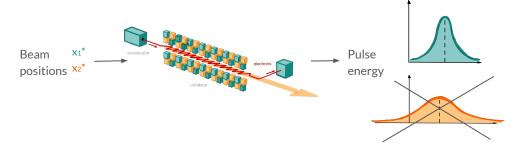
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#### **Tuning accelerators**



#### coefficient of absolute risk tolerance

Mean-Variance objective

$$MV(x) = f(x) - \alpha \rho^2(x)$$

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Risk-averse regret

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Noise-perturbed evaluations

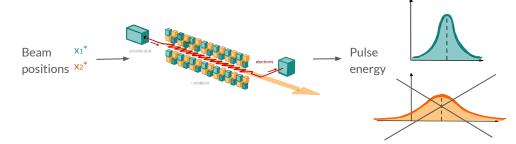
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#### **Tuning accelerators**



Mean-Variance objective  $\mathbf{MV}(x) = f(x) - lpha 
ho^2(x)$   $x^* \in rg \max_{x \in \mathcal{X}} \mathbf{MV}(x)$ 

Risk-averse regret  $R_T = \sum_{t=1}^T \left[ ext{MV}(x^*) - ext{MV}(x_t) 
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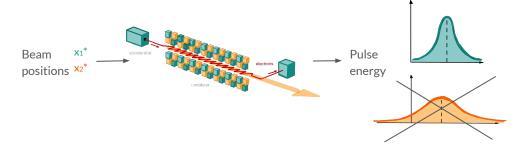
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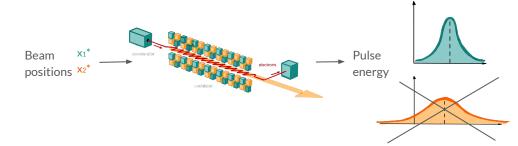
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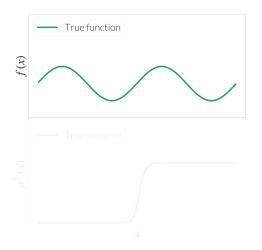
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#### **Tuning accelerators**



In the risk-averse setting the maximizers for mean-variance (RAHBO) and for mean (GP-UCB) might not coincide

Unknown objective (sine) with two global optimizers -- but one has much higher noise variance



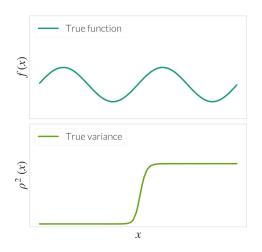
RAHBO: exploration & exploitation & risk tradeoff





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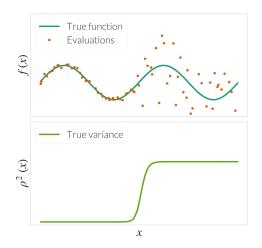
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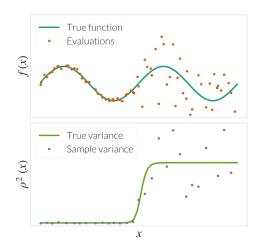
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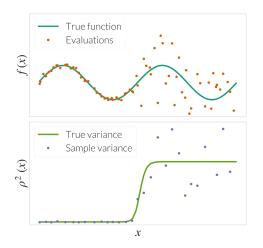
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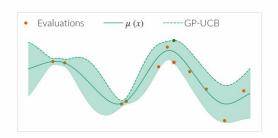
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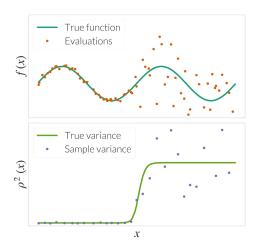
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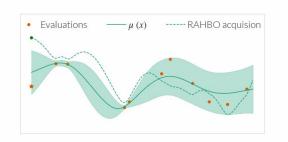


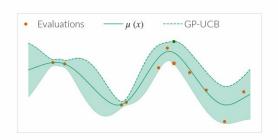
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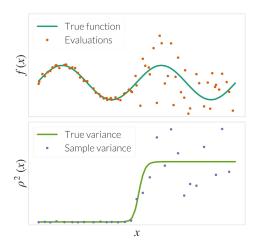
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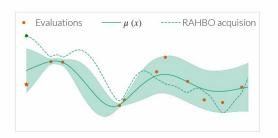


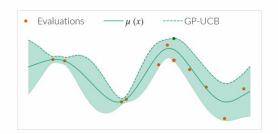
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RAHBO: exploration & exploitation & risk tradeoff

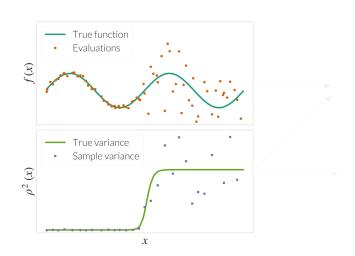


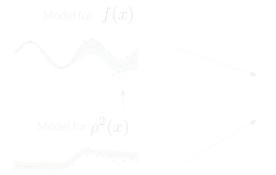




RAHBO attaints sublinear regret  $R_T = \mathcal{O}(\sqrt{T})$ 





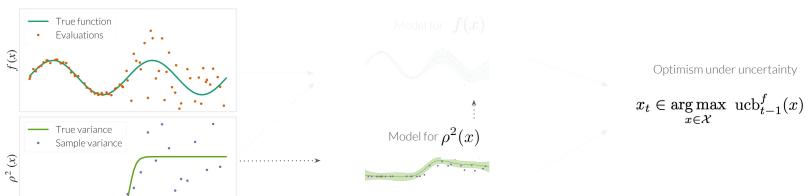


Optimism under uncertainty

$$x_t \in \operatorname*{arg\,max}_{x \in \mathcal{X}} \ \operatorname{ucb}_{t-1}^f(x) - \alpha \ \operatorname{lcb}_{t-1}^{\rho^2}(x)$$



 $\chi$ 

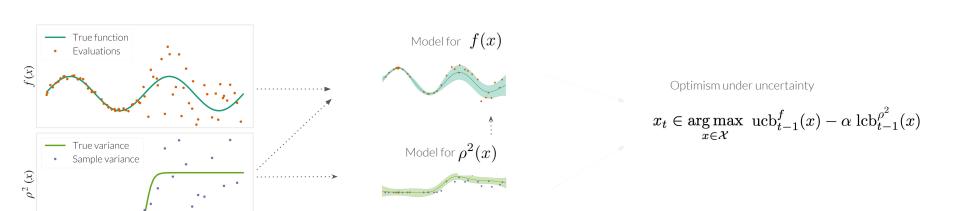


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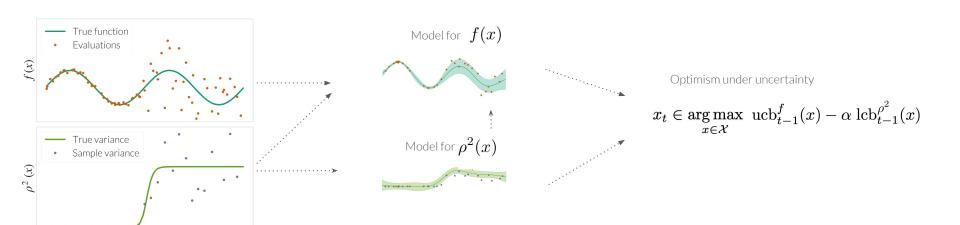
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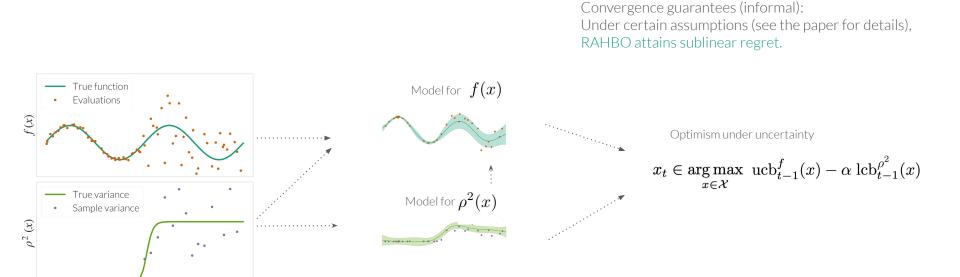


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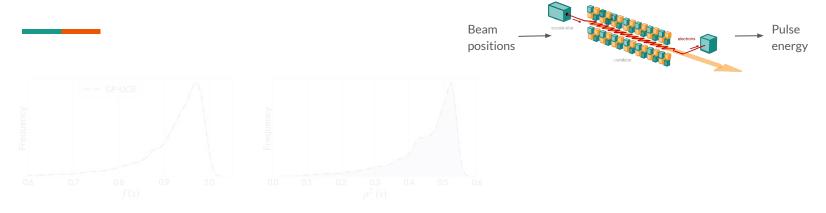
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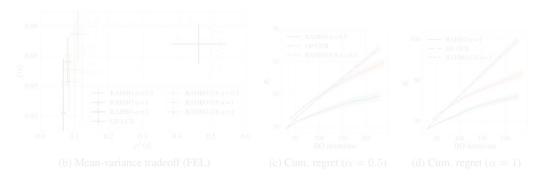
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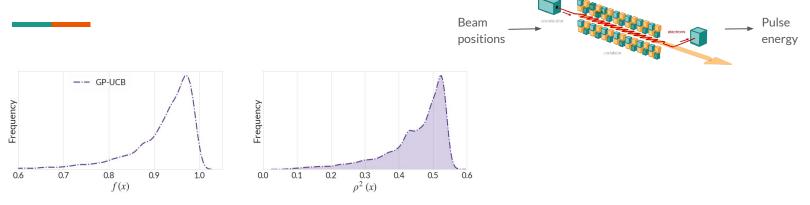
#### Tuning Swiss Free Electron Laser



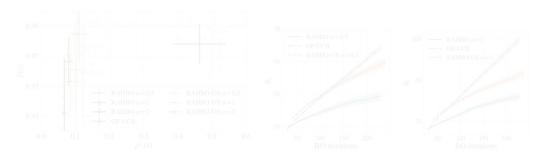
Empirical distribution of true values at acquired points



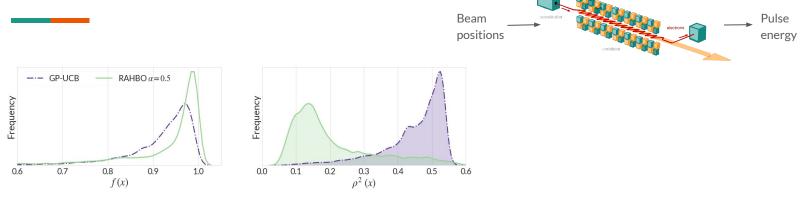
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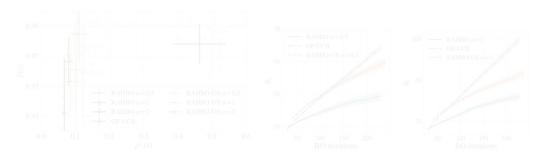
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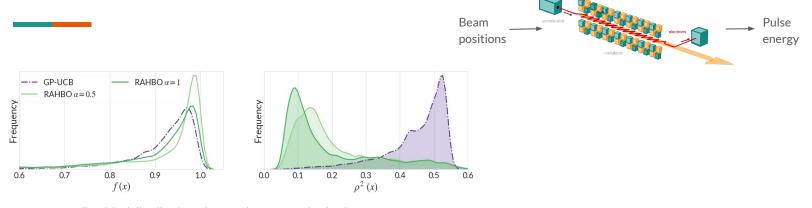
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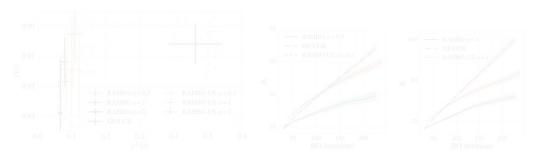
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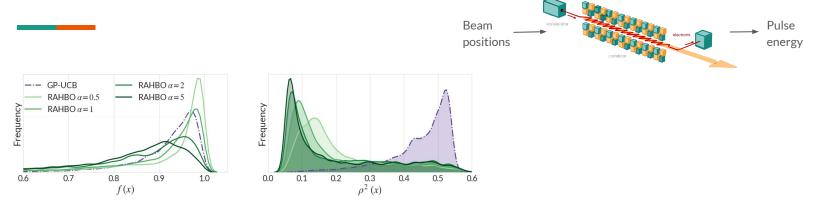
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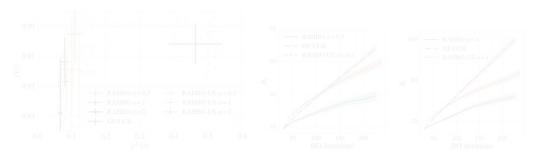
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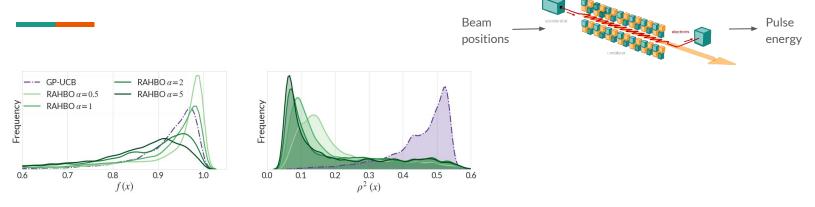
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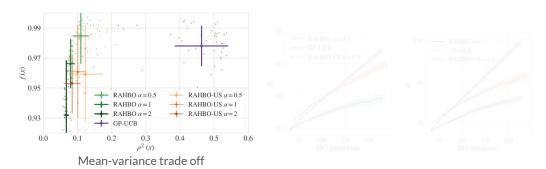
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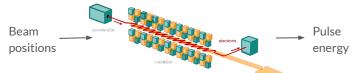


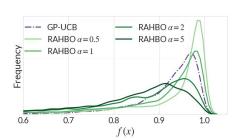
### RAHBO results for SwissFEL

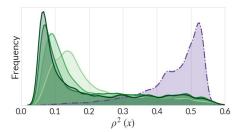


Empirical distribution of true values at acquired points

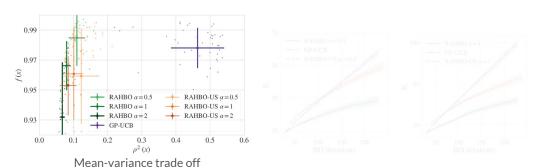




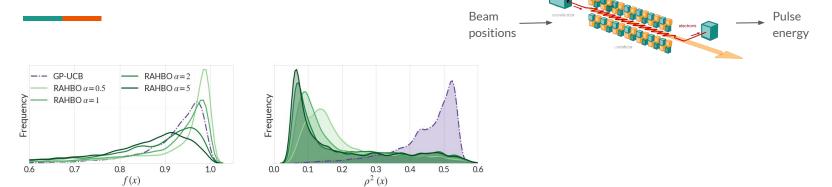




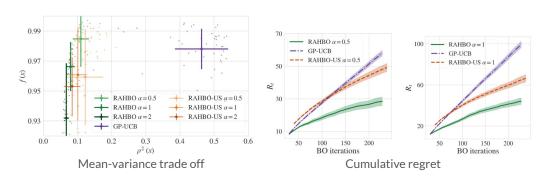
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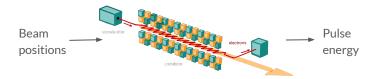
### RAHBO results for Swiss FEL



Empirical distribution of true values at acquired points



# Summary



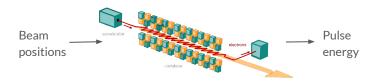
### Goal:

- Incorporate risk into exploration-exploitation trade-off

#### Our contributions:

- Mean-variance approach for Bayesian optimization
- Practical algorithm based on optimism under the face of uncertainty
- Theoretical regret bounds
- Empirical results on SwissFEL simulator and ML model tuning

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#### Goal:

- Avoid cost of failure due to noisy realizations in high-stakes applications
- Drop by our poster for more details:)

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