Runtime Analysis

a)

in	out
0	0
1	2
2	4
3	8
4	16
5	32
2^{2k}	

$$i = 2^{2^k}$$

$$2^k = \log n$$

$$\log(2^k) = \log(\log n)$$

$$k = \log(\log(n))$$

$$\{ \quad \theta(1)$$

$$k=1$$

$$\boxed{\Theta(\log(\log(n)))}$$

b) for → n times $k < i^3$

if → $\sqrt{n}$ times

i	$k <$
1	1
2	8
3	27

$n = 1$

$\sqrt{n} = 1$

$2\sqrt{n} = 2$

$3\sqrt{n} = 3$

$i = k\sqrt{n}$

inner

$$\sum_{k=0}^{i^3} \Theta(1) = \Theta(i^3)$$

$$\text{if} = \Theta(\sqrt{n})$$

$$\text{outer}: \sum_{i=1}^{n} (\Theta(1) + \Theta(i^3))$$

$$= \Theta(n) + \sum_{i=1}^{n} \Theta(i^3)$$

$$\sum_{i=1}^{n} \Theta(1) + \sum_{i=1}^{n} \sum_{k=1}^{n} (k\sqrt{n})^3$$

$$(\sqrt{n})^3 \sum_{k=1}^{\sqrt{n}} k^3 = \left(\frac{\sqrt{n}(\sqrt{n}+1)}{2}\right)^2 \cdot \sqrt{n}^3 + \Theta(n)$$

$$= \frac{n^2 + 2\sqrt{n} + n}{4} \cdot n\sqrt{n} + n$$

$$= \frac{n^3\sqrt{n} + 2n^2 + n^2\sqrt{n} + n}{4}$$

$n^{3/2}$ most sig

$$\boxed{\Theta(n^{3/2})}$$

c) for → n times

for ⇒ n times

if → true atmost n times (n^2)

for → log n times

$m = 1, 2, 4, 8, 2^k$

$2^k = n$

$k = \log n$

$A[k] = i$

$A[1] = 1$

$A[2] = 1$

$A[\ldots k] = 1$

Assume contents of A[ ] dont change bc loop happens k=n times

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \left(\Theta(1) + \Theta\left(\sum_{n=1,2,4}^{n} \Theta(1)\right)\right)$$

$n = 1, 2, 4, 8$

$$\sum_{i=1}^{n} \Theta(n) + \sum_{k=1}^{n} \Theta(\log n) = \Theta(n^2) + \Theta(n \log n)$$

$$= \boxed{\Theta(n^2)}$$

d) for → runs n times

$i = 10 \times \frac{3^k}{2}$

$n = 10 \times \frac{3^k}{2}$

$S = \log_{\frac{3}{2}} \frac{n}{10}$

$$= \sum_{i=0}^{n} \left(\Theta(1) + O\left(\sum_{j=1}^{10 \times \frac{3^k}{2}} \Theta(1) \right) \right)$$

$$= \sum_{i=0}^{n} \Theta(1) + \sum \Theta \sum_{j=1}^{10k \frac{3^k}{2}} \Theta(1)))$$

$$= \Theta(n) + \sum_{k=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \sum_{j=0}^{10 \times \frac{3^k}{2}} \Theta(1)$$

$$\Theta(n) + \sum_{k=0}^{\log_{\frac{3}{2}} \frac{n}{10}} \Theta\left(10 \times \frac{3^k}{2}\right)$$

$$= \Theta(n) + 10 \left(\frac{3}{2}\right)^{\log_{\frac{3}{2}} \frac{n}{10}}$$

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1} = \Theta(c^n)$$

$$n = \log_{\frac{3}{2}} \frac{n}{10}$$

$$= \Theta(n) + \Theta\left(10\left(\frac{n}{10}\right)\right) = \Theta(2n)$$

$$= \boxed{\Theta(n)}$$