1. unusual

$1 v \rightarrow 5! = 120$

$2 v \rightarrow \dfrac{5!}{2!} \times 4 = 246$

$3 v \rightarrow \dfrac{5!}{3!} \times 6 = 120$

1 subset when $\frac{1}{5}$ is v and rest open

2vs: 4 subsets when 3 spots for 4 non v leters

3vs: 6 subsets for 2 open spots for 4 non v

11 total subsets

$120 + 120 + 240 = \boxed{480 \text{ strings}}$

2. $nCr \left(\begin{smallmatrix} 13 \\ 2 \end{smallmatrix} \right) = \dfrac{13!}{2!(11!)} = 78$

$nCr = \left(\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) = \dfrac{4!}{2! \, 2!} = 6$

$52 - 8 \rightarrow 44$

$\left(\begin{smallmatrix} 13 \\ 2 \end{smallmatrix} \right) \left(\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) \left(\begin{smallmatrix} 4 \\ 2 \end{smallmatrix} \right) \left(\begin{smallmatrix} 44 \\ 1 \end{smallmatrix} \right) = \boxed{123,552 \text{ ways}}$

3. $\binom{n+k-1}{k}$

$_{cups}^{6}$ $\binom{6+16-1}{16} = \binom{21}{16} = \dfrac{20!}{15! \; 5!} = 15503.999$

at most 1 $\binom{6+15-1}{15} = \binom{20}{15}$
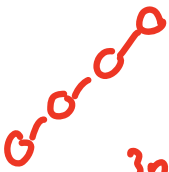
$$\dfrac{21!}{16! \; 5!} = 20349$$

$$\binom{20}{15} + \binom{21}{16} = 35852.99u$$

$\boxed{35853 \; ways}$

4. 2 nodes 2 ways

3 nodes 5 ways

4 nodes 2 ways + 2ways = 4 ways

6 ways
3nodes & 3nodes = 10 ways

10+4 = 14 ways

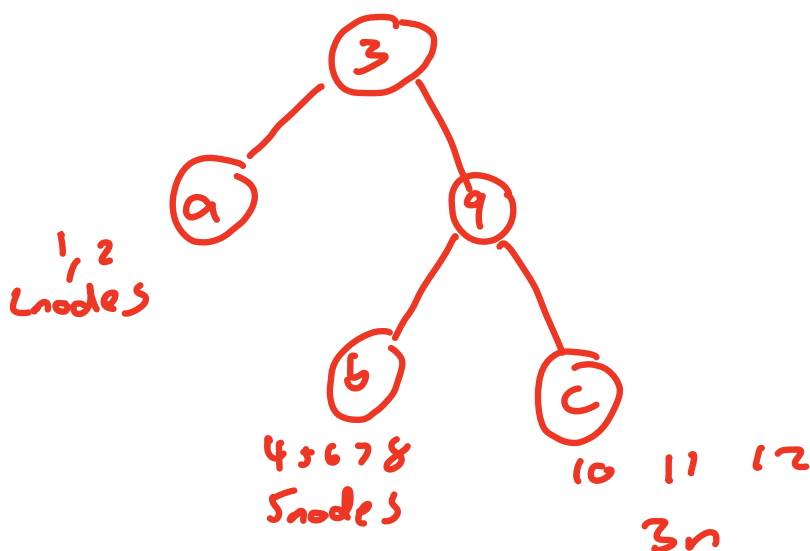5 nodes

O

4 nodes = 14 ways

q

4 nodes   14 ways

O
2   2

14 ways

$14 + 14 + 14 = \boxed{42 \text{ ways}}$

(3)
a   q
1, 2
2 nodes
b   c

4 5 6 7 8   10 11 12
5 nodes   3n

Check 5 cases:

Case 1: a=root   14 ways

Case 2: 3=root

1   3   5 ways

Case 3: c=root

4 ways
2   2

Case 4: d= root   5 ways

Case 5: e=root   14 ways

$$14 + 5 + 4 + 5 + 14 = 42 \text{ ways}$$

Sub a: 2 ways   node 1 2

Sub b: 42 ways   5 nodes   (2)(42)(5)=420

Sub c: 5 ways → 3 nodes → 5 ways

$$total = 42 \cdot 2 \cdot 5$$

420 ways

5.

4 nurses

7	1	1	1
6	2	1	1
5	3	1	1
5	2	2	1
4	4	1	1
4	3	2	1
3	3	2	2

3 nurses

8	1	1
7	2	1
6	3	1
6	2	2
5	4	1
5	3	2
4	4	2
4	3	3

31 3 3
2 2 2 4

$8 + 9 = 17$

17 combinations