Topics:

heads; tails; either

- coin fair or biased (contd) + Terminology
- 2-sided vs 1-sided tests
- [Framework] for Hyp-testing

- Z-test
- T-test
- Z-proportions test

Toss 100 times → 65 heads

Previous class

$$\text{p-val} = P(\underbrace{T \geq T_{obs} = 65}\,|\,H_0) = 0.17 \% < \alpha = 5\% \text{ (default)}$$

↳ coin is fair

↳ significance level

⇓

reject H_0 and accept H_a

⇓

coin is biased towards heads ✓

[disb of T under H_0

↳ $Bin(n=100; p=0.5)$

biased towards tails ✗

$$\overbrace{T_{obs} = 20}$$

$$\text{p-val} = P(T \geq 20 \mid H_0) = \overbrace{0.91}^{91\%} \checkmark > \alpha = \overbrace{0.05}^{5\%}$$

$$\Downarrow$$

$$\text{p-val} > \alpha$$

$$\Downarrow$$

accept $\underline{\underline{H_0}}$ ✓

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H_0 : coin is fair

H_a : coin is biased towards heads

(later)

H_0 : coin is fair

H_a : coin is unfair → heads
→ tails

4 / 4

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$$\text{p-val} < \alpha = 5\%$$

5 / 5

α (default : 5%)

$$p\text{-val} < \overbrace{0.05}^{\alpha}$$

↓

reject H_0

↓

neednot be 5%

cases: 1%

T_{obs} = let 56

5%

17%

$0.17 = \text{p-val} > \alpha = 0.05$

$\boxed{H_0}$ | H_a

cannot reject H_0

$\Downarrow$

accept H_0

17%

not small enough

Expt ✓

H_0, H_a ✓

p-val ✓

α ✓

H_0: coin is fair

H_a: coin is biased towards heads

H_0	H_a

$\boxed{H_0}$: coin is biased towards heads

H_a: coin is fair

let $T_{obs} = 65$

$P(H) = 0.51$ or 0.52 or 0.53 or 0.54 ... ∞-many

Cannot compute → $P(T \geq 65 \mid H_0) = ?$

$Bin(n = 100; p)$

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(*) Choose $\overline{H_0}$ carefully s.t you can compute

p-val

$P(T \geq T_{obs} \mid H_0) = \cdots$

disb of T under H_0

10 / 13

One-sided tests:

T = #heads in 100 tosses

— 100 tosses

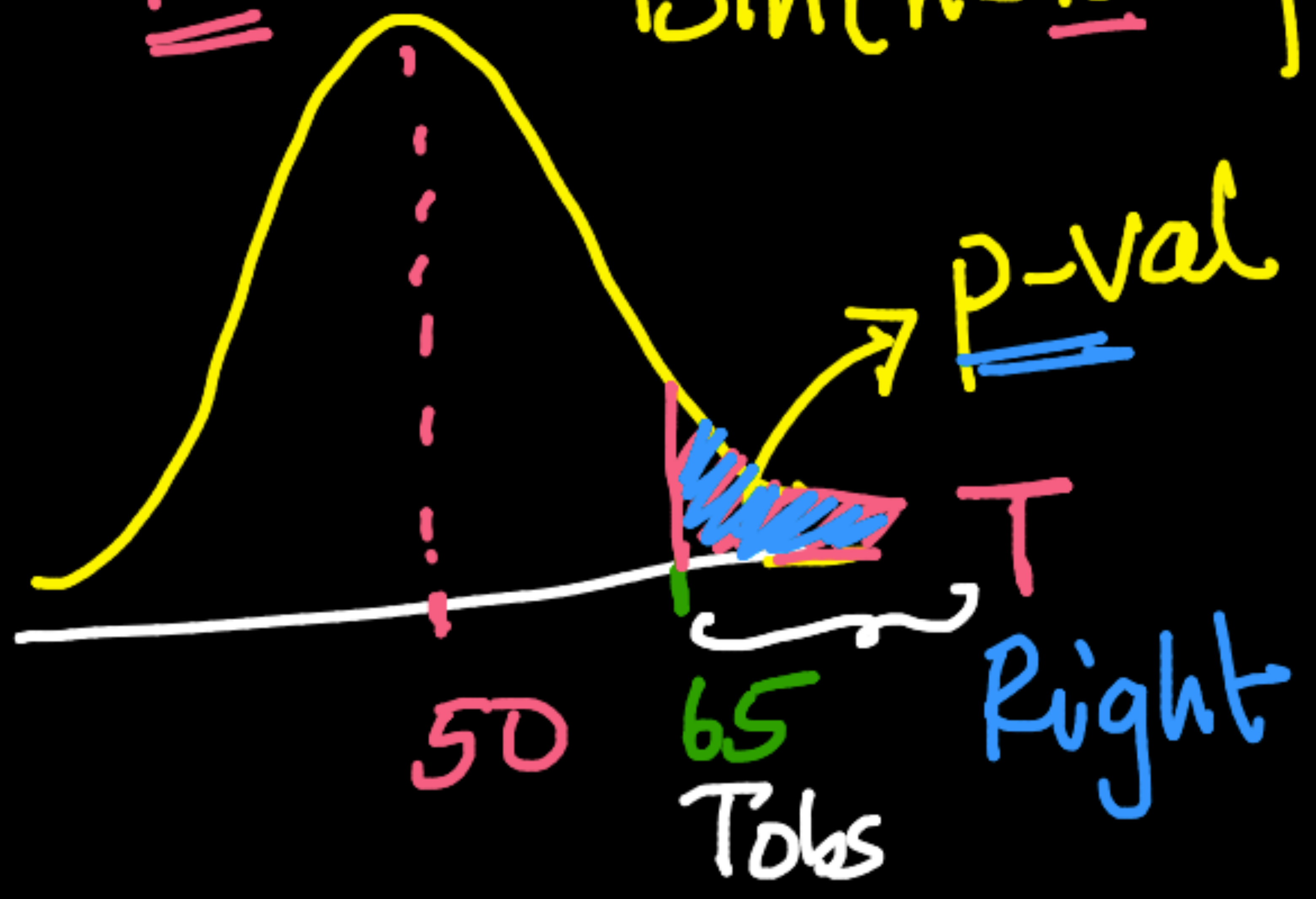
—

[H_0, H_a; Expt; T

↳ right tailed test (one-sided)

T under H_0

PDF: $Bin(n=100;\ p=0.5)$



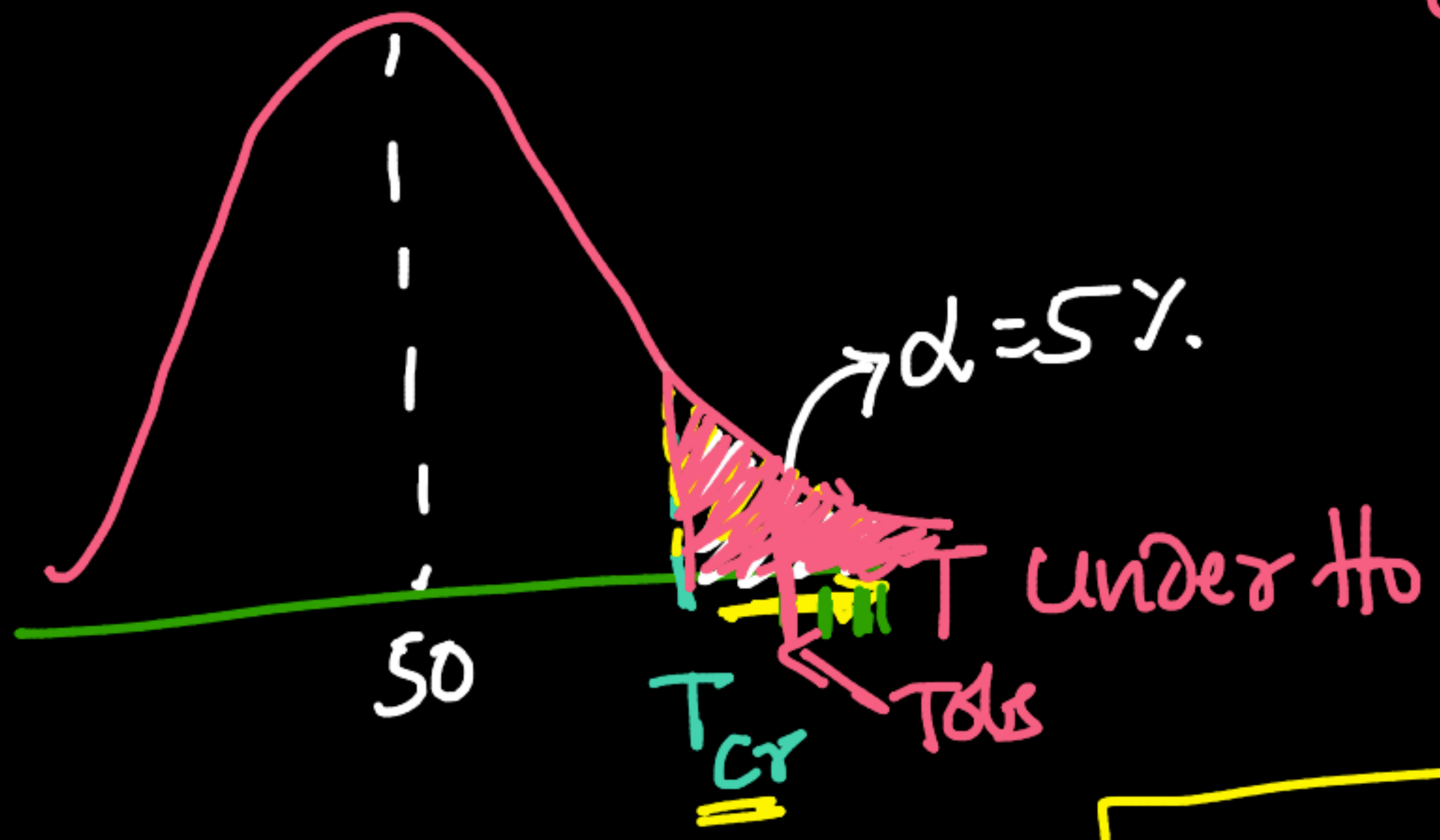
$P(T \geq 65 \mid H_0)$ = p-val =

$\boxed{T_{obs}}$

significance level
$\alpha = 5\%$

Right-tailed test

max $T = 100$



T_{cr} = critical value

$T_{obs} > T_{cr}$

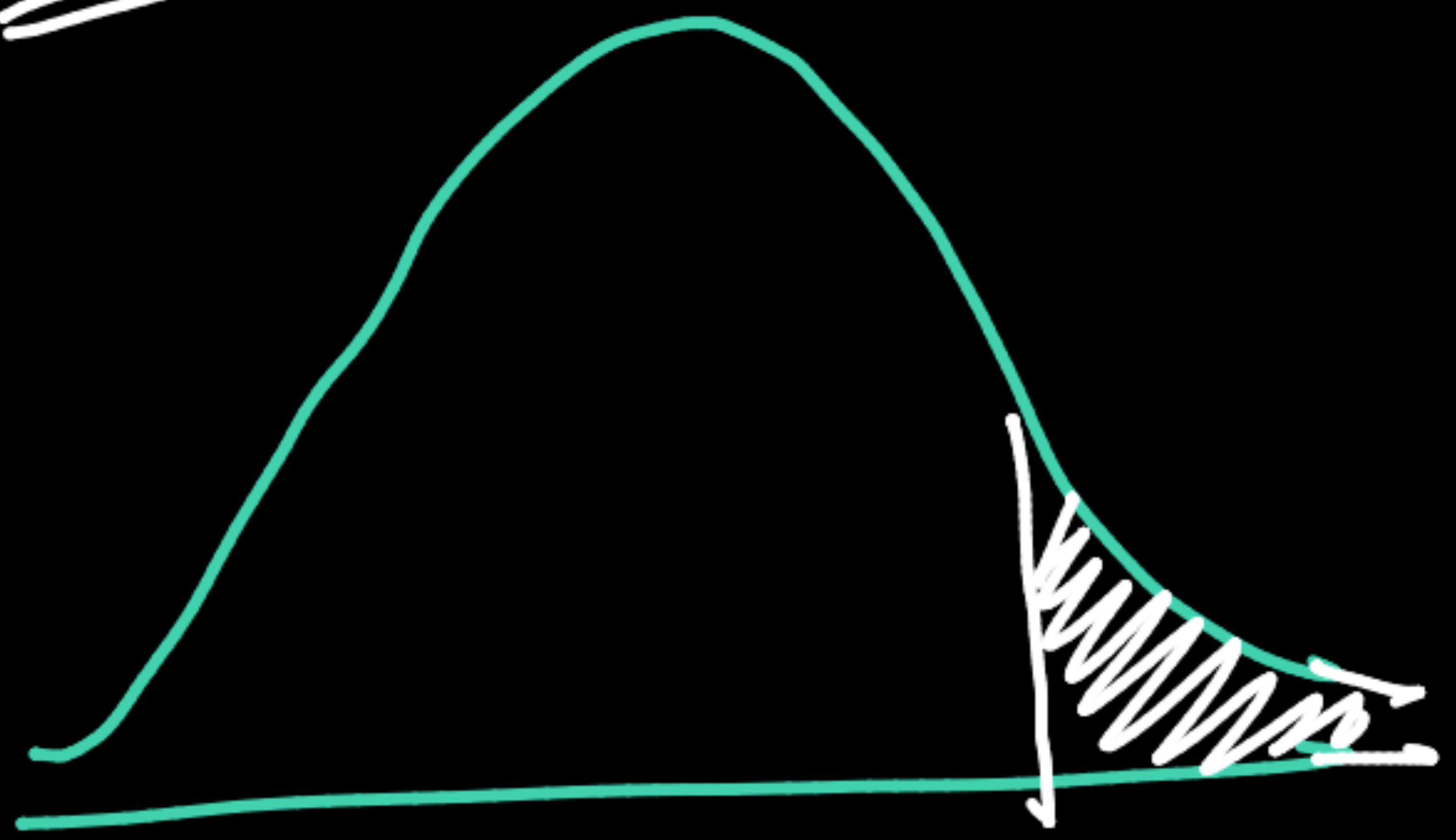
$\Downarrow$

$p\text{-val} < \alpha$

↳ reject H_0

real-valued T

Gaussian



AUC

$T_{cr} = 1.23 \rightarrow 4.989\%$ ✓

$1.24 \rightarrow 5.01\%$

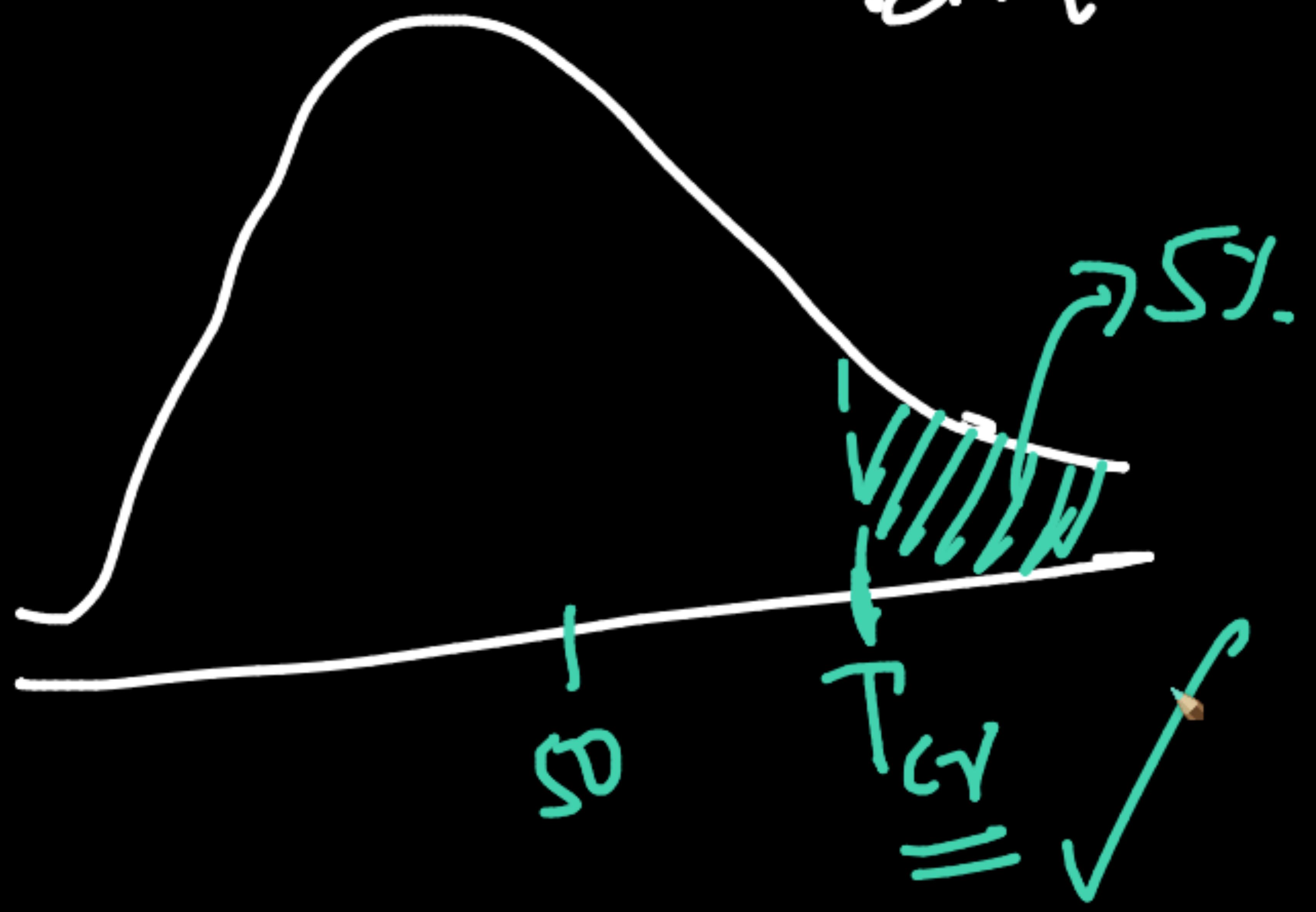
$\alpha = 5\%$

p-val vs α

$1 - CDF(64)$

$\boxed{T_{obs} = 65}$ → p-val $= 0.17\%$

$Bin(n = 100 ; p = 0.5$



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T_{cr} is the T_{obs}-value where

$$p\text{-val}_{(T_{obs})} = \alpha$$

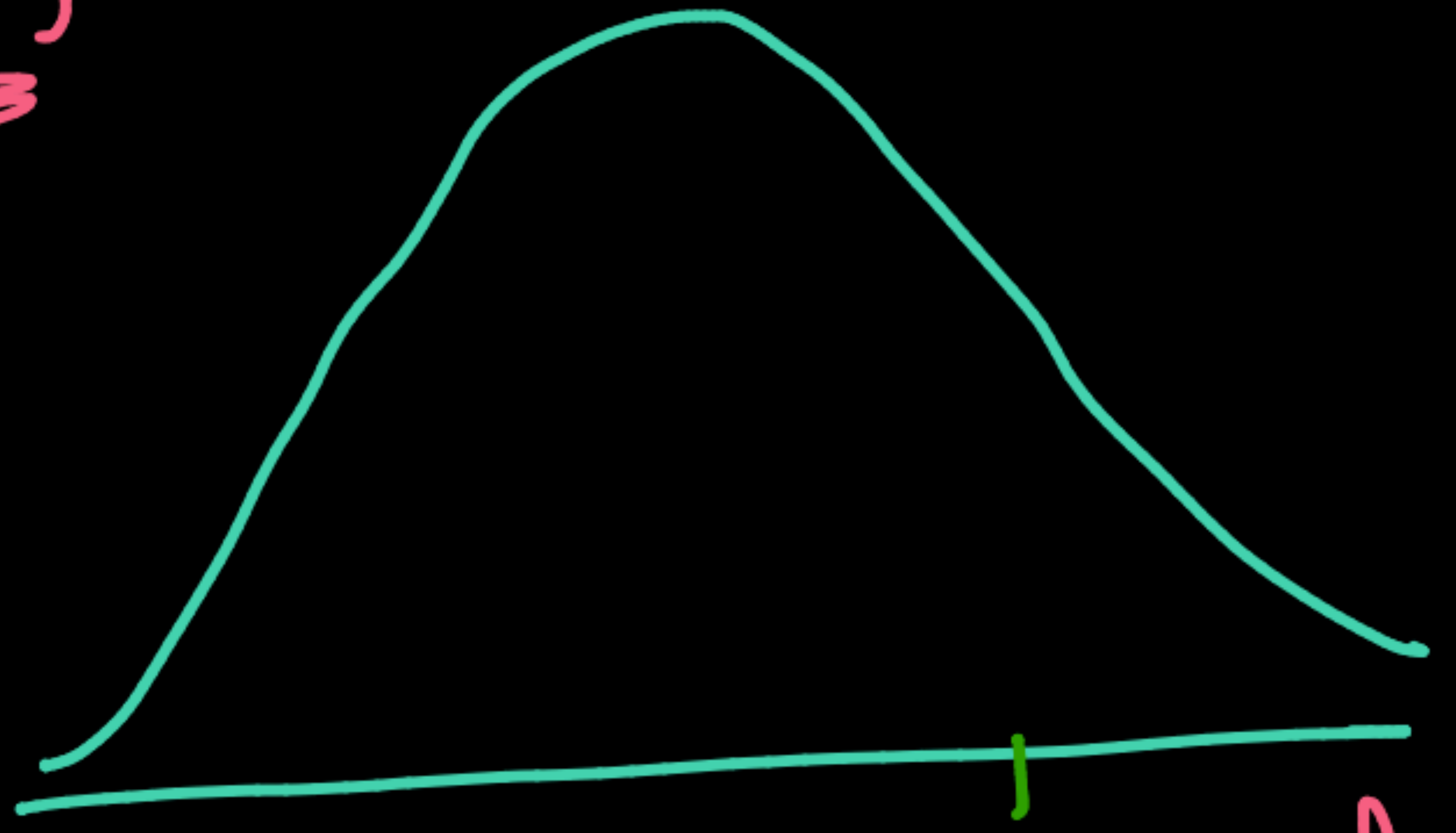
16 / 16

$T_{obs} = 57$

$\alpha = 5\%$

$T_{cr} = \cancel{57}$

58 ✓



T

dont reject H_0

$T_{obs} : 57 \rightarrow$ p-val $= 6.6\%$

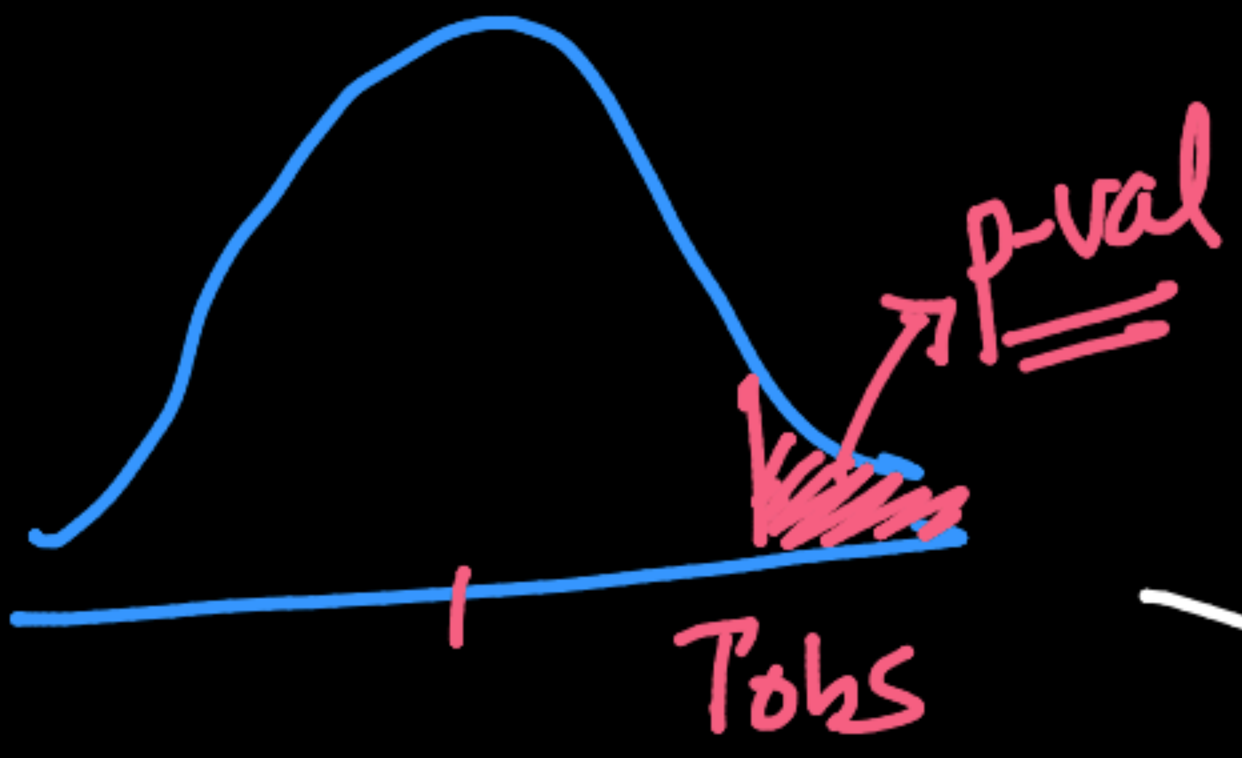
$T_{obs} : \boxed{58} \rightarrow$ p-val $= 4.4\%$

reject H_0

(e.g)

{T under H_0

(H_0, H_a)

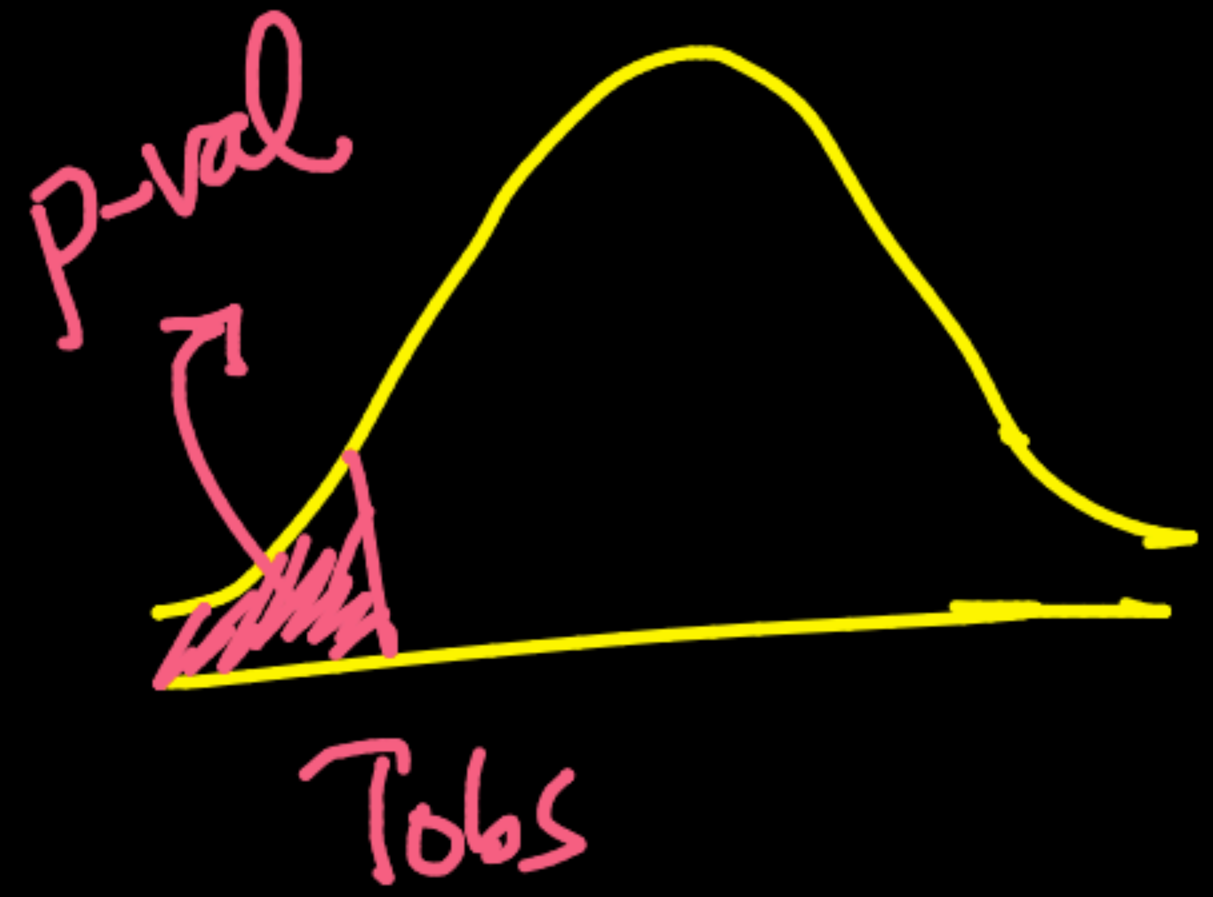


p-val = ...

→ Right tailed

More examples:

H_0, H_a, T,
T under H_0;
p-val



→ Left tailed

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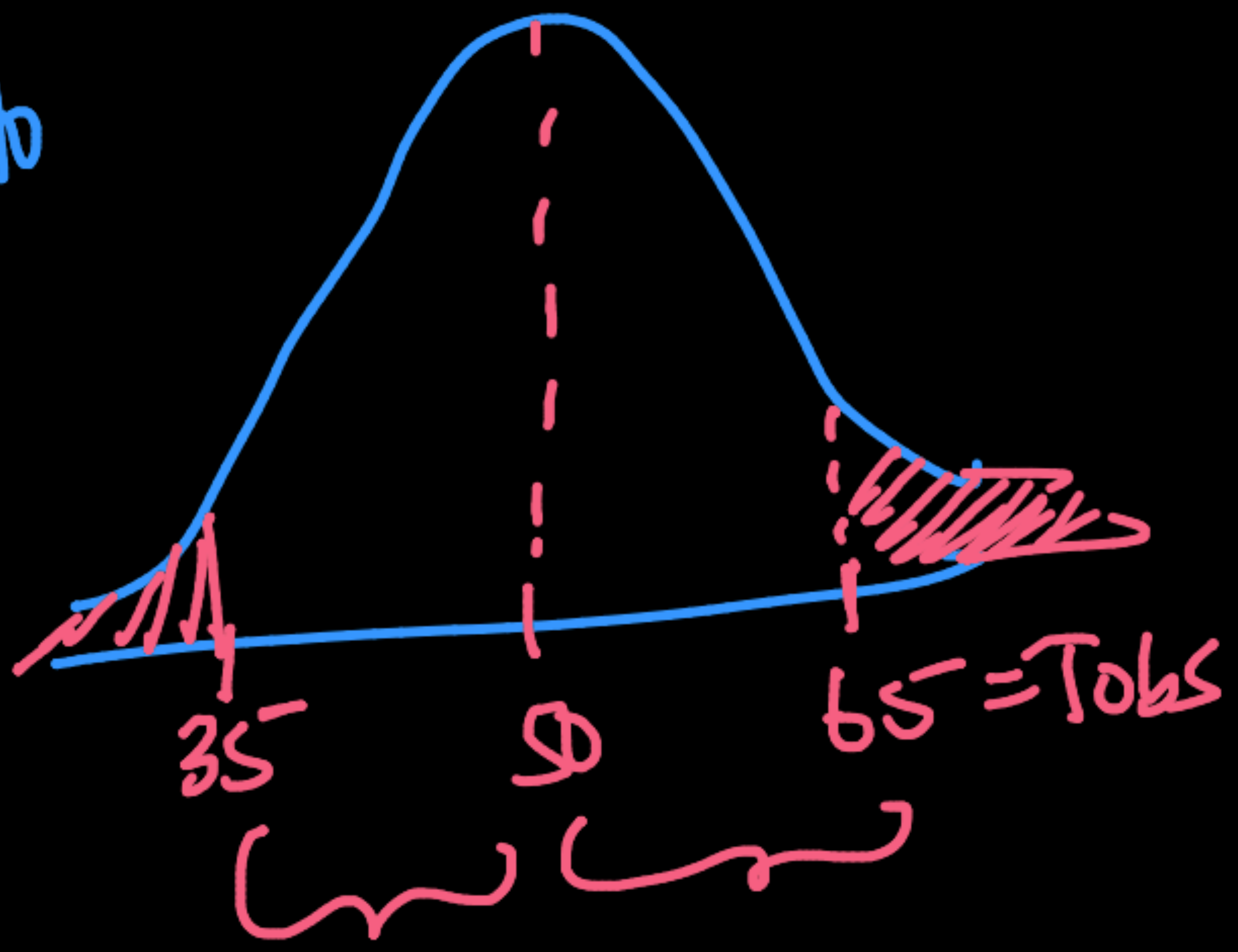
(later)

Ho, Ha, T under Ho

p.val



▨ : p-val

→ 2-sided/tailed test

Prev. example:

H_0: coin is fair

H_a: coin is biased towards heads

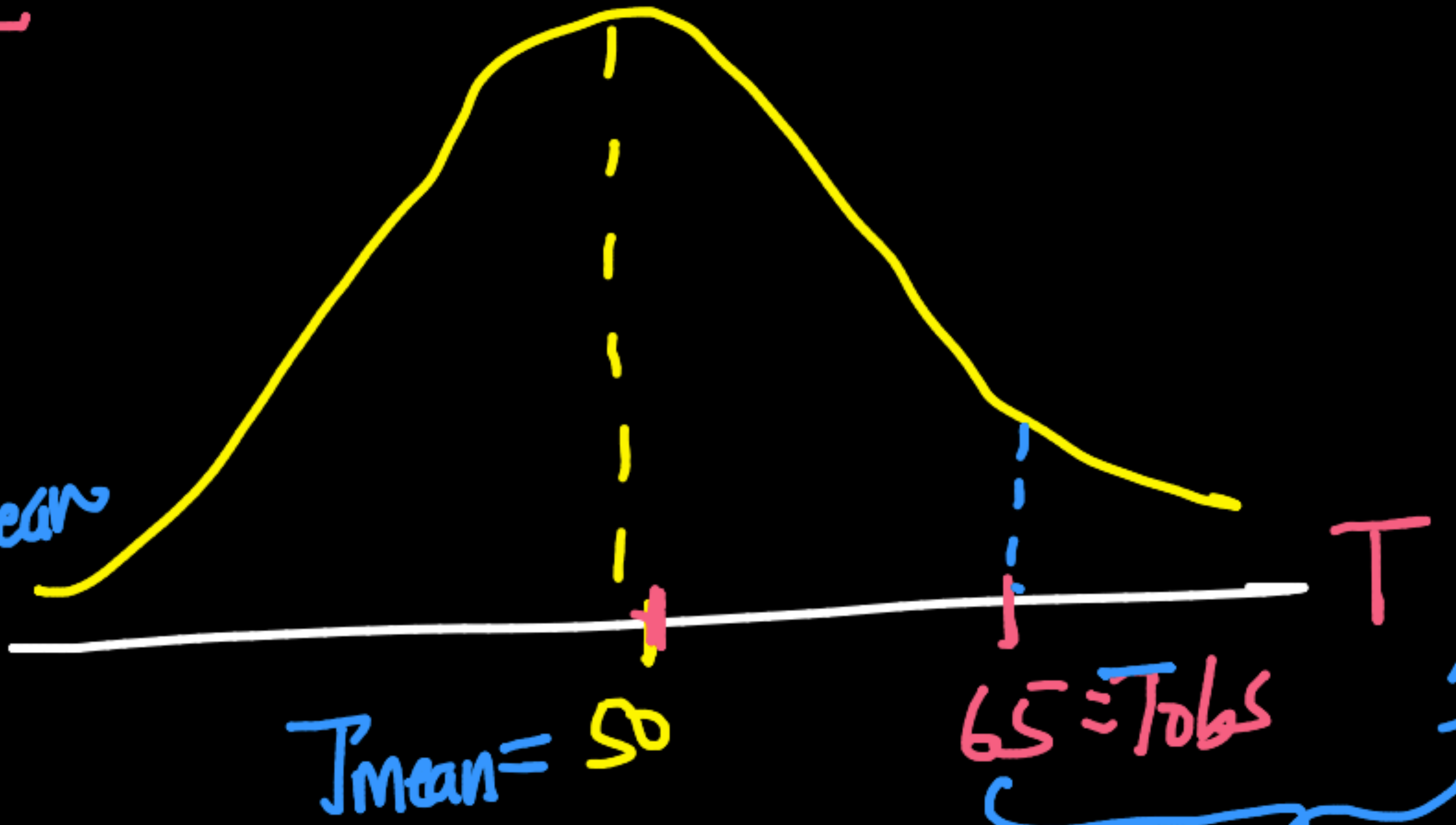
T = # heads in 100 tosses

$T_{obs} = 65$

if H_a were true:

T_{obs}

$T_{obs} > 50 = T_{mean}$



right tailed test

(e.g)

H_0: coin is fair

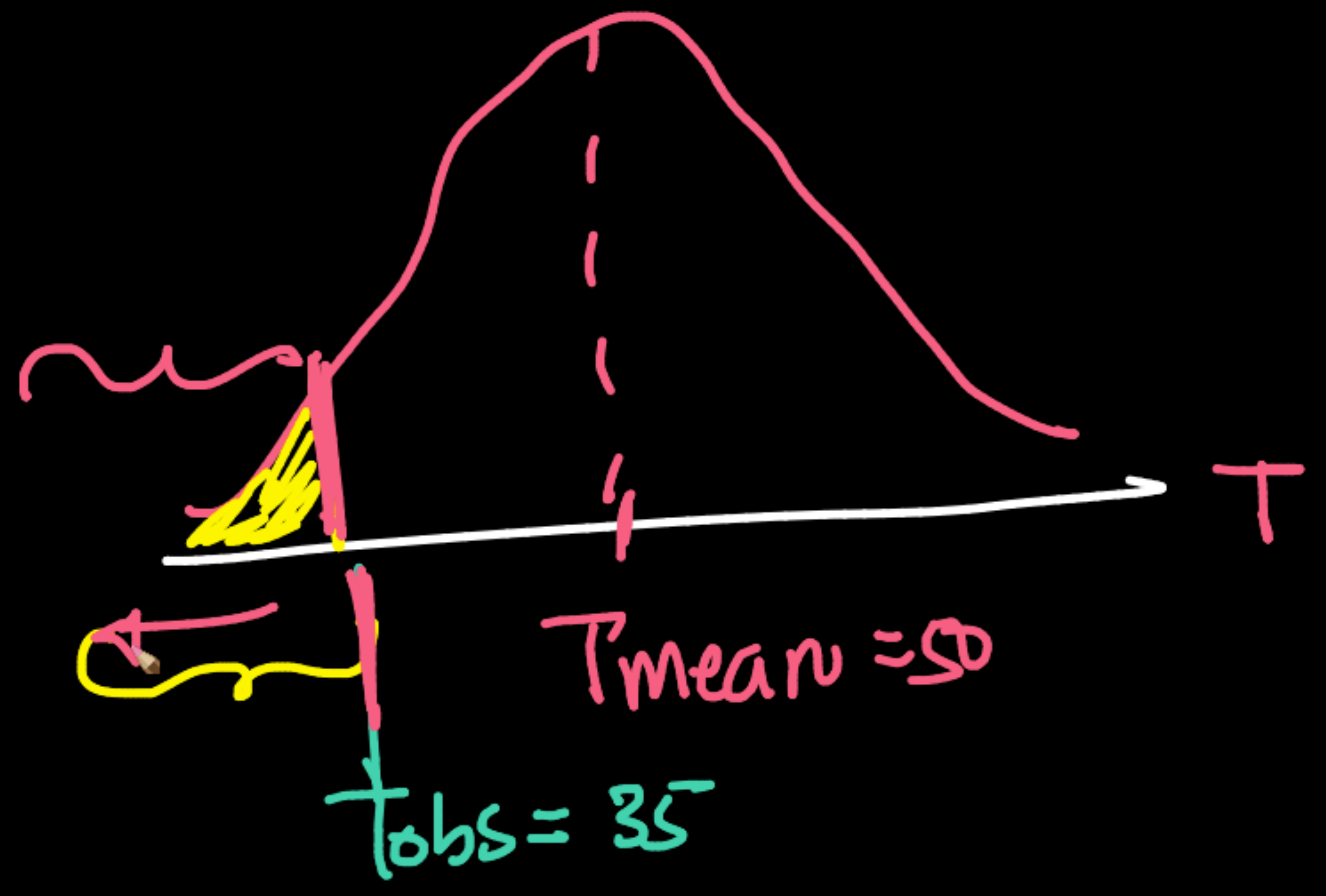
H_a: coin is biased towards tails

T = # heads in 100 tosses

left-tailed test

Under H_a:

✓ $T_{obs} < T_{mean}$



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$$\text{p-val} = P(T \boxed{\leq} 35 = T_{obs} \mid H_0)$$

$\hookrightarrow P(\text{observing } T \text{ "as extreme as" } T_{obs} \mid H_0)$

24 / 24

TWIST

H_0: coin is fair

H_a: coin is $\boxed{\text{unfair}}$ → heads, tails

T: # heads in 100 tosses

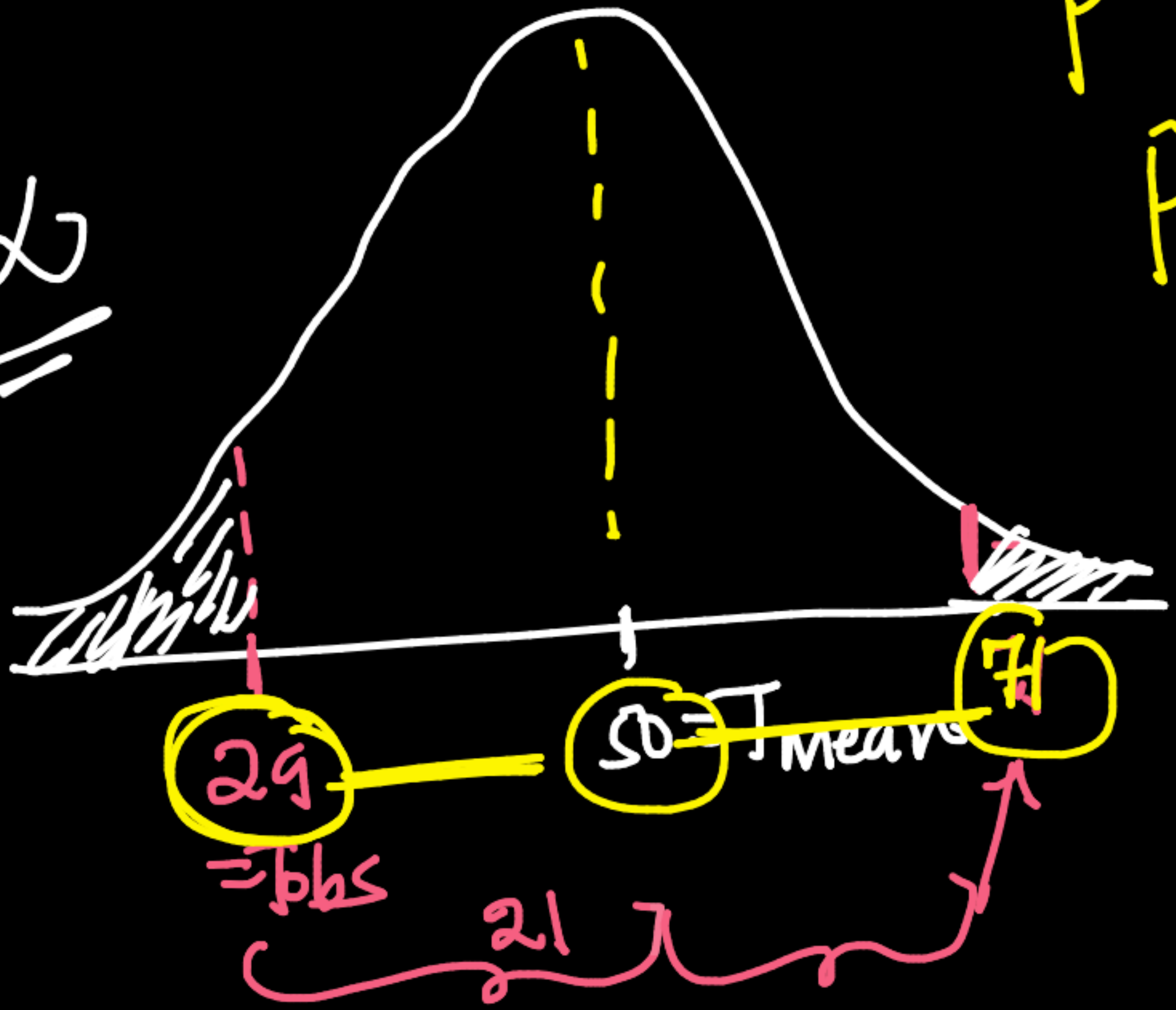
$T_{obs} = 29$ (let)

**

$$p\text{-val} = P(\text{Obs'ing } T \text{ as extreme as } T_{obs} = 29 \mid H_0)$$

$$p\text{-val} = P(T \le 29 \text{ or } T \ge 71 \mid H_0)$$

$= p\text{-val} < \alpha$



2-sided test

$T_{obs} = 29$ heads $\longrightarrow$ 71 tails

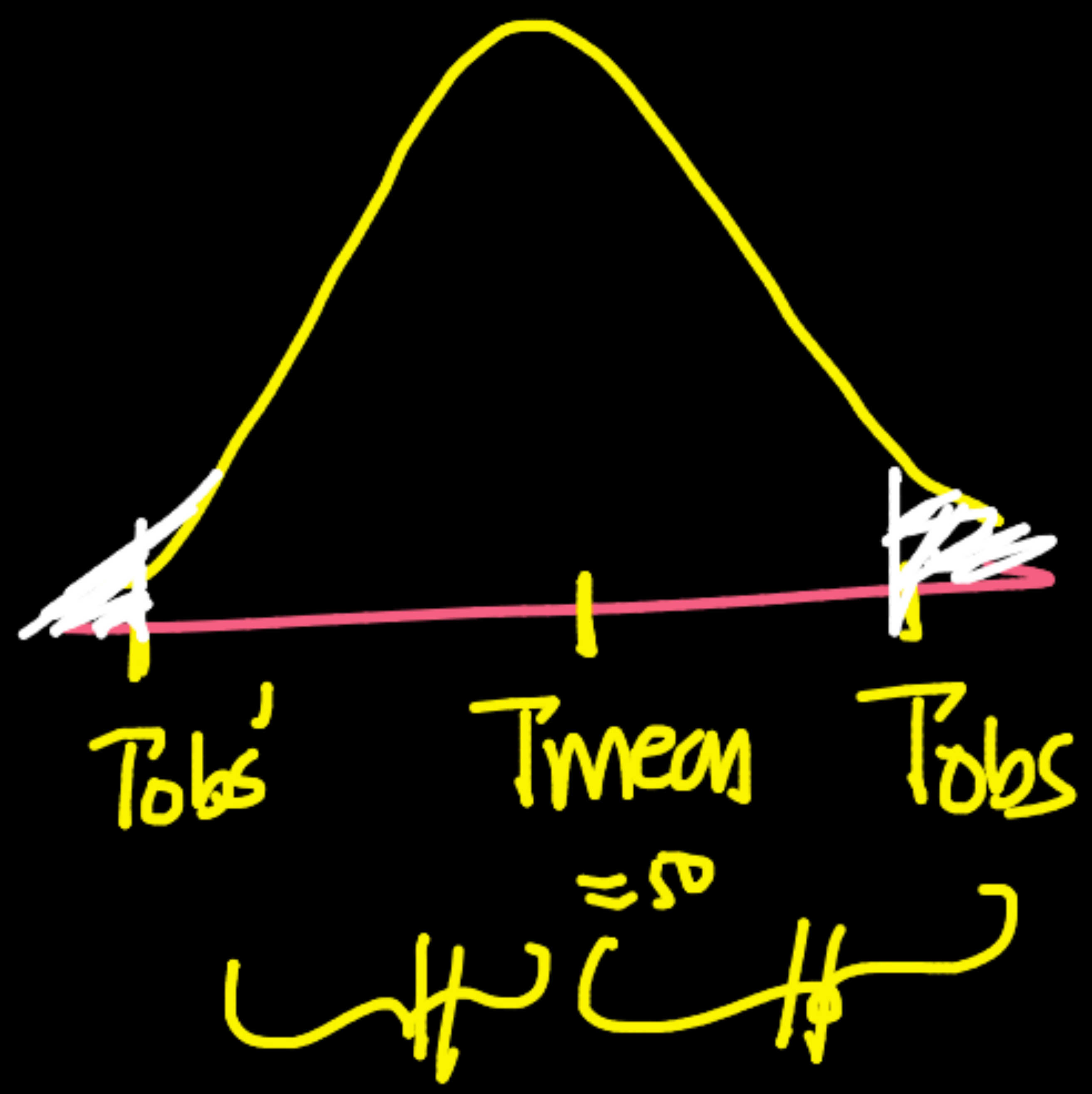
as extreme as

observing 29 tails $\longrightarrow$ 71 heads

✓ $\left[\; \text{p-val} = P(T \geq 71 \text{ or } T \leq 29) \right.$

2-sided test

$$T_{mean} - T_{obs}'$$
$$= T_{obs} - T_{mean}$$



$= \text{p-val}$

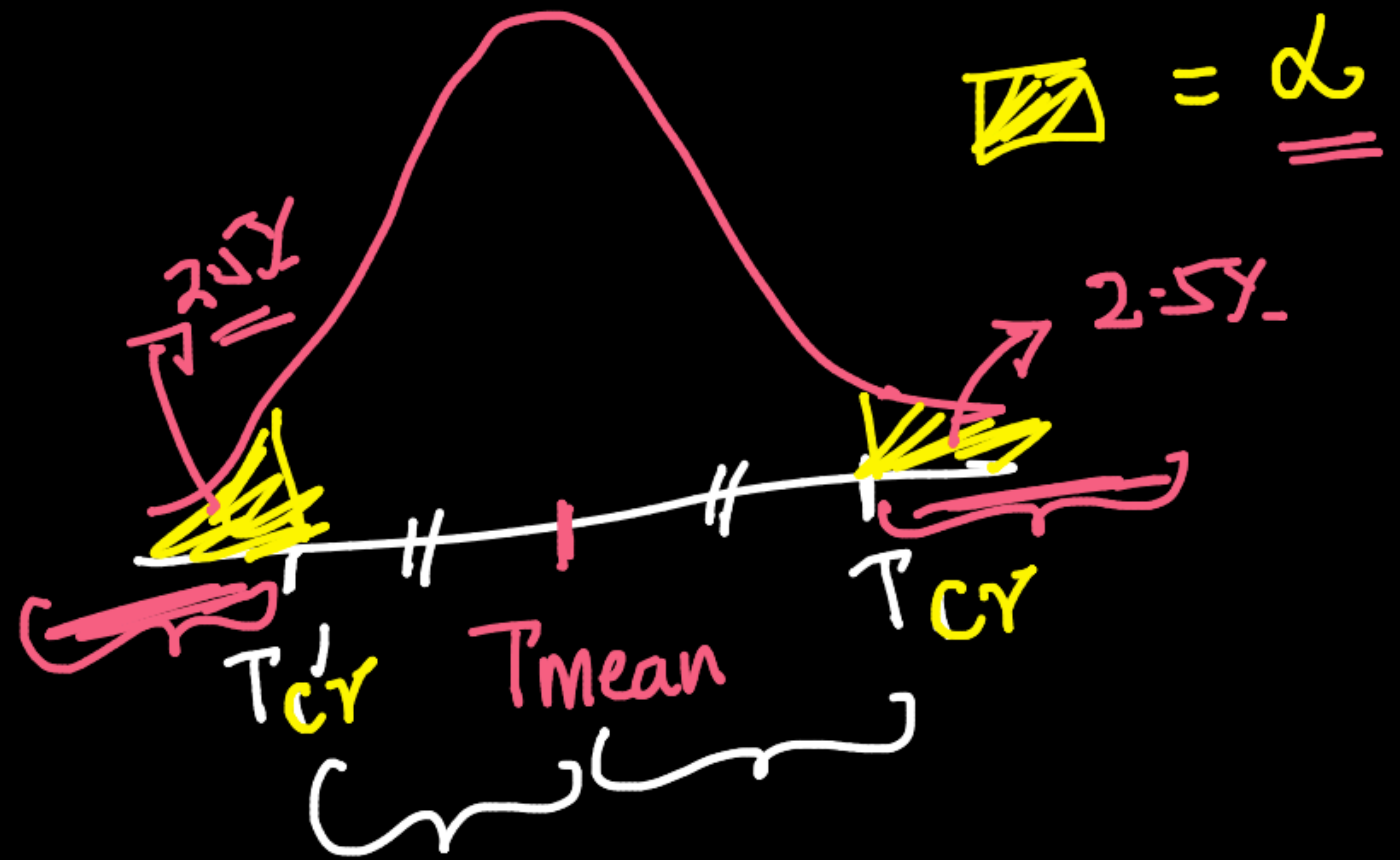
$< \alpha$

$\Downarrow$

reject H_0

Critical values:

Symmetric



$T_{obs} > T_{cr}$

or

$T_{obs} < T_{cr}$

Then T_{cr} & T'_{cr}

(*) what if PDF is not symmetric

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coin-loss ← [Framework]

covid

Test;

1. Expt; 'Sensible Test-statistic'; obtain T_{obs}

2. Ho; Ha 'Carefully'

p-val; distb of T under Ho

- Z-test
- T-test → ecommerce
- Z-prop. test
- χ^2 test
- KS test
- ANOVA

(Code)

30 / 31

3. determine Left or Right or 2-tailed test

→ H_0, H_a: T under H_0; T's behavior under the H_0

4. p-val = $P(T$ is as extreme as $T_{obs} \mid H_0)$

5. $\alpha = 5\%$ (default) → can change (medical contexts)

6. Compare p-val & α → reject H_0 or accept H_0

$\boxed{Q}$

H_0: coin is fair

H_a: coin is biased → heads

→ tails

T = # tails in 500 tosses

L, R, or $\boxed{\text{2-tailed test}}$

Task

7,30

[Covid medicines:

$n_1 = 100$ → M_1 → $x_{11}, x_{12} \dots x_{1,100}$ (x_{1i}) → $\bar{x}_1$: sample Mean

recovery time

$n_2 = 90$ → M_2 → $x_{21}, x_{22} \dots x_{2,90}$ (x_{2j}) → $\bar{x}_2$: sample mean

→ are they 'same'?

ICMR/WHO/FDA ...

one way!

mean-rec-time

$[\mu_1 = \mu_2]$

95% C.I

Hyp. testing

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Hyp-testing

Task ✓

1

✓ $H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$

2 → 'sensible'

$$\boxed{T} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

~~sensible~~

distb

T under H_0

$T \sim Z(0,1)$

$N(0,1)$

σ_1 & σ_2 are pop. std-dev

↳ $s_1 \approx \sigma_1$ as n_1 is not too small

$s_2 \approx \sigma_2$ as $n_2 = 90$

H_0: $T \to 0$

H_a: T : large +ve

large -ve

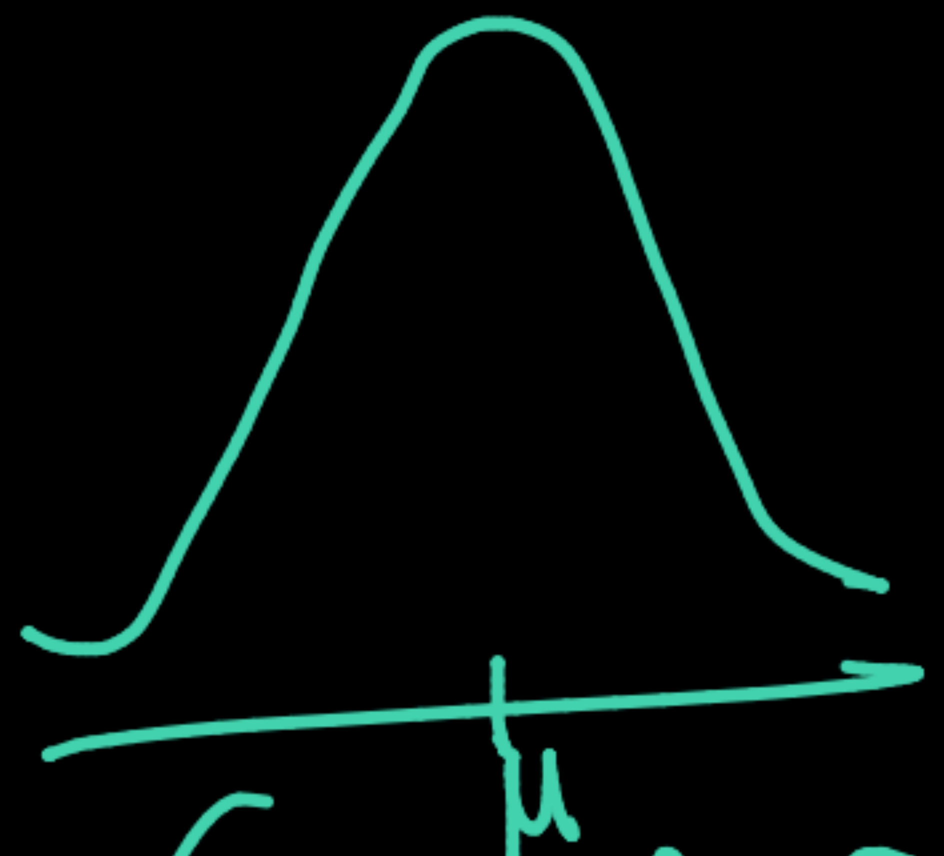
34 / 43

If $T = \bar{x}_1 - \bar{x}_2$ then we do NOT know distb of T under H0

$$\text{Std. err}^2 :- \left(\frac{\sigma_1}{\sqrt{n_1}}\right)^2 = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

den

$\cancel{s} = \frac{\sigma}{\sqrt{n}}$ → CLT



→ std-dev of (distb of sample-means)

as $n \to \infty$

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\overset{s_1}{\cancel{\sigma_1}}{}^2}{n_1} + \frac{\overset{s_2}{\cancel{\sigma_2}}{}^2}{n_2}}}$$

$\sigma_1 \approx$ Sample Std-dev of group 1 $\to s_1$

$\sigma_2 \approx$ Sample-std-dev of gp-2 $\to s_2$

why?

n_1 & n_2 are not too small

T is sensible

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$H_0: \longrightarrow \approx 0$

$H_a: \longrightarrow$ large +/- ve value

T_{obs}

means $\longrightarrow$ $\bar{x}_1$ & $\bar{x}_2$

$$\boxed{T} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\ldots}} \quad \text{vs} \quad T = \frac{Med_1 - Med_2}{\sqrt{\ldots}}$$

donot know disb under H_0

Sensible

disb of T under H_0

Test-statistic

tons of them

each test has a different test-stati

diff distributio
too

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Terminology: 2 sample ~~z~~-test for difference of means

x_{1i} $i: 1 \to 100$

x_{2j} $j: 1 \to 90$

T under H_0

$Z(0,1)$

Task

σ_1 & σ_2 $\to$ known

(or)

estimate them well $\to$ n_1 & n_2 are not too small

41 / 42

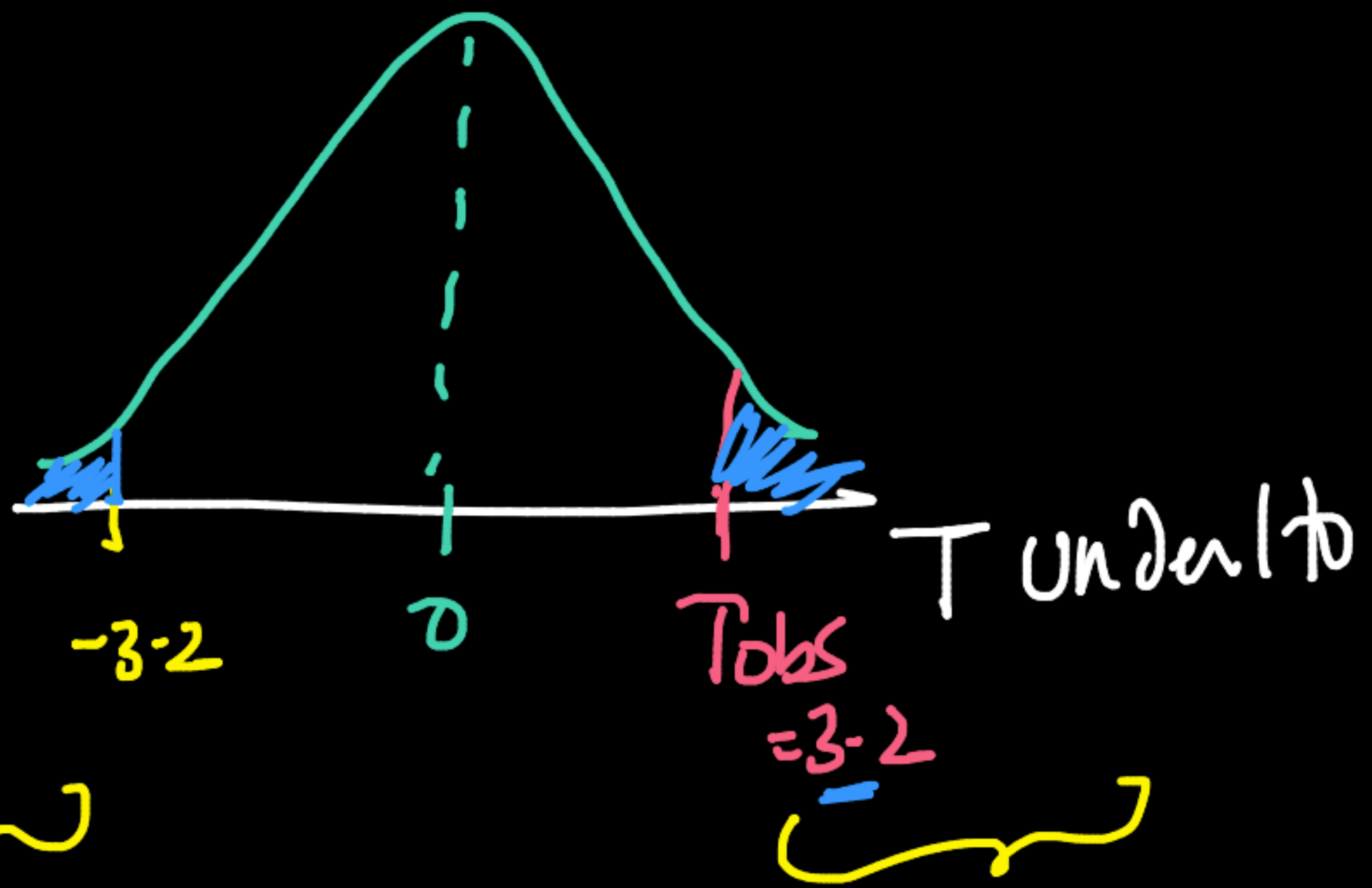
3 ✓ $\left\{ T_{obs} = \boxed{3.2} \right.$

$x_{11}\ x_{12}\ \dots\ x_{1,10} \rightarrow \bar{x}_1$ (n_1)

$x_{21}\ x_{22}\ \dots\ x_{2,10} \rightarrow \bar{x}_2$ (n_2)

$$T_{obs} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$N(0,1) = Z$



4. R or L or 2-sided test

under $\boxed{H_a:}$ $\mu_1 \neq \mu_2$

$T = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\cdots}}$: large +/- values

5. p-val < 1%

6. $\alpha = 5\%$

p-val < α
⇒ reject H_0 ✓

$N(0,1)$
68-95-99 rule

2-sample Z test

Comparing diff. of means

$$T_{H_0} \sim Z(0,1)$$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim Z(0,1)$$

Conditions for z-test

1. popl has finite μ & σ — CLT

   Sample means are Normally d/sb

   $T \sim Z(0,1)$

2. σ_1 & σ_2 → known

   (or)

   estimate well → n_1 & n_2 are not small

   (pop ~N)

   > 30

$\boxed{\text{Param}}$

Non-param

bootstrapping

CLT

Z-Test

why?

assumption on population

NOTHING to do with $Z(0,1)$

X

n_1 & n_2 have to be Same ✗

$n_1 = 2000$

$n_2 = 30$

$\sigma_1 = s_1$

$\sigma_2 \approx s_2$

$n_1 = 100$

$n_2 = 20$

Z-test ?

σ_2 Cannot be estimated well

T - - .

μ_1 vs μ_2 $\longrightarrow$ Z-test if n_1 & n_2 are not too small

$\left\{ \mu_1 \text{ vs } \mu_2 \text{ vs } \mu_3 \cdots \mu_k \right.$ $\longrightarrow$ (ANOVA)

Z-Test

(2) assumptions are true

Test for difference in means

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim Z(0,1) \text{ under } H_0$$

$\boxed{Q}$

Z-test

Pgg$_1$ vs Pgg$_2$ → no

~~CLT~~

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\cdots}}$$

do not know distAT under H_0 → ~~later~~: Permutation Resampling Tests (post-read)

Same "framework" $\longrightarrow$ hyp. tests

T under H_0

Choose α?

p-val = $P($ observing T as extreme as $T_{obs} \mid H_0)$

$\alpha = 5\%$ (let)

p-val $= 4.99999\% < \alpha \Rightarrow$ reject H_0

T_{obs}

~5%

$\boxed{H_0}$

H_a

$\exists$ a ~~0.9999%~~ probability of obs. a value as extreme as T_{obs}

reject H_0

α is the max threshold probability of making a 'mistake' by reject H_0

$\alpha =$ Max. probability of incorrectly rejecting H_0

M_1 (placebo)

M_2 (remdensu)

Z-test

Case 1:

$p\text{-val} = 4.9999\% < \alpha = 5\%$ (choose $\alpha = 5\%$)

$\downarrow$

reject $\boxed{H_0}$ $\Rightarrow$ $\mu_1 \neq \mu_2$

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Concluding $\mu_1 \neq \mu_2$

eventhough $\exists$ a ~5% probability of observing T as extreme as this under H_0 $(\mu_1 = \mu_2)$

If $\boxed{H_0}$ (oracles) were true

$\alpha = 5\%$

p-val = 4.999% → ....

57 / 58

$\alpha = 5\%$

Could 'incorrectly' reject H_0 with a prob of $\alpha = 5\%$

In medicine,

$H_0: \mu_1 = \mu_2$

M_1: placebo

M_2: remdensivir

$H_a: (\mu_1 \neq \mu_2)$

$\alpha = 5\%$

reduced / stringent

$\alpha = 1\%$ or 0.1%

We conducted a double-blind, randomized, placebo-controlled trial of intravenous remdesivir in adults who were hospitalized with Covid-19 and had evidence of lower respiratory tract infection. Patients were randomly assigned to receive either remdesivir (200 mg loading dose on day 1, followed by 100 mg daily for up to 9 additional days) or placebo for up to 10 days. The primary outcome was the time to recovery, defined by either discharge from the hospital or hospitalization for infection-control purposes only.

0.1%

RESULTS

A total of 1062 patients underwent randomization (with 541 assigned to remdesivir and 521 to placebo). Those who received remdesivir had a median recovery time of 10 days (95% confidence interval [CI], 9 to 11), as compared with 15 days (95% CI, 13 to 18) among those who received placebo (rate ratio for recovery, 1.29; 95% CI, 1.12 to 1.49; $P<0.001$, by a log-rank test). In an analysis that used a proportional-odds model with an eight-category ordinal scale, the patients who received remdesivir were found to be more likely than those who received placebo to have clinical improvement at day 15 (odds ratio, 1.5; 95% CI, 1.2 to 1.9, after adjustment for actual disease severity). The Kaplan–Meier estimates of mortality were 6.7% with remdesivir and 11.9% with placebo by day 15 and 11.4% with remdesivir and 15.2% with placebo by day 29 (hazard ratio, 0.73; 95% CI, 0.52 to 1.03). Serious adverse events were reported in 131 of the 532 patients who received remdesivir (24.6%) and in 163 of the 516 patients who received placebo (31.6%).

CONCLUSIONS

Our data show that remdesivir was superior to placebo in shortening the time to recovery in adults who were hospitalized with Covid-19 and had evidence of lower respiratory tract infection. (Funded by the National Institute of Allergy and Infectious Diseases and others; ACTT-1 ClinicalTrials.gov number, NCT04280705.)

$\boxed{\alpha}$: acceptable ~~rate~~ probability of making an error:

rejecting H_0 when H_0 is true

for 'critical' tasks:

↳ lower prob of an error

$\alpha \downarrow$

another type of error

prob. of erroneously accepting H_0 when H_0 is false

$\alpha \downarrow$

$\alpha = 1\%$

placebo

M_1 & M_2 rem → ...

same

p-val = 1.001%

accept H_0 ✓

pre-covid : exp-drugs

covid: rem ✓

t-test

z-prop. test

χ^2 test

KS-test

ANOVA

$\vdots$

1:57 10-Jul-2018

[How To Find The Z Score Given The Confidence Level of a ...](#)
YouTube · The Organic Chemistry Tutor
28-Oct-2019

3 key moments in this video

[Finding z* Multiplier in StatKey and ME (No Sound)](#)
YouTube · Whitney Zimmerman
10-Jul-2017

View all

People also ask

What is the Z multiplier?

Multiplier, denoted as z*, is **the standardized score such that the area between −z* and z* under the standard normal curve corresponds to the desired confidence level.**

https://www.ics.uci.edu › ~jutts PDF
[Estimating Proportions with Confidence - ICS UCI](#)

Search for: What is the Z multiplier?

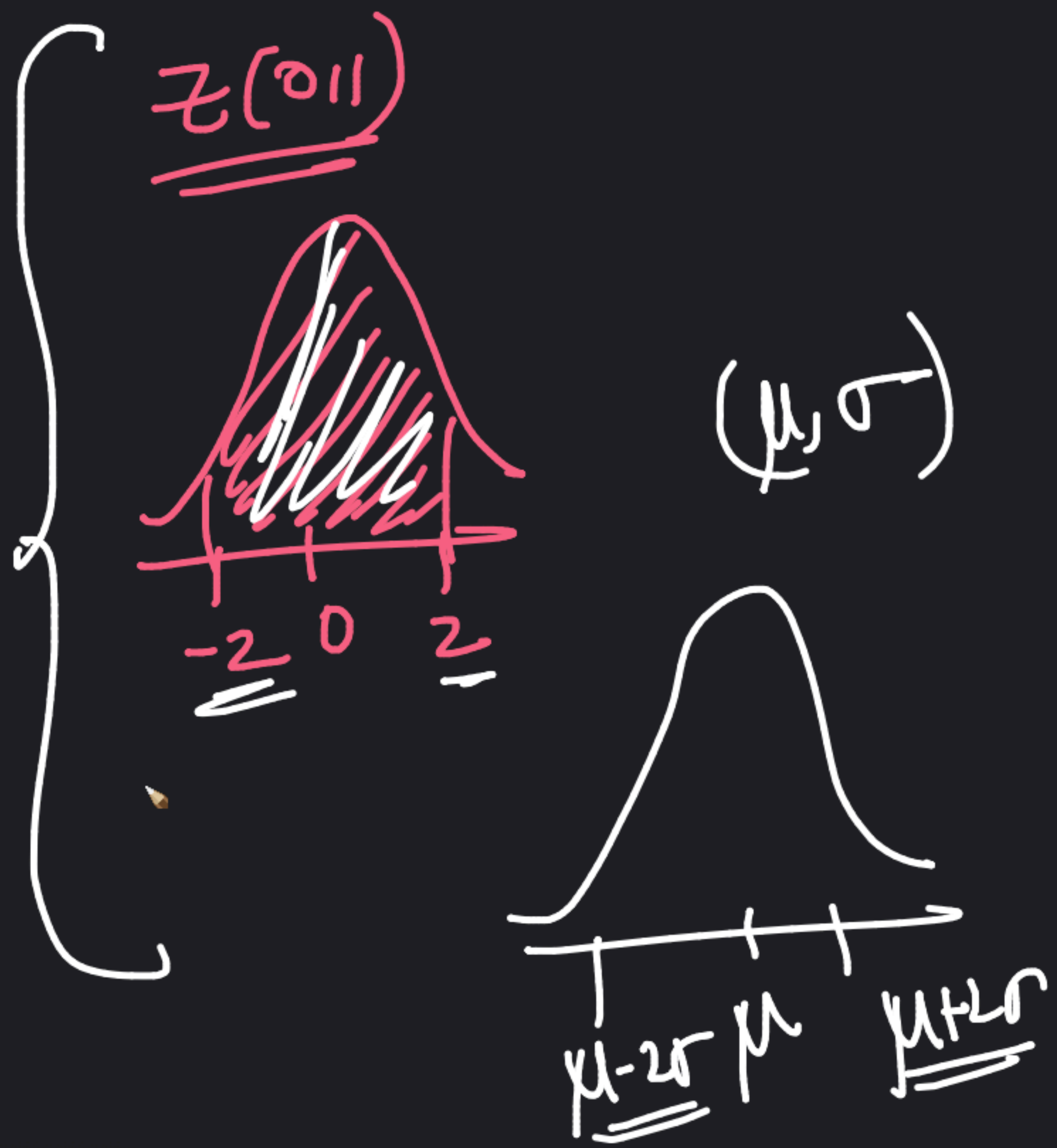
What is the Z multiplier for a 95% confidence interval?

How do you find the Z value multiplier?

How do you find the multiplier?

What is Z for a 99 confidence interval?

What does 1.96 mean in statistics?



revise → Tues

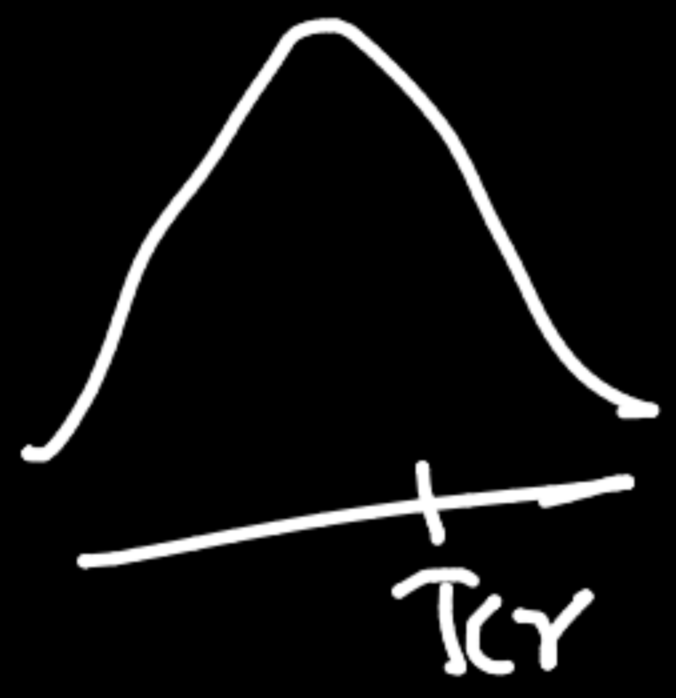
Q&A

Coin Toss: right tailed test

$T_{obs} = 58 \longrightarrow$ p-val $= 4.4\%$

$T_{obs} = 57 \longrightarrow$ p-val $= 6.6\%$ $> \alpha = 5\%$

T_{cr}

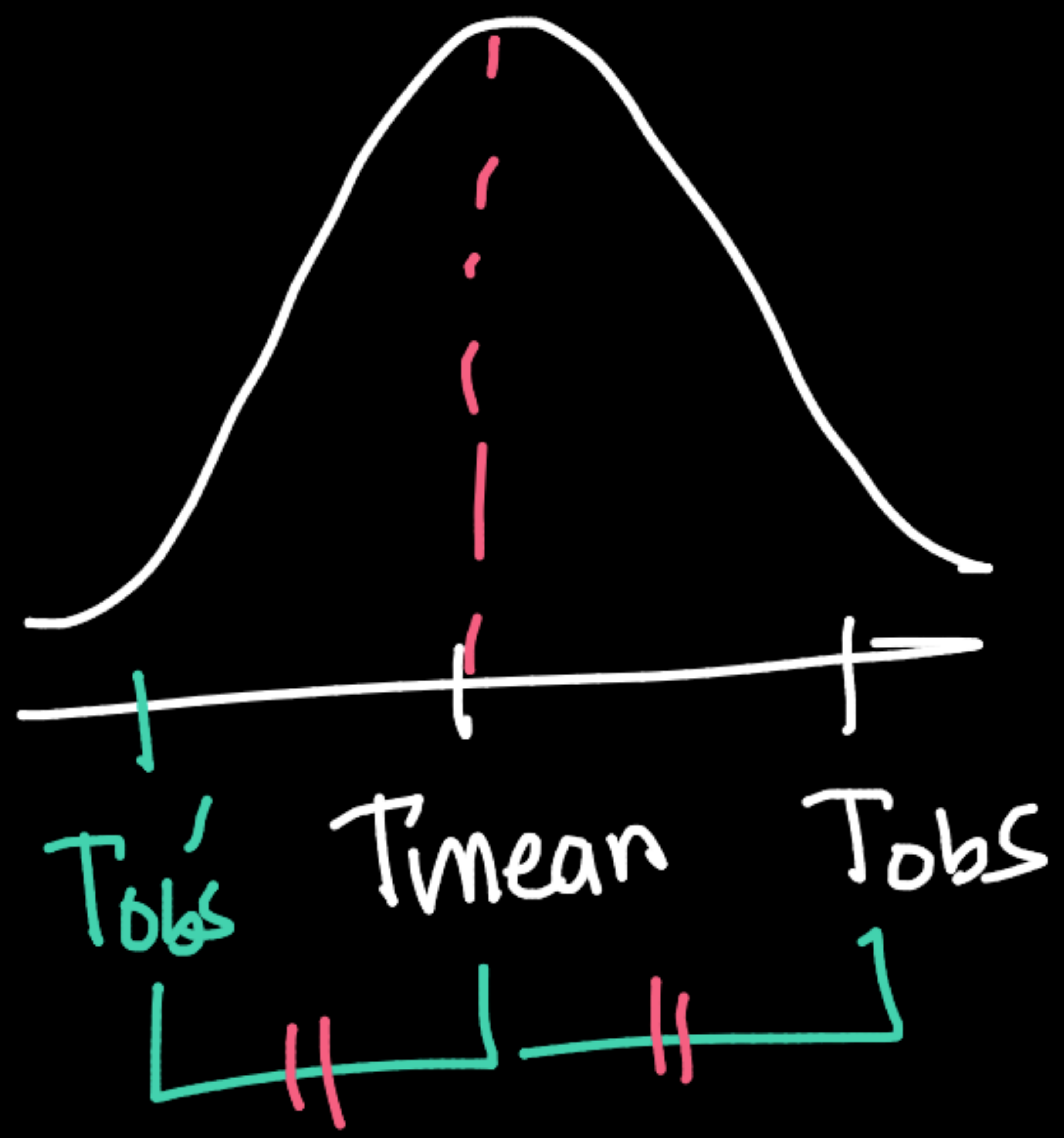


$T_{cr} \le T_{obs} = \boxed{57} \longrightarrow$ reject H_0 @ 5%

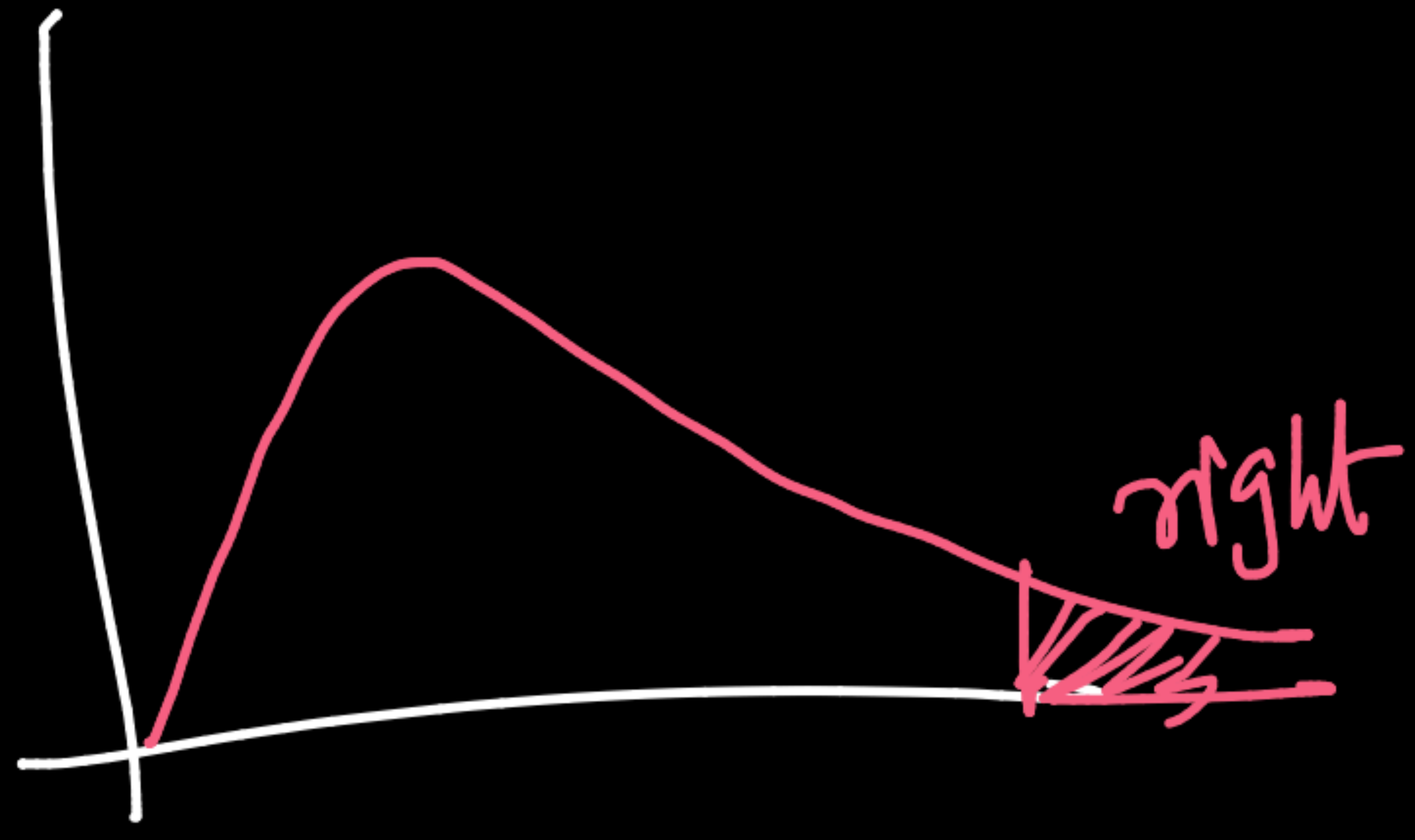
non-symm disb

2-tailed t test

typically avoided



e.g KS-test



not-symm

Covid-example :-

[$\mu_1 = \mu_2$ or not]

Medicine 1   Medicine 2

x_{11} x_{21}

x_{12} x_{22}

$\vdots$ $\vdots$

$x_{1,10}$ n_1 $x_{2,40}$ n_2

Z-test

$\boxed{\text{T-test}}$

$n_1 < 30$

$n_2 < 30$

Task: ecommerce

A 1 200 2

0.9 * Amzn price

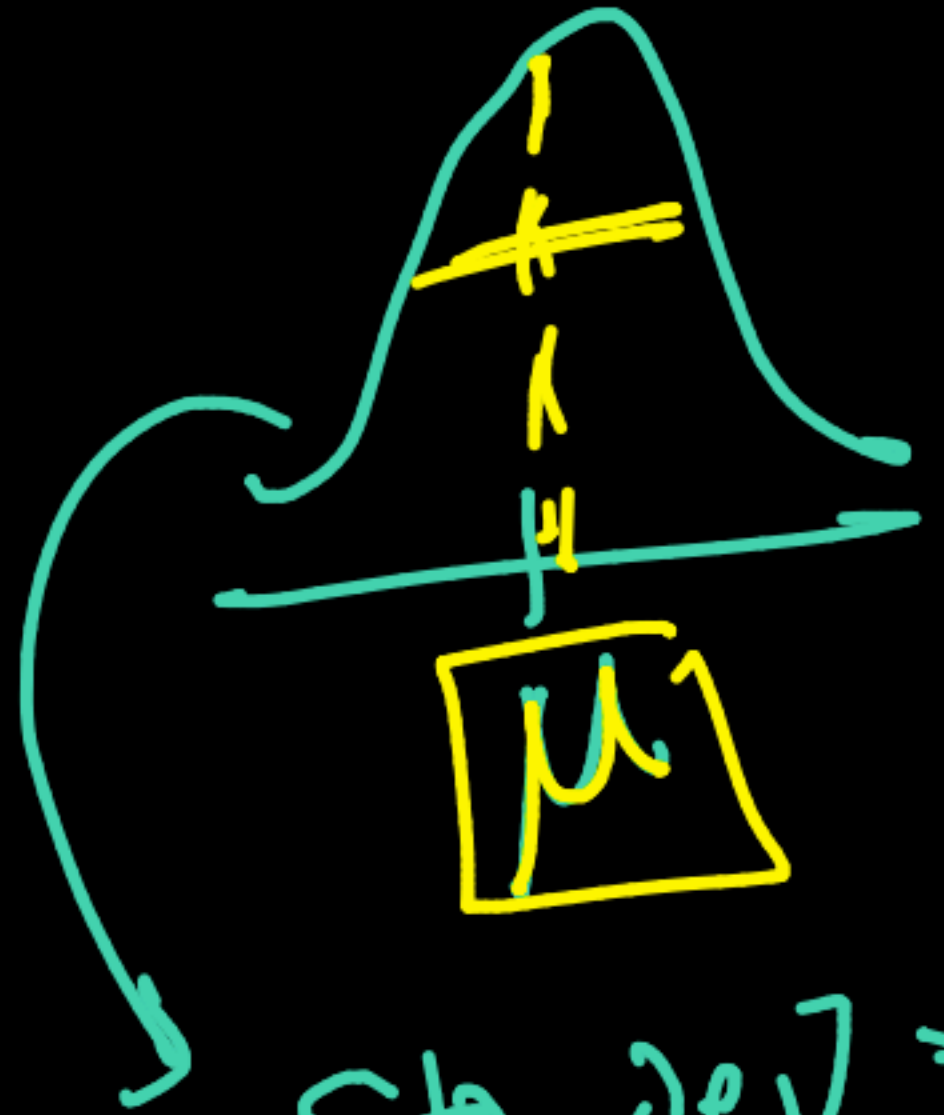
Amzn price

B 2 → 200

{ A/B Testing }

CLT: disb of sample means

$$S.E = \frac{\sigma}{\sqrt{n}}$$



$$\text{std-dev} = \frac{\sigma}{\sqrt{n}}$$

H_0: ~~avg~~ mean temp in BLR is 25°C → pop. Mean

H_a: not 25°C

T_{obs} = mean temp across 1yr → 26°C

↳ sample mean

1-Sample

$$T = \frac{\bar{x} - 25^{\circ}}{\sigma/\sqrt{n}} \sim N(0,1) = Z\text{-test}$$

↳ 365 days

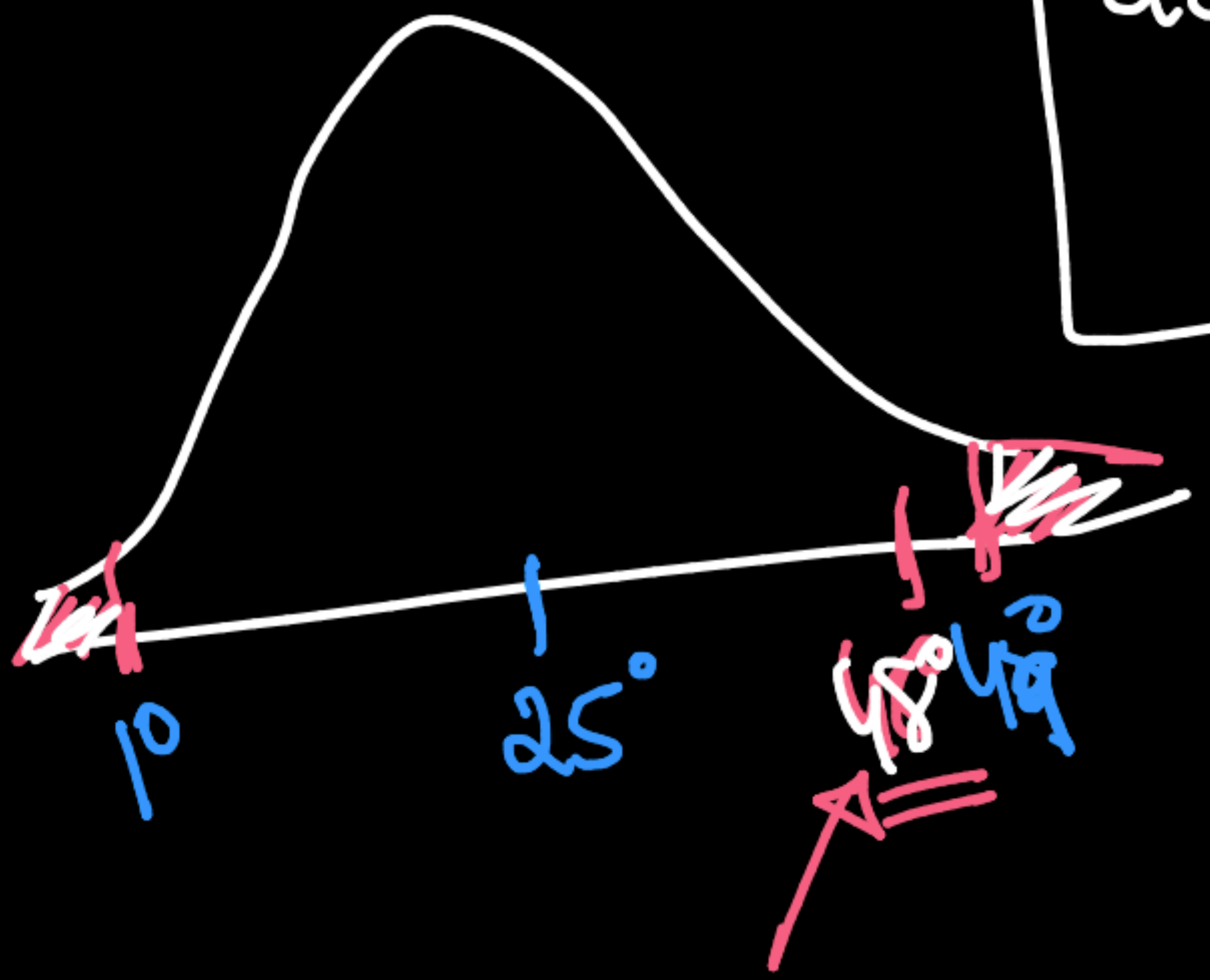
α

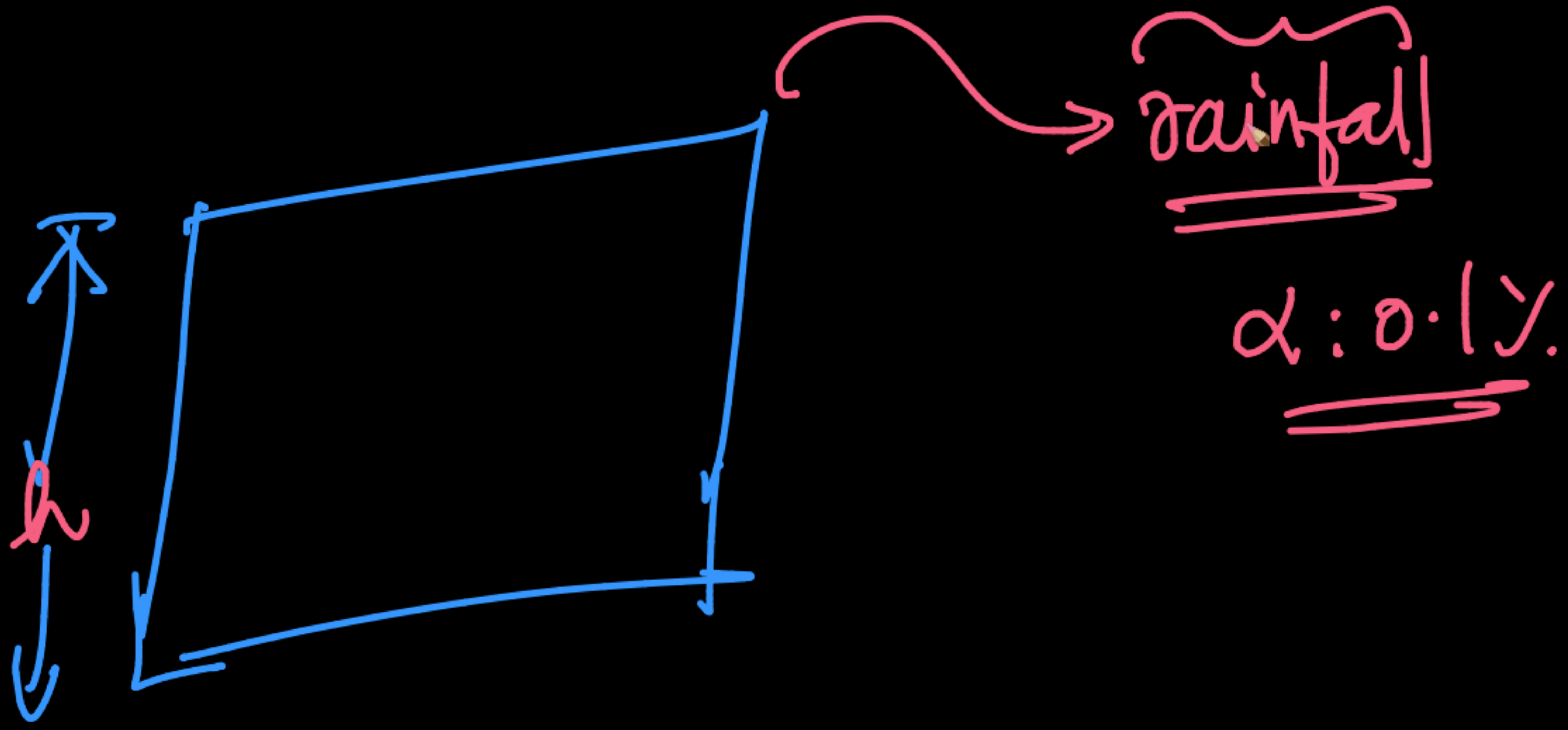
accept H_0

$\alpha = 1\%$

Biz - decision

$\alpha = 1\%$




h
rainfall
α : 0.1%

Z-test → population μ & σ finite

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\cdots}} \sim Z(0,1)$$

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