Maths for Machine Learning
Revision notes

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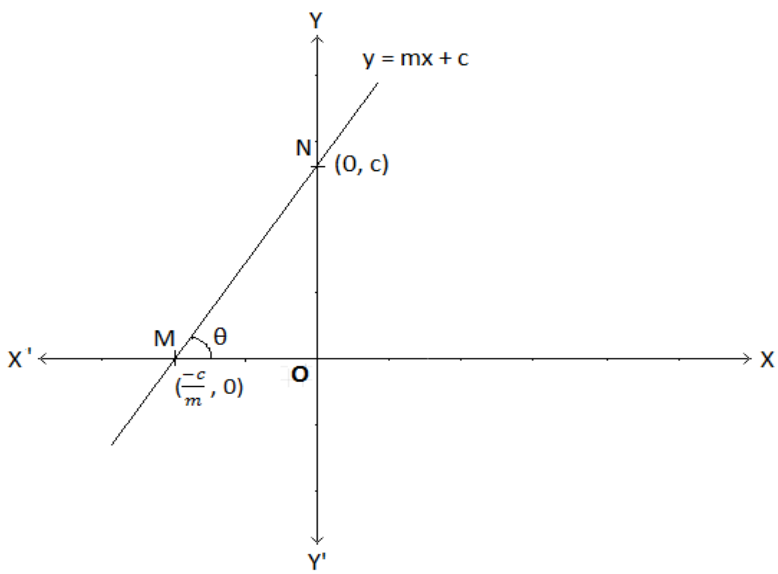
- **Equations of a straight line:**

1. **Slope-intercept form** of a straight line is given as:

$$y = mx + c$$

Where, m is the slope of the line and c is the y-intercept.
And $m = tan\theta$ where θ is the angle which the line makes with the positive x-axis.



2. **Point-slope form:**

$$y - y_1 = m(x - x_1)$$

where m is the slope of the line and (x_1, y_1) are the coordinates of a point on the line.

3. **Two-point form:**

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Where, (x_1, y_1) and (x_2, y_2) are the coordinates of two points on the line.

4. **Intercept form:**

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where *a* and *b* are the intercepts of the line on the x-axis and y-axis respectively.

5. **General form:**

$$ax + by + c = 0$$

where *a, b,* and *c* are real numbers.

- Two lines are called **parallel** to each other if the values of the slope are equal. Let's consider two lines $y = m_1x + c_1$ and $y = m_2x + c_2$.

  The above two lines are parallel if $m_1 = m_2$.

  The above two lines are **Perpendicular** to each other if:

$$m_1 = -\frac{1}{m_2}$$

- **Hyperplane** is a linear surface in n-dimensions.
  The general equation of a hyperplane is given as:

$$w_1x_1 + w_2x_2 + w_3x_3 + ... + w_nx_n + w_0 = 0$$

  Where, $w_1, w_2, w_3, ..., w_n$ are called the **weights/coefficients** and

  $x_1, x_2, x_3, ..., x_n$ are the **features**.

  The equation of a **plane** in **3-D** is given as:

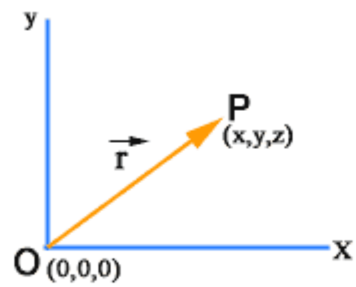
$$w_1x + w_2y + w_3z + w_0 = 0$$

- **Vector form of a hyperplane is:**

$$w^Tx + w_0 = 0$$

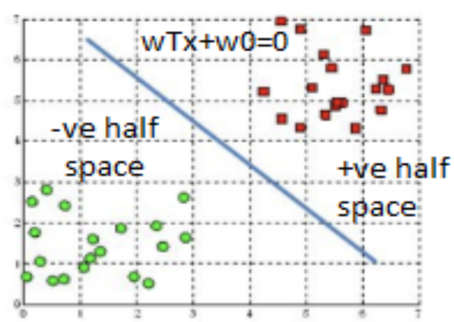
Where, $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ . \\ . \\ . \\ w_n \end{bmatrix}$ and, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix}$

- **Vectors** can be interpreted as coordinates as well as a line segment from the origin to the coordinate.
  For example, the below-given vector $\vec{r}$ can be considered as coordinates of point P(x,y,z) as well as a line segment from the origin to point P(x,y,z).



- **Half Spaces:** In geometry, a half-space is either of the two parts into which a plane divides the three-dimensional Euclidean space.

  Example: Let's assume that a hyperplane $w^T x + w_0 = 0$ is classifying the data points of two different classes in a space.



Let's say we got a point x_0 in the space.
Now, **If:**

$w^T x_0 + w_0 > 0$ => the point is in the +ve halfspace

$w^T x_0 + w_0 < 0$ => the point is in the -ve halfspace.

- The **transpose** operation changes a column vector into a row vector and vice versa.
  For example,

  if $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_n \end{bmatrix}$ then, $\vec{a}^T = \begin{bmatrix} a_1 & a_2 & a_3 & . & . & . & a_n \end{bmatrix}$

- **Dot product** of two vectors $\vec{a}$ and $\vec{b}$ is given as :

$$\vec{a} . \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + ... + a_n b_n$$

  Where, $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ . \\ . \\ a_n \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} b_1 & b_2 & b_3 & . & . & . & b_n \end{bmatrix}$

  Also, $\vec{a} . \vec{b} = \vec{b} . \vec{a}$

  Geometrically, it is the product of the magnitudes of the two vectors and the cosine of the angle between them.

  i.e. $\vec{a} . \vec{b} = |\vec{a}| . |\vec{b}| . cos(\theta)$

  where θ is the angle between the two vectors.

  If the dot product of two vectors is **zero**, then the vectors are **perpendicular** to each other.

- **Unit vector** is a vector that has a magnitude of 1.
  To convert a vector $\vec{u}$ into a unit vector, we divide the vector by its magnitude.

  i.e. unit vector = $\hat{u} = \frac{\vec{u}}{||\vec{u}||}$

➔ We can multiply any scalar value with the unit vector to get the desired magnitude (equal to that scalar value) in the same direction.

➔ All vectors with the same unit vector are **parallel**

- **Distance between two points** having coordinates (x_1, y_1) and (x_2, y_2) in an x-y plane is given as:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

- **Norm** or **Magnitude** of a vector is calculated by taking the square root of dot product with itself.

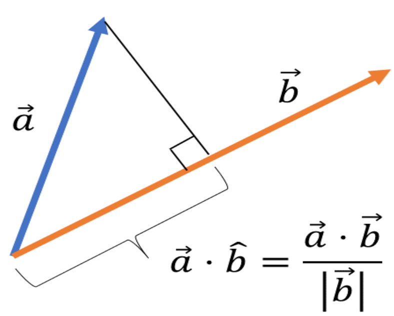
  i.e. $||\overrightarrow{a}|| = \sqrt{\overrightarrow{a}\,.\,\overrightarrow{a}}$

  It represents the **length** of a vector or **distance** of $\overrightarrow{a}$ coordinate from the origin.

- **Angle between two vectors** is given as :

$$\theta = cos^{-}\left(\frac{\overrightarrow{a}\,.\,\overrightarrow{b}}{||\overrightarrow{a}||.||\overrightarrow{b}||}\right)$$

- **Projection of vector** $\overrightarrow{a}$ on $\overrightarrow{b}$ = $\dfrac{\overrightarrow{a}\,.\,\overrightarrow{b}}{||b||}$



- At the **point of intersection** of two lines, both lines will have the same coordinates.

  Example: Let's say we have two lines, y = x+2 and y = 2x+1.
  We need to find the point of intersection of these two lines.
  We assume that the lines intersect at a single point (a,b).

Therefore, this point will satisfy both the line's equations.
i.e. b = a+2 — i)
b = 2a+1 — ii)
Solving above two equations, we get, a = 1 and b = 3.
Therefore, the given two lines intersect at the point (1,3).

- **Distance of a Hyperplane from the origin:**

  Let's assume a hyperplane $\vec{w}^T\vec{x} + w_0 = 0$.

  Its distance from the origin is given as: $d = \dfrac{w_0}{||\vec{w}||}$

- **Distance of a point** $\vec{x}_0$ from a **hyperplane** is given as:

$$d = \frac{|w^T x_0 + w_0|}{||w||}$$

  i.e. Just put the point in the hyperplane's equation and divide by the square root of the summation of coefficients' square (or norm of the *w* vector)

- **Distance between two parallel hyperplanes**
  Given two parallel hyperplanes, $w^T x + w_0 = 0$ and $w^T x + w_1 = 0$,
  Distance between them is given as:

$$d = \frac{|w_1 - w_2|}{||w||}$$

- The **equation of a circle** in the x-y plane is given as:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

  where, (x_0, y_0) are the coordinates of the center of the circle and *r* is the radius of the circle.