

31/01/22

D S M L Intermediate - DSA  
Session 13 - Arrays & Maths

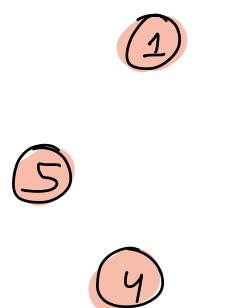
- (1) Josephus Problem
- (2) Majority Element

## Q1 War Stories

Josephus  
\_\_\_\_\_

N people

$N=5$



If there are  $N$  people in the circle, what should be the position where Josephus should stand in order to save his life.

Josephus should stand in order to save his life.

③ ← → remains alive.

ans

1

1

$N=1$

①

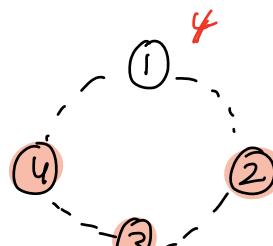
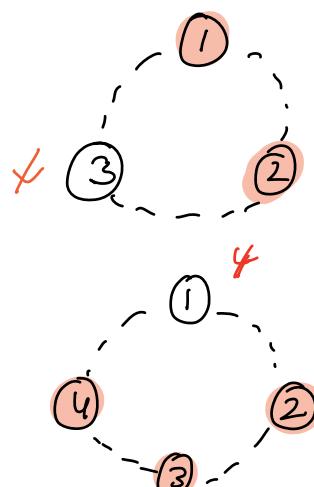


$N=2$

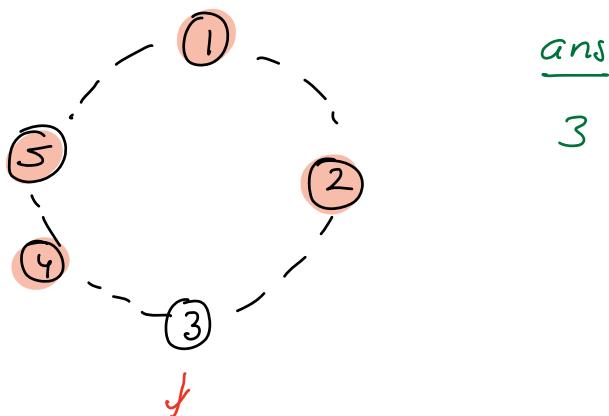
3

$N=4$

1

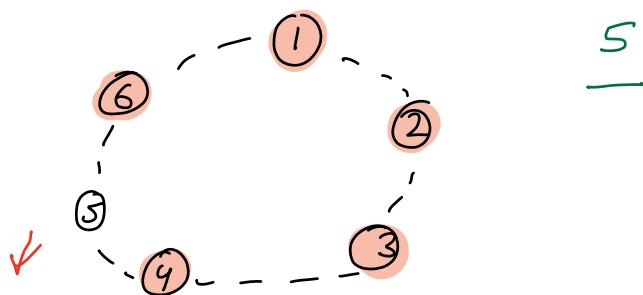


$N=5$



ans  
3

$N=6$

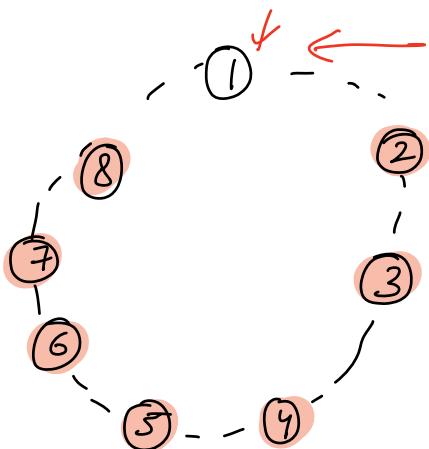


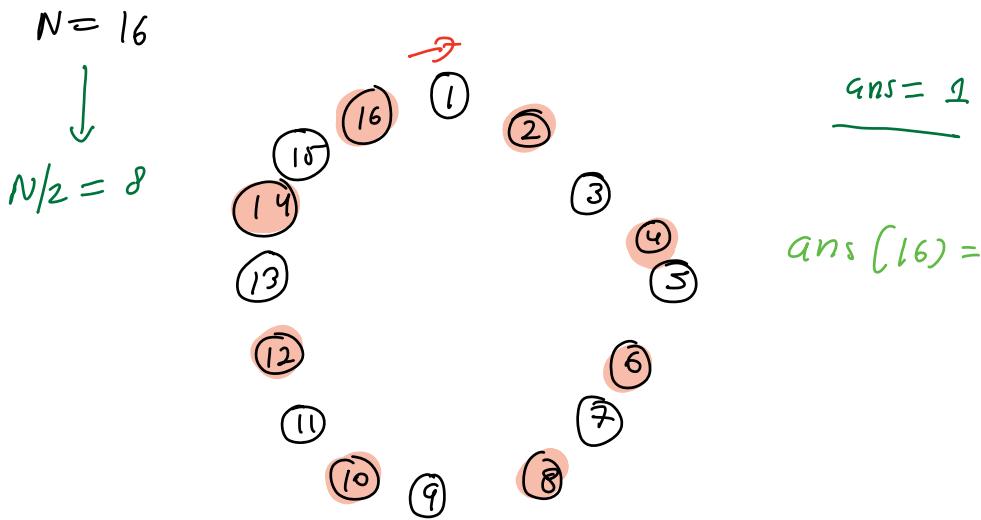
5

Obs 1 : Prime No X

Obs 2 : It will be an odd number ✓ .

$N=8$





$$N = 32 \quad 2, \underline{4, 6, 8, \dots, 32} \quad \frac{3^2}{2} = 16.$$



After step 1

$$\underline{\underline{N=16}} \quad \underline{\text{square back to 1}}$$

Observation 1: If  $N$  is a power of 2,

$$N = 2^k$$

The person who remains =  $\boxed{1}$ .

0000

How to check if  $N$  is a power of 2?

$$\begin{array}{r} \cdots \\ \underline{\underline{00000}} \end{array}$$

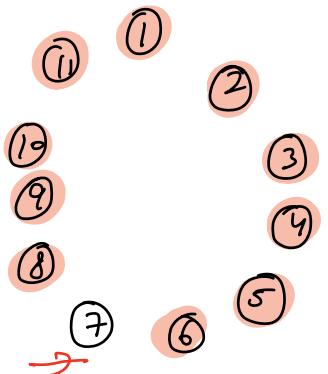
$$\textcircled{N} = \begin{array}{r} 1 \\ - \\ 0 \ 0 \ 0 \ 0 \ 0 \end{array}$$

$$\& \textcircled{N-1} = \begin{array}{r} 1 \\ - \\ 0 \ 1 \ 1 \ 1 \ 1 \ 1 \\ \hline 0 \ 0 \ 0 \ 0 \ 0 \ 0 \end{array}$$

$$\begin{array}{r} N \& (N-1) = 0 \\ \hline 1 & 0 = 0 \end{array} \quad \begin{array}{l} \text{Edge} \\ \text{case.} \end{array}$$

$0 \neq -1 \text{ ???}$

$N = 12$



$$\text{ans} = 7 \quad \# \text{ person remains}$$

1 step      10

2 steps      9

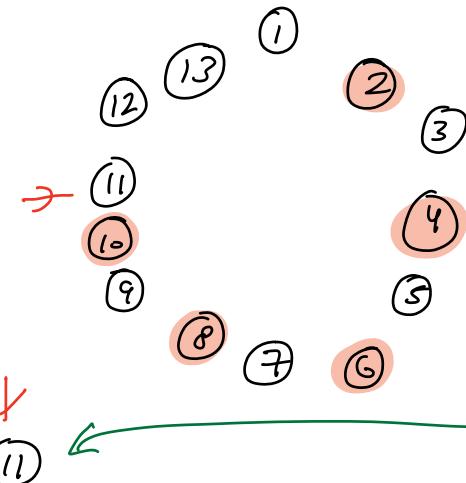
3 steps

8

$$11 - 8 = 3$$

Closest power of 2 for 11  
# person remain

$N = 13$



1 step      12

2 steps      11

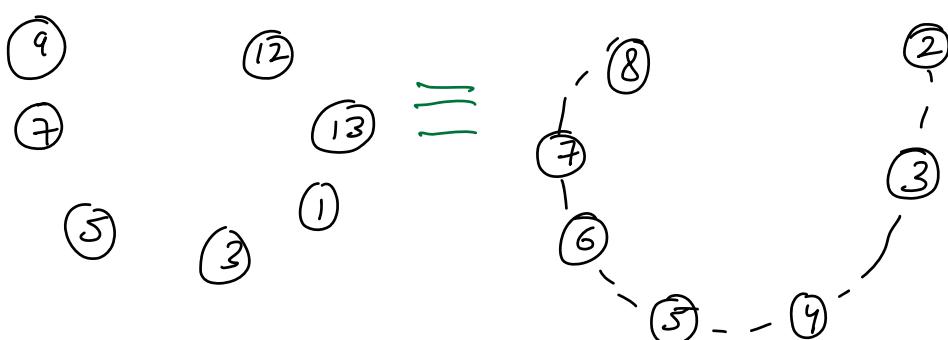
3 steps      10

4 steps      9

5 steps

8

$$13 - 8 = 5$$

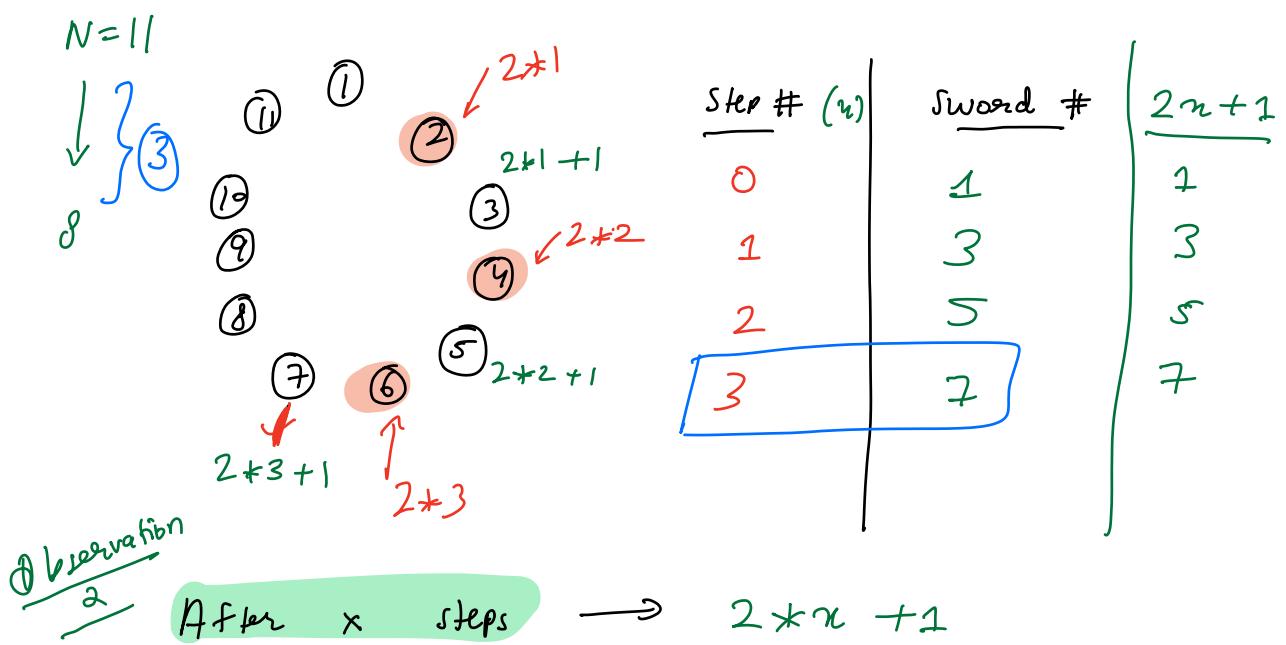


$N = 13$

$\downarrow 8$

$$13 - 8 = 5$$

$$\text{ans} = 2 + 5 + 1 = \underline{11}$$



$N=17$

Step 1. Find nearest power of 2

$$= 16$$

Step 2. Find the difference (no. of steps)  
 $17 - 16 = 1$  to set to power of 2.

Step 3.  $ans = 2 + \text{diff} + 1$

$$= 2*1 + 1$$

$$= 3$$

$N=13$

① Nearest power of 2 = 8

② Steps =  $13 - 8 = \underline{\underline{5}}$

③ Power =  $2 + 5 + 1 = \underline{\underline{11}}$

## Pseudo Code

```
# n > 0
def lastPersonAlive ( n ) :
    if n & (n-1) == 0 : # power of 2.
        return 1
    # When n is not a power of 2
    ⇐ x = nearestPowerOf2(n) ⇒ Todo ↴
    d = n - x
    return 2**d + 1
```

```
def nearestPowerOf2 (n) :
    logValue = int(log2(n))
    return 2**logValue
```

$$\begin{aligned} 11 &: 3 \cdot x = \frac{\text{int}}{3} \\ 13 &: 3 \cdot y = 9 \\ 17 &: 4 \cdot z = 4 \end{aligned}$$

5 min break

## Q2 Google Interview Problem

1 hr

### Majority Element

Given a list of size  $N$ , return the element that has a frequency  $> N/2$ .  $\neq$  T.C.  $O(N)$ , S.C.  $O(1)$



$$arr = [1, 6, 1, 1, 2, 1]$$

$$N = 6$$

$$N/2 = 3$$

$$\text{freq}[1] = 4 > N/2$$

$$\text{ans} = 1.$$

$$N = 11$$

Qn 2-1

$$3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3 \quad N/2 = 5$$

$$\underline{\text{Ans} = 3}$$

Qn 2-2

$$4, 6, 5, 3, 4, 5, 6, 4, 4, 4 \quad N = 10$$

No majority element.  $N/2 = 5$

$$\text{ans} = -1.$$

### Brute Force

For all elements

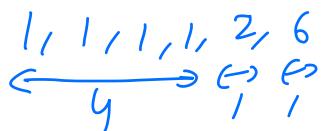
$O(n^2)$

check if it is a majority

2) Sort

$$[1, 6, 1, 1, 2, 1]$$

$O(N \log N)$



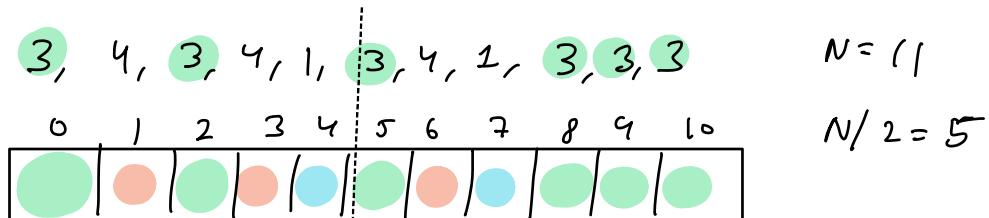
$1, 2, 3, 4, 5, 6, 7, 8$

-1

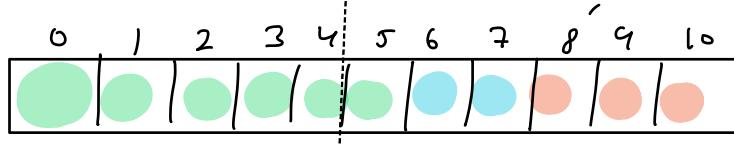
$$8/2 = 4$$

Observation

①



$3, 3, 3, 3, 3, 3, 1, 1, 4, 4, 4$



$\leftarrow \rightarrow > N/2$

# majority elements can be at most 2.

②

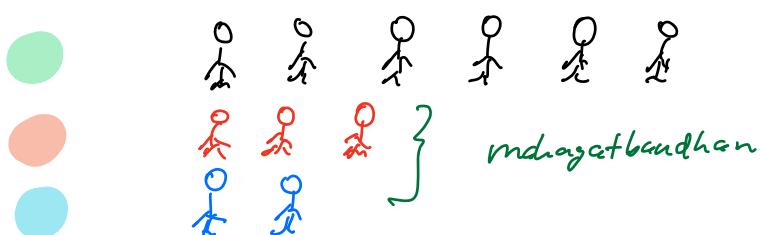
If suppose, we have a majority elt.

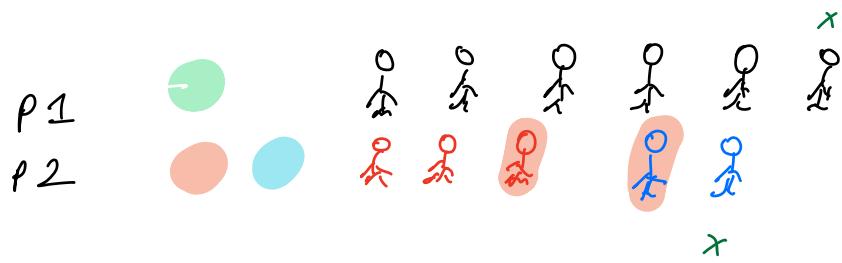
• freq  $> N/2$

• All the other elements combined have freq.  $< N/2$

MLA

• Freq of majority element  $>$  all the other elements combined

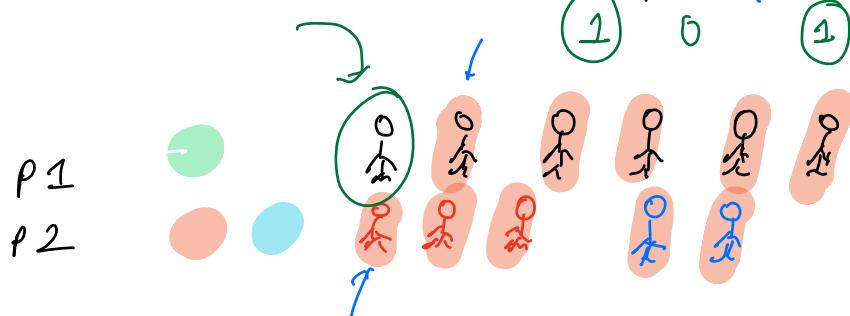




Observation 3 :

If we delete 2 distinct entries, majority element does not change.

$N$	$N/2$	# P1	# P2
11	5	6	5
9	4	5	4
7	3	4	3
5	2	3	2
3	1	2	1
	0	1	0



✓ Boyer Moore's Voting Algorithm  $\Rightarrow$  prospect

3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3  
 ↑      ↑  
 ♀      ♂

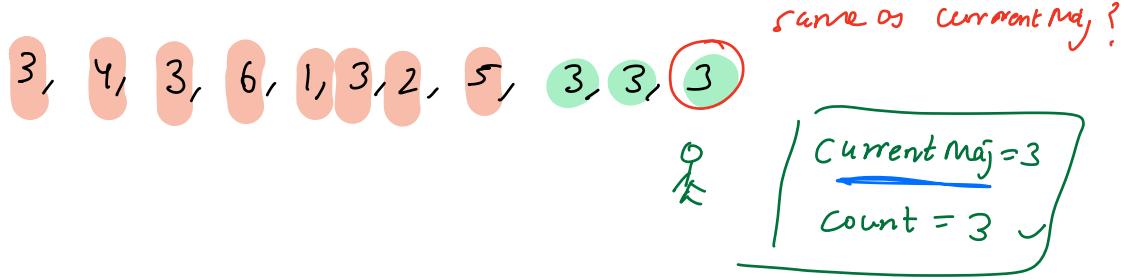
3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3  
 ↑      ↑  
 ♀      ♂

3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3  
 ↑      ↑  
 ♀      ♂

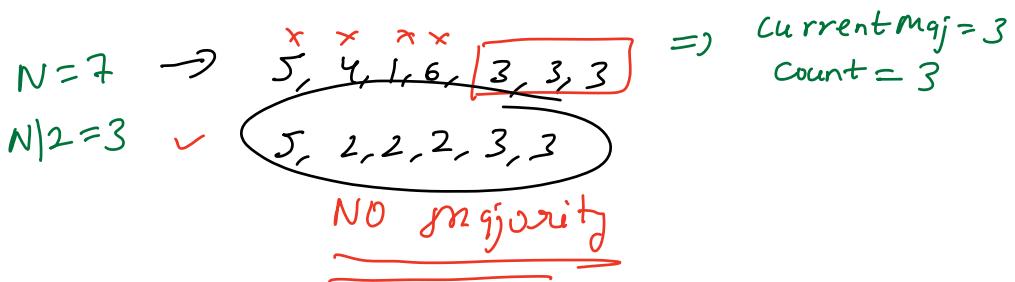
3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3  
 ↑      ↑  
 ♀      ♂

Current Maj = 3  
 Count = 2

3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3  
 ↑      ↑  
 ♀      ♂



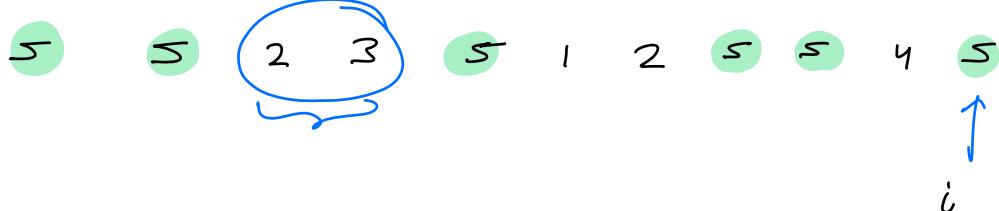
If there was a majority element then  
current Maj is a prospect answer.

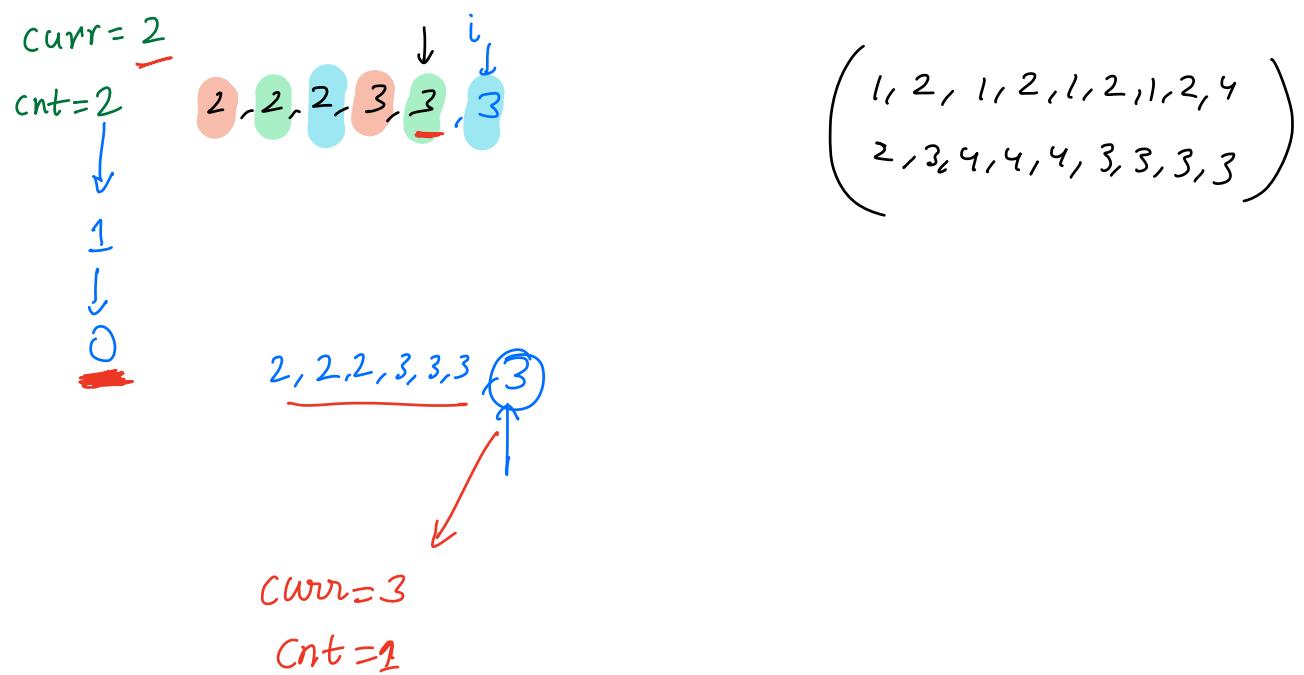


Ex 2.



currentMaj = 5  $\rightarrow$  5  $\rightarrow$  5  $\rightarrow$  5  $\rightarrow$  5  $\rightarrow$  5  $\rightarrow$  2  $\rightarrow$  2  $\rightarrow$  2  $\rightarrow$  5  $\rightarrow$  5  
 $Cnt = 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 1$





## Pseudo Code

①

```

maj = arr[0], Cnt = 1
for i in range (1, n):
    if (arr[i] == maj):
        Cnt++
    else if (Cnt == 0):
        maj = arr[i], Cnt = 1
    else:
        Cnt--

```



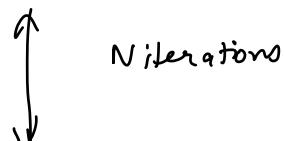
②

Check if maj is actual majority/not.

```

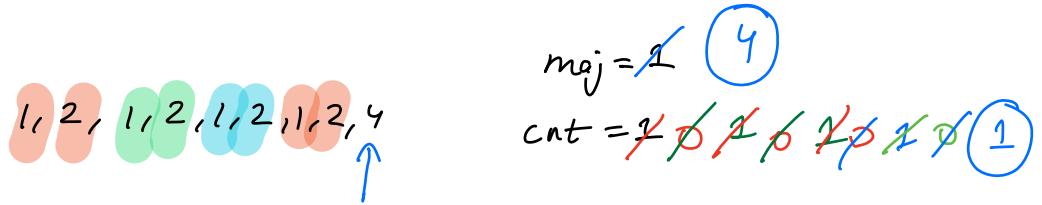
(Cnt=0
for i in range (0, N):
    if arr[i] == maj:
        Cnt++
    if (Cnt > N/2) { ✓ }
    else { ✗ }

```



$$TC = N + (N-1) = 2N-1 \Rightarrow O(N)$$

$$SC = O(1)$$




---

$\text{maj} = \text{arr}[0], \text{cnt} = 1$

for  $i$  in range ( $1, n$ ):

if ( $\text{arr}[i] == \text{maj}$ ):

$\text{cnt}++$

else if ( $\text{cnt} == 0$ ):

$\text{maj} = \text{arr}[i], \text{cnt} = 1$

else:

$\text{cnt}--$

---

~~X~~

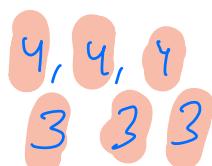
~~X~~



$\text{maj} = 2 \rightarrow 3$

$\text{cnt} = \cancel{1} \cancel{0} \cancel{1} \cancel{2} \cancel{3} \cancel{4} \cancel{5} \cancel{6} \cancel{7} \cancel{8} \rightarrow 2$

$\text{maj}:$



$N = 9$

$N/2 = 4$

$\text{cnt}(3) = 5 > N/2$

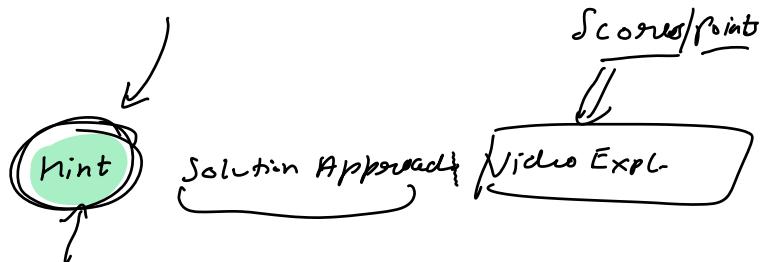
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Next session = Sorting Applications



$\Rightarrow$  Stuck on Problem  $> \underline{20-40 \text{ mins}}$

✓ Unlock Hints



for  $> N/3$

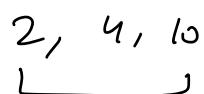
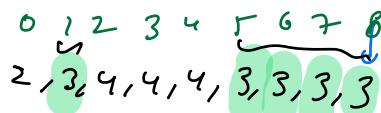
Take as many distinct examples

to build ideas & observations

Always understand all the possible approaches.

(When & where  
to use a  
technique)

$$(N-1) * \frac{(x - x_{\min})}{(x_{\max} - x_{\min})}$$



$$N=9$$



$$x_{\max} = 4$$

$$\frac{x-2}{2}$$

$$x_{\min} = 2$$

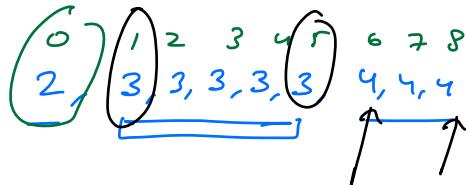
$$x=4 \Rightarrow \frac{4-2}{2} = \frac{2}{2} = 1$$

$$1 * (N-1) = 8$$

$$2 : 0$$

$$3 : 8 * \frac{1}{2} = 4$$

$$4 : 8 * \frac{2}{2} = 8$$



$$2 : 1$$

$$3 : 5$$

$$4 : 3$$

Standard Scaling