CD XX VI + C XL IV = D LXX

100 500 10 5 1 50 570

400 20 6 144

426 + DC

600

↳ 426 + 144 =

Roman Number System

I II III IV V

D → 500 CD → 400

Number Representation are important.

Number System → way of representing no.

Expand 1307 base are we taking in

↓

1000 + 300 + 0 + 7

base 10 — decimal no system.

↳ defining → 10 digits

10 different symbols that can be used.

digits

0 1 2 3 4 5 6 7 8 9

base - 10 → how many symbols do we have

base - 10 → how does the value of a particular digit change?

th

4 3 2 1st

1 3 0 7 $2 \leftarrow R$

face-value of 1

place-value of 1000 10^3

$$(1307)_{10} = \underbrace{1 \times 10^3 + 3 \times 10^2 + 0 \times 10^1 + 7 \times 10^0}$$

$$(1307)_{25} = \underbrace{1 \times 25^3}_{25^3} + \underbrace{3 \times 25^2}_{3 \times 625} + \underbrace{0 \times 25^1}_{0} + \underbrace{7 \times 25^0}_{7}$$

$$= (17507)_{10}$$

$$(1307)_{25} = (17507)_{10}$$

Octal number system   Deci → 10

$\underbrace{\text{Octal}}$ → 8

$(\quad\quad)_8$ → how many symbols do we have?

8 → {a, b, c, d, e, f, g, h}

→ {0, 1, 2, 3, 4, 5, 6, 7}

base − k → digits → $\{\underbrace{0 \ldots k-1}\}$

$(1001)_{10}$

1 0 0 0 0 1 $\rightarrow$ ✓ valid in base-2.

2 5 8 7 $\rightarrow$ $8 \notin \{0 .. 7\}$

2 1 6 — ✓

≡ — ✓

1326 $\rightarrow$ what base is this no written in?

$\cancel{(1326)_3}$

10

7

6

Can't tell

8, 10

max 6

7 -- ∞

$(1326)_5$

$(1326)_7$ $\rightarrow$ valid choice

$$(a\ b\ c\ d\ e)_k = e \cdot k^0 + d \cdot k^1 + c \cdot k^2 + b \cdot k^3 + a \cdot k^4$$

$(1\,0\,1\,1\,0)_2 = (?)_{10}$ $(22)_{10}$

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 0 + 4 + 2 + 0 = 22$

$(02101)_3 = (\ \ 64\ \ )_{10}$

$0 \cdot 3^4 + 2 \cdot 3^3 + 1 \cdot 3^2 + 0 \cdot 3^1 + 1 \cdot 3^0$

$0 \cdot 81 + 2 \cdot 27 + 1 \cdot 9 + 0 \cdot 3 + 1 \cdot 1$

$= \quad 54 + 9 + 1$

$= (64)_{10}$

$(125)_8 = (\ ?\ )_{10}$

$1 \cdot \underbrace{\cancel{8^2}}_{64} + 2 \cdot \underbrace{\cancel{8^1}}_{8} + 5 \cdot \underbrace{\cancel{8^0}}_{1}$

$64 + 16 + 5 = (85)_{10}$

<u>Computers</u>

<u>0</u> fingers?

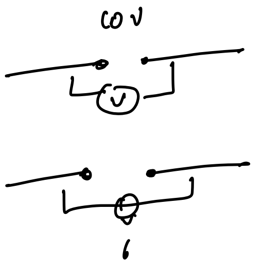
↳ circuits → electronic

Use voltage levels to represent different symbols.

<u>Humans</u>

10 fingers → decimal no. system

10 volts → digit 7



→ digit 3

Simplest (with the least no of digits)

base-k has k digits

base-1 has 1 digits

base-0 has 0 digits.

base-0 → one possible value

∵ no digit → no way to represent a no.

base-1 → valid

↳ digits — {0}

0 = 0

00 — 1

000 — 2

0000 — 3

00000 — 4

⋮

$$(k)_{10} = (\underbrace{000000000}_{k+1 \text{ zeros}})_1$$

base-2 (binary no. system)

~2.5V → 0 low

~5V → 1 high

given $(\_\_\_\_)_k = (\_\_\_\_)_{10}$

$(1379)_{10} \rightarrow (\quad)_8$

$(1 \cdot 10^3 + 3 \cdot 10^2 + 7 \cdot 10^1 + 9 \cdot 10^0)$

$(1307)_8 \rightarrow (\quad)_{10}$

$1 \cdot 8^3 + 3 \cdot 8^2 + 0 \cdot 8^1 + 7 \cdot 8^0 = (\quad)_{10}$

why does writing like this gives us base 10?

$8^3 = (512)_{10}$
rules of base 10

$(\quad)_k \rightarrow (\quad)_m$

$(\quad)_k \rightarrow (\quad)_{10} \rightarrow (\quad)_m$

program

$(1037)_{10} \xrightarrow{\text{expand}} (1037)_{10} \rightarrow (\ldots d\ c\ b\ a)_m$

$1037 = a \cdot m^0 + b \cdot m^1 + c \cdot m^2 + d \cdot m^3 + \ldots$

$m = 2$

divisible by $4 = (2^2)$

$1037 = a \cdot 2^0 + \underbrace{b \cdot 2^1 + c \cdot 2^2 + d \cdot 2^3 + \ldots}_{\text{divisible by } 2}$

only value not divisible by 2

$\frac{1037}{2}$ → take remainder = a

1037 % 2
↳ remainder.

1037 − 1
1036 / 4 → 6

1037 % 2 → ($\square$ $\boxed{a}$)

0th digit from the right
Least Significant digit

1037 % 4 → (b a) → quotient.
2

$\frac{5a}{2} = 6 \frac{a}{2}$
↑
quotient

(1039) % 4 = 3 = b a
↓
$(1\ 1)_2$
b a

$(1037)_{10}$ → 1037 ÷ 2 = 518 $\frac{1}{2}$ → remainder.
↳ quotient

518 ÷ 2 = 259 $\frac{0}{2}$ b

259 ÷ 2 = 124 $\frac{1}{2}$ c

⋮

2	1037	1
2	518	0
2	259	1
2	129	1
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	1

$(1037)_{10}$

$= (1000\ 000\ 11\ 01)_2$

given a decimal no → print the binary representation,

```
def binarize(n):
    while (n > 0):
        r = n % 2
        q = n / 2
        print(r)
        n = q
```

→ binary no in reverse.

$\frac{5}{2} = 2.5$

```
def binarize(n):
    l = []
    while (n > 0):
        r = n % 2
        n = n // 2
        l.append(r)
    l = reversed(l)
    print(l)
```

→ in python

→ most other language = 2 integer division.

$n/2 \rightarrow$ float in pyt

$n//2 \rightarrow$ inte in python

$\underset{1}{b} \underset{1}{a} = \frac{2 \cdot b + a}{2} = b \frac{a}{2}$

$$(\quad)_k \longrightarrow (\quad)_m$$

$$(\quad)_k \longrightarrow (\quad)_{10} \longrightarrow (\quad)_m$$

$$(57)_8 \longrightarrow (\quad)_3$$

digit by digit

$$3 \,\big|\, 5\cdot 8^1 + 7\cdot 8^0$$

write a program for this

$$(57)_8 \longrightarrow (\quad)_{10} \longrightarrow (\quad)_3$$

$$(37)_{10} =$$

2	37	1
	18	0
	9	1
	4	0
	2	0
	1	

$$(100101)_2 = (37)_{10}$$

Binary (2) $\{0-1\}$

Octal (8) $\{0-7\}$

Hexadecimal (16) $\{0-15\} \Longrightarrow \begin{Bmatrix} 0 \,-\,-\,-\, 9 \\ a\ b\ c\ d\ e\ f \end{Bmatrix}$

Decimal (10) $\rightarrow \{0-9\}$

$$(\underbrace{1\,1\,5}_{\text{ambiguous}})_{16} \rightarrow (1\cdot 16^1 + 15\cdot 16^0)$$
$$\rightarrow (1\cdot 16^2 + 1\cdot 16^1 + 5\cdot 16^0)$$

in CSS → color → background: #$\underbrace{\overset{R}{37}\,\overset{G}{37}\,\overset{B}{15}}_{\text{Hexadecimal}}$

---

an N-digit no (base-2)

N bits

$$(\underset{n}{\_\_}\ \ \_\ \ \cdots\ \ \underset{3}{\_}\ \ \underset{2}{\_}\ \ \underset{1}{\_})$$

$$\underbrace{\ }_{2^{n-1}}\ \ \cdots\ \ 8\ \ \ 4\ \ \ \underbrace{2}_{2^1}\ \ \ \underbrace{1}_{2^0}$$

N-digit binary no in which MSB is 1 → Most Significant

all other things are 0

= value ? =

$$(\underbrace{1\,0\,0\,0\,0\,0\,\cdots}) _2 = 2^{n-1}$$

n

N digit no in base b

↳ how many possible values can you represent? → b^N different values

$(\_ \ \_ \ \_)_2$ =

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

8 distinct values.

3 digit binary

$2^3 = 8$

$b \times b \times b \times b \times b \times b$

_ _ _ _ _ _ n digits $= b^n$ choices.

$\left.\begin{matrix} 0 \\ 1 \\ 2 \\ \vdots \\ b-1 \end{matrix}\right\}$ b choices.

---

4 digit no in base 3 → how many diff values can you make

0, 1, 2

$3 \times 3 \times 3 \times 3 = 81$ possibilities.

2	0	1	0
1	1	1	1
2	1	2	1

3 choices
0, 1, 2

Addition / Multiplication / Substraction

$$\begin{array}{r} {}^{1}\ \\ 1\,3\,7 \\ +\ 2\,5\,6 \\ \hline 3\,9\,3 \end{array} \qquad \begin{array}{r} 1\ \ 1\ 1\ \ \\ 1\,1\,0\,1\,1 \\ +\ \ 1\,0\,1\,1 \\ \hline 1\,0\,0\,1\,1\,0 \end{array}$$

0 — 0
1 — 1
2 — 10
3 — 11

$$\begin{array}{r} (1\,6\,5\,4)_7 \\ +\ (5\,1\,6\,6)_7 \\ \hline (1\,0\,1\,5\,3)_7 \end{array}$$

(7)

$6 \to 6$
$7 \to 10$
$8 \to 11$
$9 \to 12$
$10 \to 13$

$10-7=3$

$(12)_{10} = (15)_7$

$(12)_{10} \longrightarrow (\ )_7$

$$\begin{array}{r|l} 7 & 12 \quad 5 \\ \hline 7 & 1 \quad 1 \\ \hline & 0 \end{array}$$

$(15)_7$

base - 3

$$\begin{array}{r} 2\,1\,1\,2\,1 \\ \times\ \ 2\,0\,1\,2 \\ \hline 1\,2\,0\,0\,1\,2 \\ 1\,2\,1\,1\,2\,1\,\times \\ 0\,\times\,\times \\ 1\,2\,0\,0\,1\,2\,\times\,\times\,\times \\ \hline 1\,2\,1\,1\,2\,0\,2\,2\,2 \end{array}$$

$$\overset{k^4}{a}\ \overset{k^3}{b}\ \overset{k^2}{c}\ \overset{k^1}{d}\ \overset{k^0}{e}\ {}_{k} = x$$

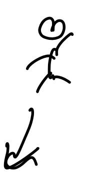
multiply by k.

$$(\underset{k^5}{a}\ \underset{k^4}{b}\ \underset{k^3}{c}\ \underset{k^2}{d}\ \underset{k^1}{e}\ \underset{k^0}{0})_k = k \cdot x$$

$$(\overset{k^4}{\_}\ \overset{k^3}{\_}\ \overset{k^2}{c}\ \overset{k^1}{\_}\ \overset{k^0}{a})_k = x$$

$$(\underset{k^5}{\_}\ \underset{k^4}{\_}\ \underset{k^3}{c}\ \underset{k^2}{\_}\ \underset{k^1}{a}\ \underset{k^0}{0})_k = k \cdot x$$

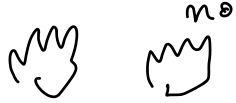
Man Martian



→ that's funny

we also use base 10

we use base 10
to work with
nos



10:37 → 10:50

$$\begin{array}{r|ll} 6 & 6 & 0 \\ 6 & 1 & 1 \\ & 0 & \end{array}$$

$(6)_{10} = (10)_6$

$$\begin{array}{r|ll} 9 & 9 & 0 \\ 9 & 1 & 1 \\ & 0 & \end{array}$$

$(9)_{10} = (10)_9$

6 in base 6 $= (10)_6$

$(0 - 6-1)$

26

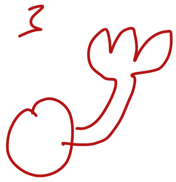
35 – 00

36 – 10

$(10)_6 = 0\cdot 6^0 + 1\cdot 6^1$

$= 6$

3



base-10

$3 \rightarrow 10$

(10)   10

base-10

Martian → base -3

$0 \rightarrow 0$
$1 \rightarrow 1$
$2 \rightarrow 2$
$\boxed{3} \rightarrow 10$
$4 \rightarrow 11$

every box is base-10

Binary Number System

→ operates on the entire number.

decimals → $\underbrace{+ \;\; * \;\; \div \;\; - \;\; \% \;\; -}$ —

binary → additional operators → bitwise operators.

$$\begin{array}{r} 1\,2\,3 \\ +\; 7\,8\,9 \\ \hline 9\,1\,2 \end{array}$$

$$\begin{array}{ccc} 1 & 2 & 3 \\ 7 & 8 & 9 \\ \hline 8 & 10 & 12 \end{array} = 81012$$

Not → flips the bit

$1 \qquad \sim 1 = 0$

$0 \qquad \sim 0 = 1$

$\overline{0}$

$!0$

Truth Table
↳ gives us the value when we apply the operator

$\sim(x) = y$

UNARY

NOT	x	y
	0	1
	1	0

$\sim \; 11011$

$ 00100$

---

operators → identity.

binary operators.

$x \; \_\_\_\_ \; \underset{id}{\_\_\_\_} = x$

Not

$x + 0 = x$

$x * 1 = x$

operator	identity
+	0
$\times$	1

$x \sim \_\_\_\_$

$\sim x$

Not → unary operator.

OR → either a or b.

if any bit is 1
OR → 1

OR	x	y	x|y
	0	0	0
	0	1	1
	1	0	1
	1	1	1

1011 →
+ 1101 →

11010 → 16 + 8 + 0 + 2 + 0 = 26
OR 10011 → 16 + 0 + 0 + 2 + 1 = 19
11011 → 16 + 8 + 0 + 2 + 1 = 27

a = 26
b = 19
print (a | b) → 27

26 | 19

26
+ 19

26 → binary
OR 19 →
(2|1) (6|9) ✗ incorrect.
bitwise

binary

a = 11
print (a | 1)

$$\begin{array}{r} 1\,1 \\ \text{OR} \quad 0\,1 \\ \hline 1\,1 \end{array}$$

in corr

$(11)_{10} = (1011)_2$

$(1)_{10} = (0001)_2$

OR $(1011)_2 = (11)_{10}$

correct

---

a = 10
print (a | 1) → 11

$a = (10)_{10} = (1010)_2$

$(1)_{10} = (0001)_2$

OR $(1011)_2 = (11)_{10}$

---

a = 13
b = 10
a | b = ?

13 | 10 = 15

$$\begin{array}{rl} & 8\;4\;\;\;1 \\ (13)_{10} = & (1101)_2 \\ & 8\;\;\;2 \\ (10)_{10} = & (1010)_2 \\ \hline \text{OR} & (1111)_2 = (15)_{10} \end{array}$$

$x \mid \boxed{0} = x$

identity for | ?

$10 \mid 1 = 11$

1 cannot be the identity

$x \mid (-111111)_2 = (-11111)_2$

AND   & $\cdot$ $\rightarrow$ only true when both are true.

x	y	$x \& y$
0	0	0
0	1	0
1	0	0
1	1	1

$a = 13$

$b = 10$

$a \& b = ?$

$(13)_{10} = (1101)_2$

$(10)_{10} = (1010)_2$

AND $(1000)_2 = (8)_{10}$

$13 \& 10 = 8$

a = 789013

print(a & 1)

$\rightarrow$ $(\_ \_ \_ \_ \_ \_ \_ \boxed{})_2$

$(0\ 0\ 0\ 0\ 0\ 0\ 1)_2$

& 0 0 0 0 0 0 k

if a is even $\rightarrow$ (~~~~ 0)

↳ 0

if a is odd $\rightarrow$ 1

1

OR / AND / NOT

NOR

NAND

x NOR y = !(x OR y)

x NAND y = !(x AND y)

x	y	~x	x|y	x · y	x NOR y	x NAND y
0	0	1	0	0	1	1
0	1	1	1	0	0	1
1	0	0	1	0	0	1
1	1	0	1	1	0	0

Universal gates.

XOR → Ex - OR

→ Exclusive OR

A or B → A or B or both A & B (OR)

→ either A or B but not Both. (XOR)

Harpreet or Venkata are Software Engineers.

Harpreet has either done the assign or not
both outcome cannot hap

^ → caret   ⊕

x	y	x|y	x ^ y
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	0

$a = 13 \qquad (1101)_2$

$b = 10 \qquad (1010)_2$

$a \wedge b = ?$

$\text{XOR} \quad (0111)_2 = (7)_{10}$

XOR → acts like lock & key.

$$(x \wedge x) \rightarrow \begin{array}{r} 1\,1\,0\,1\,1\,1\,0 \\ 1\,1\,0\,1\,1\,1\,0 \\ \hline 0\,0\,0\,0\,0 \end{array}$$

$x \wedge x = 0$

it cancels itself out.

$x \wedge 0 = x$

$$\begin{array}{r} 1\,1\,0\,1\,1\,0\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0 \\ \hline 1\,1\,0\,1\,1\,0\,1\,1 \end{array}$$

$x \wedge x \wedge x \wedge x \wedge x = x$

$0 \quad x \quad 0 \quad x$

$x \wedge x \wedge x \wedge x \wedge x$

	OR	AND	XOR
0 op x	x	0	x
0001 op x	$x+1$ if even, x if odd	0 if even, 1 if odd	$x+1$ if even, $x-1$ if odd
...111 op x	...11111	x	$\sim x$
x op x	x	x	0

$$\begin{array}{r} 1\,1\,0\,1\,\cancel{0} \\ 0\,0\,0\,0\,1 \\ \hline 1\,1\,0\,1\,1 \end{array}$$

$$\begin{array}{rr} & 1\,0\,1\,1\,1\,0\,0\,0\;k \\ \wedge & 0\,0\,0\,0\,0\,0\,0\,1 \\ \hline & \sim k \end{array}$$

$$\begin{array}{rr} & 1\,1\,1\,1\,1\,1\,1\,1 \\ & 1\,0\,1\,1\,0\,1\,0\,0 \\ \hline \text{flip} & 0\,1\,0\,0\,1\,0\,1\,1 \end{array}$$

$$\begin{array}{rcl} x & \Rightarrow & 1\,1\,0\,1\,1\,1\,0\,1\,0 \\ \wedge\; 1 & \Rightarrow & 0\,0\,0\,0\,0\,0\,0\,0\,1 \\ \hline & & 1\,1\,0\,1\,1\,1\,0\,1\,1 = x+1 \end{array}$$

$$\begin{array}{rcl} x & = & \boxed{1\,1\,0\,1\,1\,0\,1} \\ \wedge\; 1 & = & 0\,0\,0\,0\,0\,0\,1 \\ \hline & & 1\,1\,0\,1\,1\,0\,\boxed{0} = x-1 \end{array}$$

arr = [10, 8, 8, 9, 12, 9, 6, 11, 10, 6, 12, 17]

```
ans = 0
for i = 0; i < 12; i++
        ans = ans ^ arr[i]
print ans
```

→ XOR all values of the array.

$(11)_{10} = (01011)_2$

$(17)_{10} = (10001)_2$

$\wedge \quad (11010)_2 = (26)_{10}$

16 8 2

$a = (10011)_2$

$b = (01001)_2$

$+ \quad (11100)_2$

---

last digit → ← 1st digit

5 4 3 2 1 → n^{th} digit has what place value $= 10^{n-1}$

(12378)

4 3 2 1 0 → x^{th} digit place value $= 10^x$

ask on clarify

a) a & b = b & a
b) a|b|c = a|c|b
c) a ^ b = b ^ a
d) a ^ b ^ c ^ d = d ^ b ^ a ^ c
e) One of the above is wrong → ans.

Which is incorrect?

^ | & Commutative → $a \odot b = b \odot a$

associative → $(a \odot b) \odot c = a \odot (b \odot c)$

+ − % ÷ * → not digitwise operators

| ^ & → operator on bits → valid for binary nos.

→ bitwise operators

⊕ → bitwise add → defined specially

4th 3rd 2nd 1st

(_ _ _ □ □ □)

10^2 10^1 10^0

1

↓ nth digit

1 — 10^0

2 — 10^1

$$3 \rightarrow 10^2$$

$$x \rightarrow 10^{x-1}$$