

## Arrays - 1

↳ Lists?    Python    C++, Java: Homogeneous

- collection of elements → Python: Heterogeneous.
- Data structure → data → access  
→ operations/method.

Initialize     $l_1 = [1, 2, 5, -2, 4, 6]$

$l_2 = ["akhil", 1, 2, 5, -2, 1.23]$

Declare an empty list

[name\_of\_list = []]

a=list()    # declaration

Defining

Initial values

a = [1, 2, 5, -3]

For eg.  $\Rightarrow -6 \ -5 \ -4 \ -3 \ -2 \ -1$   $\Leftrightarrow$ : Python  
 marks = [99, 98, 87, 23, 92, 100]  $\Rightarrow$  index  
 $\underline{1} \ 2 \ \underline{3} \ 4 \ \underline{5} \ 6$

\* First index = 0  
 size = N In this eg N=6

\* Last Index = N-1

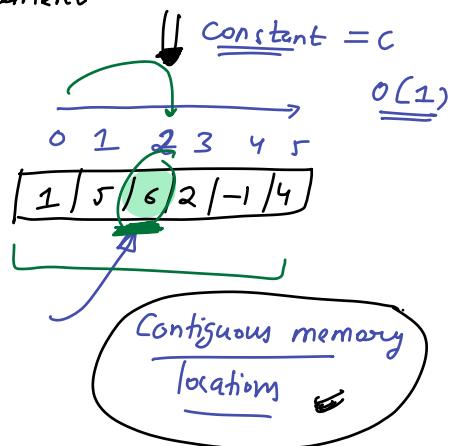
Sum of 1st & 5th elements = marks[0] + marks[4]

i<sup>th</sup> element  $\Rightarrow$  (i-1)<sup>th</sup> index.

NOTE: T.C. to access i<sup>th</sup> index element

arr[i]

Index  $\stackrel{i}{=}$   $\rightarrow$  arr  $\rightarrow$  arr[i]  
arr[2]



Q) Print all the array elements.

$\underline{0} \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ \underline{N=8}$   
 $\text{arr} = [1, 2, -3, 4, 6, 2, 7, 5]$

Output

1  
2  
-3  
4  
6  
2  
7  
5

$\Rightarrow N = \underline{\underline{\text{len}}(\text{arr})} \ # \ \underline{\underline{\text{range}(N)}} \Rightarrow \underline{\underline{\text{start}=0}}$  Default.  
{ for  $i$  in  $\underline{\underline{\text{range}(0, N)}}:$   
    print  $(\text{arr}[i])$  }  $\rightarrow \# O(1)$

$$\text{No. of iterations} = N - 0 = N = [0, N]$$

Time taken in each iteration =  $C$  (constant)

$$\text{Total time} = \underbrace{C + C + C + C + \dots + C}_{N \text{ times}} = CN$$

$$\text{Big-O Notation} = \underline{\underline{O(N)}}$$

$\Leftarrow$  for  $\underline{\underline{\text{my-ele}}}$  in  $\underline{\underline{\text{my-list}}}:$   
    print( $\text{my-ele}$ )

Index

- ✓ Q.1 Given  $N$  elements in a list, count the no. of elements having atleast 1 element greater than itself.

i)  $\begin{array}{ccccccccc} 2, & 5, & 1, & 4, & \textcircled{8}, & 0, & \textcircled{8}, & 1, & 3, & \textcircled{8} \\ \downarrow & \downarrow \\ \checkmark & \checkmark & \checkmark & \times & \checkmark & \times & \checkmark & \checkmark & \times \end{array}$

$\text{res} = 10 - 3 = 7$

$N = 7$

0  
7  
6  
3

ii)  $-3, -2, 6, \textcircled{8}, 4, \textcircled{8}, 5$   $\text{res} = 7 - 2 = 5$

$N = 7$

✓ Observation: ① Maximum element will not have any element greater than itself. (cnt)

✓ ② For all non-maximum elements, atleast 1 elt (max of array) is greater than it.

↳  $\text{res} = N - \underline{\text{Cnt of maximum}}$

iii)  $\text{arr} = [1, 1, 1, 1, 1, 1]$

$\text{maxi} = 1$

$\text{cnt of maxi} = N$

$\text{res} = N - N = 0$

Pseudo Code

- ✓ 1 - Iterate & Get max  $\xrightarrow{\quad\quad\quad} O(N)$
- ✓ 2 - Iterate & Cnt maxi  $\xrightarrow{\quad\quad\quad} O(N)$
- ✓ 3  $\rightarrow (N - \text{cnt})$  as ans.  $\xrightarrow{\quad\quad\quad} \frac{O(1)}{2N+1} = O(N)$

Linear

$\boxed{1 * 10^9}$   
 $\boxed{2 * 10^9}$

$\Rightarrow 1 \text{ sec}$   
 $\Rightarrow 2 \text{ sec}$

10 min.

①

```

1 maxi = arr[0] # 1 3
2 N = len(arr)
3 for i in range(1, N):
4     if arr[i] > maxi:
5         maxi = arr[i]
# maxi

```

1. no. of iterations

$$= \underline{\underline{N-1}}$$

i	0	1	2	3	4	5	6	7	8	9	10	11
arr[i]	1	3	2	4	2	5	1	6	2	7	1	2
maxi	1	3	3	4	4	5	5	6	6	7	7	7

2. Ignore any constants

$$Tc: \underline{\underline{O(N)}}$$

②

$$cnt = 0$$

for i in range(0, n):

Tc:  $O(N)$       if arr[i] == maxi:  
 $cnt += 1$

③

$$\text{return } \underline{N - cnt}$$

Hw: Try to do ① & ② in the same loop.

Q2 Given  $n$  array elements, check if there exists a pair

indices  $\leftrightarrow (i, j)$  such that  
 $\text{arr}[i] + \text{arr}[j] = K$  and  $i \neq j$   
Return True/False  
different indices.

def check-pair-sum( $\text{arr}$ ,  $K$ ):

    =

    =

    =

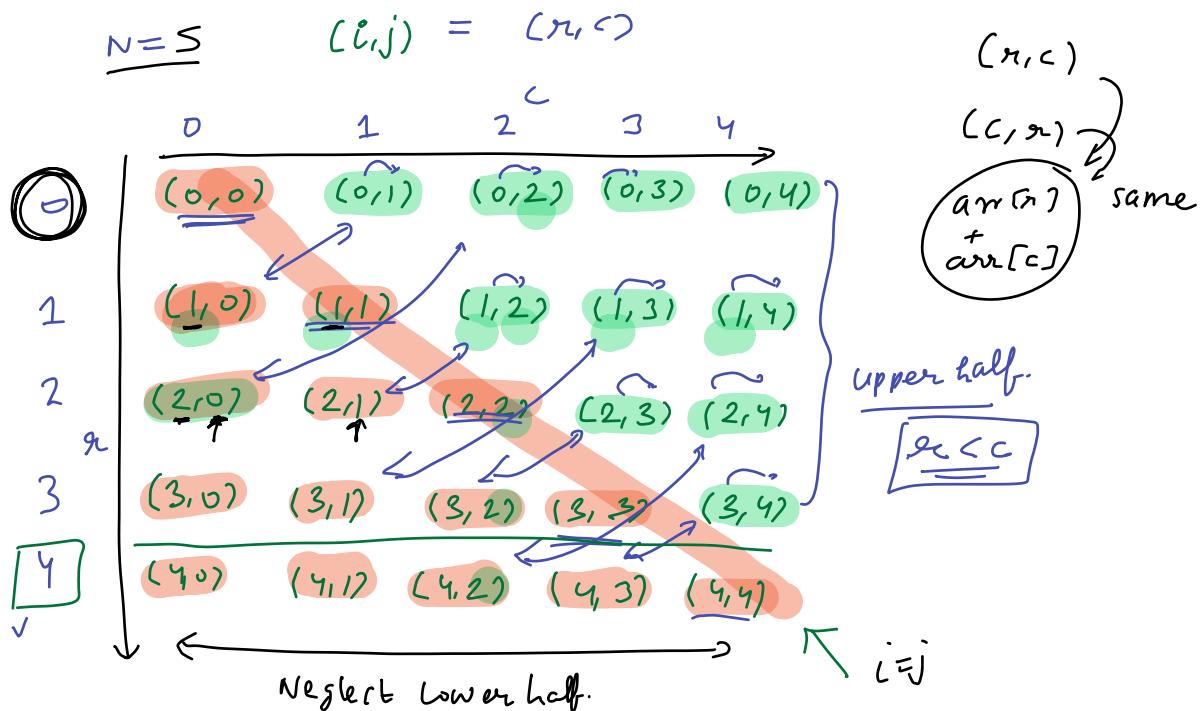
Ex  
①  $[3, -2, 1, 4, 3, 6, 8]$  True  
 $K=10$

---

②  $[5, 2, 3]$  False  
 $K=12$

### Brute Force

Idea: For every pair check if sum =  $K$ .



Iterate on all the pairs:

$$arr = [6, 2, 3]$$

$$K = 12$$

```
def check_pair_sum(arr, K):  
    for r in range(0, n):  
        for c in range(0, n):  
            # (r, c)  
            if r == c: # edge cases.  
                continue  
            if arr[r] + arr[c] == K:  
                return True  
    return False
```

T.C.  $O(N^2)$

r	c	Tot iterations
0	[0, N)	N
1	[0, N)	N
2	[0, N)	N
⋮	⋮	⋮
N-1	[0, N)	N
		$N^2$

Iterate on Upper Half:

```
def check_pair_sum(arr, K):
```

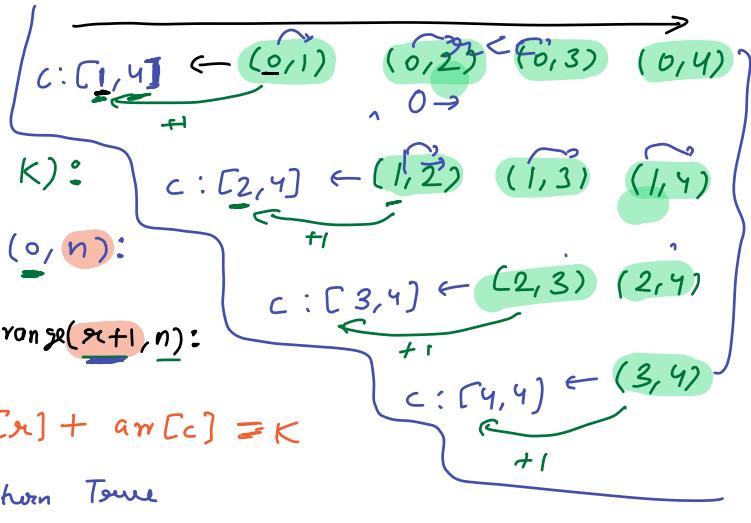
```
    for x in range(0, n):
```

```
        for c in range(x+1, n):
```

```
            if arr[x] + arr[c] == K
```

```
                return True
```

```
return False
```



x	c	Iterations
0	[1, N)	N-1
1	[2, N)	N-2
2	[3, N)	N-3
⋮	⋮	⋮
$\therefore N-1$	[N, N)	⑥

=  $1 + 2 + 3 + \dots + (N-1)$  (1).

=  $\frac{(N-1)N}{2}$  (2)

$= \frac{N^2-N}{2} = \frac{1}{2}N^2 - \frac{1}{2}N$

range(1,1) = [ ]

TC:  $O(N^2)$

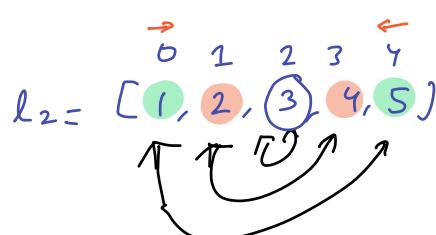
Q3. Given a list, reverse this entire list.

in the same list

$l = [-1, 4, 7, 6, -2, 7, 8, 10]$

Output:  $[10, 8, 7, -2, 6, 7, 4, -1]$

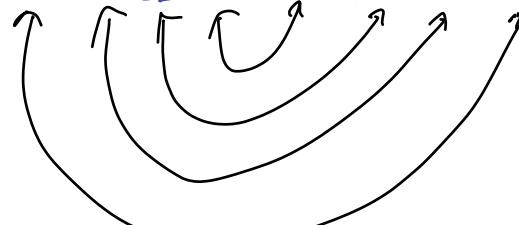
swap( $i, j$ )  
 $i, j = j, i$



$l_2 = 5, 4, 3, 2, 1$

---

$l = [-1, 4, 7, 6, -2, 7, 8, 10]$



$10, 8, 7, -2, 6, 7, 4, -1$

Constraint:

You have to  
reverse in the  
same input list.

In-place :  $O(1)$

$i \quad j$

0	4	swap
1	3	swap
2	2	-swap
3	1	

---

$i \quad j$

0	$j = N-1$	swap
1	6	swap
2	5	swap
3	4	swap

---

$4 \quad 3$

$i > j \Rightarrow$  stopping condition

def reverse (arr):

$$i=0, j = \text{len}(arr)-1$$

while ( $i < j$ ):

$$\cancel{i=j}$$

fails:

Swap( $\text{arr}[i], \text{arr}[j]$ )  $\# \underline{\text{arr}[i], \text{arr}[j] = \text{arr}[j], \text{arr}[i]}$

$$i+=1$$

$$j-=1$$

# arr is now reversed.

↑↑↑↑...↑↑↑↑↑↑

$$N/2, N/2+1$$

Iterations  $\approx N/2$

Time =  $Tc : \underline{\underline{o(N)}}$

$\Rightarrow$  Space Complexity  $\Rightarrow$  not part of input.

Big-O Notation

$\Rightarrow$  How many extra variables

$$= 2 \text{ variables} = \text{constant.} \Rightarrow O(1)$$

Precisely,

$$\frac{\text{Total Space}}{\text{Input Space}} = \frac{\text{Input Space}}{\text{Input Space}} + \frac{\text{Auxiliary/Extra Space}}{\text{Input Space}}$$

array of size  $N$ : integers

1 integers: 4 bytes

$$= (4N)$$

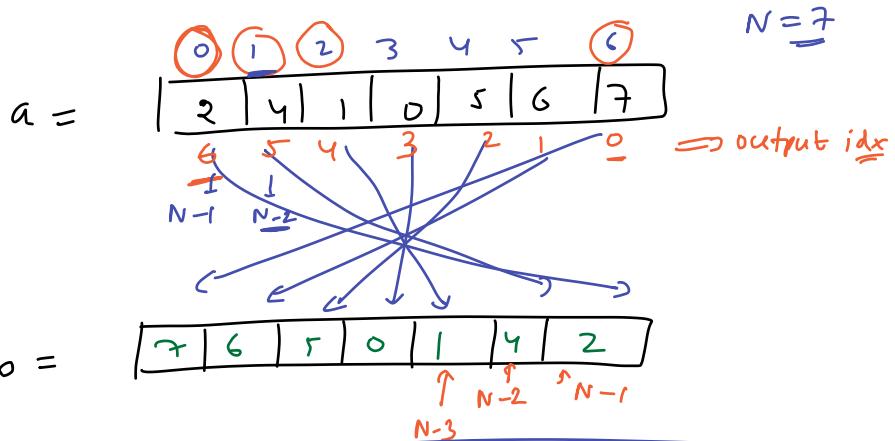
$$+ 2 * 4$$

$$= 4N + 8$$

$$= \underline{\underline{o(N)}}$$

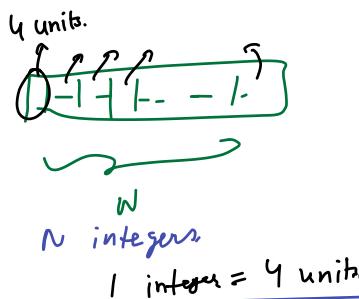
$$= \underline{\underline{o(1)}}$$

Space Complexity  
Big-O



`def rev2(arr):`

Space Complexity



$$i=0 \\ N = \text{len}(arr)$$

$$l_2 = [0] + N$$

for  $i$  in  $\text{range}(0, N)$ :

$$l_2[i] = arr[N-i-1]$$

Try to do a dry run.

$$\underline{\text{Extra space}} = 4 \underline{N}$$

$$SC: O(N)$$

Quia:

$$n = \text{len}(arr) \quad \text{Input}$$

$$\underline{\text{sum}} = 0$$

for  $i$  in  $\text{range}(0, n)$ :

$$\text{sum} += arr[i]$$

$$\boxed{2 | 1 | -3 | 4}$$

$$\boxed{2, 3, 0, 4}$$

TC: Iterations =  $n$

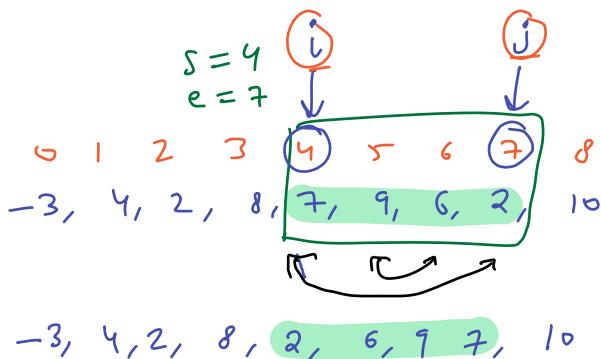
Bis-o:  $O(N)$

SC: Extra variables? = 3  $\rightarrow 3 * \underline{c}.$

Bis-o =  $O(1)$

Q4

Reverse part of the array. [s, e]



11.45 -

Doubts

TAs  $\approx$  30 min.

Slack.

$\Rightarrow$  2 days

def reverse(arr, s, e):

~~i = 0, j = len(arr) - 1~~

$i = s, j = e$

while (i < j):

Swap(arr[i], arr[j])

i += 1

j -= 1

# arr is now reversed.

Same code as last

problem works. --

with

minor changes.

No. of iterations

$$= \frac{(e-s+1)}{2}$$

No. of elements

$$N' = (e-s+1)$$

$$\begin{array}{l} s=0 \\ e=N-1 \end{array}$$

Worst case T.C. =  $O(N)$

If we want to reverse the entire array,

what is s & e?

$$\begin{array}{l} s=0, e=n-1 \\ \Rightarrow O(N) \end{array}$$

$$\begin{array}{l} \boxed{e-s+1} = n-1+1 \\ = n \end{array}$$

Q5 Given an array of  $N$  elements, rotate the array in clockwise direction by  $K$  times.

i) Given. ( $K < N$ )  $\Rightarrow$  how.

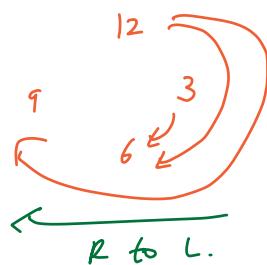
ii)  $K$  can be anything  $\rightarrow$  HW, discuss next class

Constraint: In-place  $\text{SC: } O(1)$   
(In the same array)

$N=7$

$K=1$

$[3, 2, 1, 4, 6, 9, -8]$

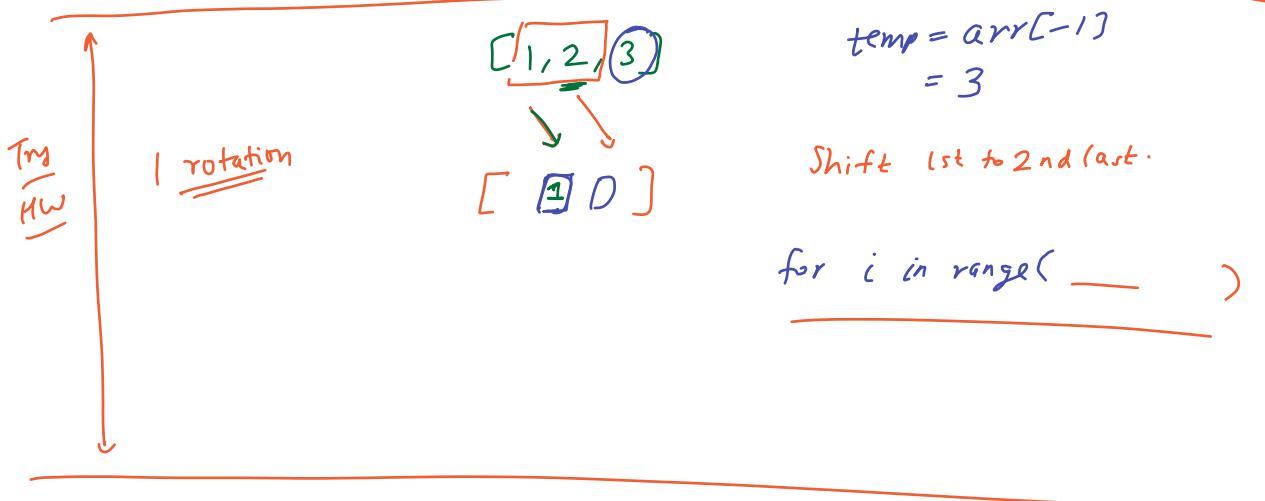


$[-8, 3, 2, 1, 4, 6, 9]$  1 - Last element goes to 1st  
2 - Shift all other by 1 to right.

$K=2$

$[9, -8, 3, 2, 1, 4, 6]$

Brute Force: Repeat the rotation  $K$  times.



## Observations

$a_{w[6]} = [3, -2, 1, \underbrace{4, 6, 9, 8}]$        $\xrightarrow{N-K} \quad \xrightarrow{K}$

$K=1$        $8 \quad 3 \quad -2 \quad 1 \quad 4 \quad 6 \quad 9$

$K=2$        $9 \quad 8 \quad 3 \quad -2 \quad 1 \quad 4 \quad 6$

$K=3$        $6 \quad 9 \quad 8 \quad 3 \quad -2 \quad 1 \quad 4$

$K=4$        $4 \quad 6 \quad 9 \quad 8 \quad 3 \quad -2 \quad 1$

1- Last  $K$  elements come to first  $K$  positions.

2- First  $(N-K)$  elements come to last  $(N-K)$  positions.

$N=6$ .  $a_{w[6]} : a_0, a_1, a_2, a_3, a_4, a_5$

$K=1$        $a_5 \quad a_0, a_1, a_2, a_3, a_4$

$K=2$        $a_4, a_5 \quad a_0, a_1, a_2, a_3$

$K=3$        $a_3, a_4, a_5 \quad a_0, a_1, a_2$        $\Rightarrow$  Output

$K$	$N-K$
1	5
2	4
3	3

reverse output!

$a_2, a_1, a_0$        $a_5, a_4, a_3$

wrt the original.

$\uparrow K$   
reverse  $1 \leq k \leq K$

$\uparrow N-K$   
reverse Last  $N-K$

$0 \ 1 \ 2$	$3 \ 4 \ r$
$a_0, a_1, a_2$	$a_3, a_4, a_5$

$N=6$   
 $K=3$

Iterations ✓ ① Reverse the first  $K$  elements

$$K \leftarrow \text{rev}(\text{arr}, 0, K-1) \quad \begin{matrix} a_2, a_1, a_0 \\ a_5, a_4, a_3 \end{matrix}$$

✓ ② Reverse the last  $(N-K)$  elements

$$N-K \leftarrow \text{rev}(\text{arr}, K, N-1) \quad \text{_____}$$

✓ ③ Reverse the entire array.

$a_3, a_4, a_5, a_0, a_1, a_2$

$$\text{rev}(\text{arr}, 0, N-1)$$

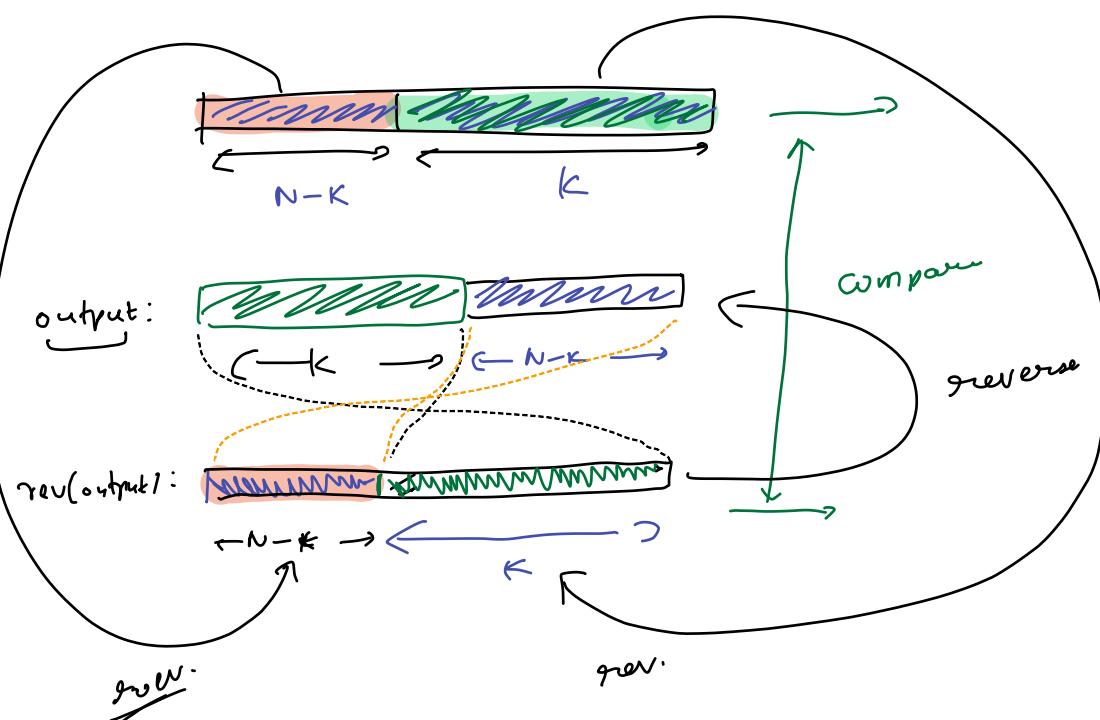
$$\frac{K}{2N}$$

$$\text{rev}(\text{output}) = \underbrace{\text{rev}(1st \ K)}_{\text{rev}(\text{output})} + \text{rev}(\text{last } N-K)$$

$$\text{output} = \text{rev}(\text{_____})$$

TC:  $O(N)$

SC:  $O(1)$



⇒

$\boxed{TC, HW}$

$\underbrace{\text{message...}}_i$

~~Doubts~~

~~Dictionary~~

$$arr = [1, 2, 3]$$

$\underline{k=5}$

(1)	3	1	2
(2)	2	3	1
(3)	1	2	3
(4)	3	1	2
(5)	2	3	1

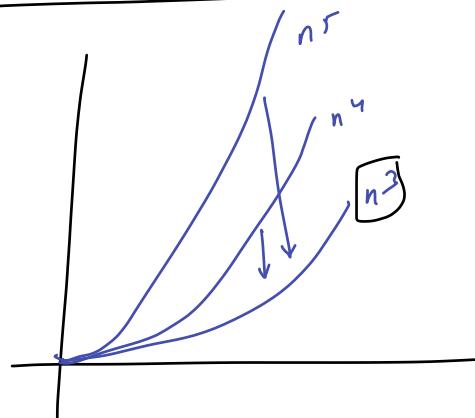
for (i) in range (1, n):

$$\underline{t = 5+i}$$

$O(1)$  space.

/ variable.

All Classes → Optional Classes



$$f(n) = O(n^3)$$

$$f(n) = O(n^4)$$

$$= \underline{O(n^5)}$$

$$\underline{n^3 \leq c \cdot n^4} \quad c \geq 1, n \geq 1$$

```

i = N
while i > 0:
    for j in range(0, i):
        # --
        i = i/2

```

i	j	Iterations
N	[0, N)	$N - 0 = \underline{N}$
$N/2$	[0, $N/2$ )	$N/2 - 0 = \underline{N/2}$
$N/4$	[0, $N/4$ )	$N/4$
1	1	1
1	1	1
1	1	1
2	[0, 1)	1
0		.

$\underline{\underline{= O(N)}}$

$$1 + 2 + 4 + \dots + N$$

Grp.

$$\begin{aligned}
& \frac{N}{1} + \frac{N}{2} + \frac{N}{4} + \dots + 1 + \dots \simeq N. \\
& \frac{a=N}{r=1/2} \quad S_\infty = \frac{a}{1-r} \quad |r| < 1 \\
& \quad = \frac{N}{1-\frac{1}{2}} = \frac{N}{1/2} = \underline{\underline{2N}}
\end{aligned}$$