

09/02/22

D SML Intermediate - D SA

Hashing - 2

Today's Content :

- ✓ (Q1) Count distinct for all subarrays of size K ↳ Sliding Window.
 - ✓ (Q2) Pair with given sum
 - * (Q3) Longest consecutive sequence
- ↓
optional + HW.

(1 hr)

Q1

Given $[N]$ array elements,

Calculate the no. of distinct elements in

Return an array every window (subarray) of size $[K]$.

Ex 1:

$N=4$

$K=1$

$\text{ans} = [1, 1, 1, 1] \rightarrow K=1$

$\Rightarrow N$ subarrays.

$N=4$

$K=2$

$[1, 1, 2, 2]$

$[1, 1] \rightarrow 1$ distinct
 $[1, 2] \rightarrow 2$ distinct
 $[2, 2] \rightarrow 1$ distinct

$\boxed{\text{ans} = [1, 2, 1]}$

$N=7$

$K=3$

$N-K=7-3=4$

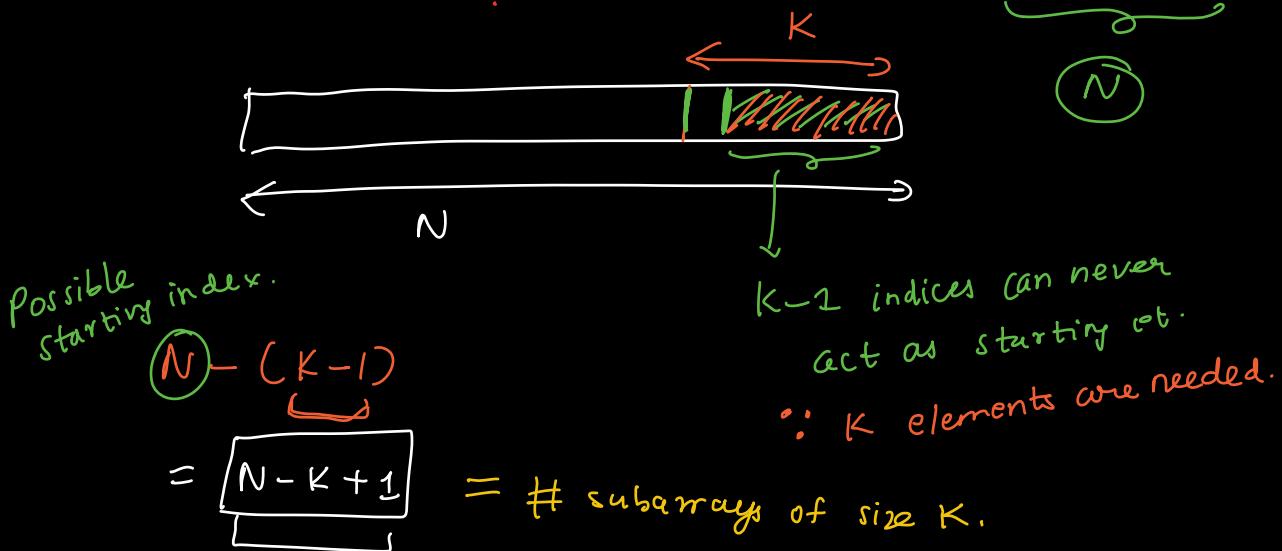
$A = [6, 3, 7, 3, 8, 6, 9]$

Sliding window

$\text{ans} = [3, 2, 3, 3, 3]$

$N-K+1 = 7-3+1 = 5$

Why $N-K+1$?



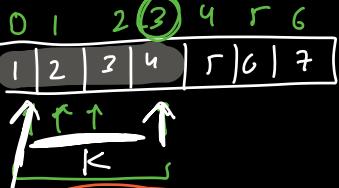
Brute Force

For every window of size K

get the no. of distinct elements

by inserting the elements in a set.

$N-K+1 = 4$	
$K = 4$	
$N = 7$	$N-K = 7-4 = 3$
$\begin{array}{ c c c c c c c } \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline \end{array}$	



$N-K+1 \rightarrow l = \underline{\text{idx}}$ for last window

```

ans = []
for start in range(0, N-K+1):
    my_set = set()
    for idx in range(start, start+K):
        my_set.add(A[idx])
    ans.append(len(my_set))

```

0	1	2	3	4	5	6
1	2	3	4	5	6	7

↑ ↑
K

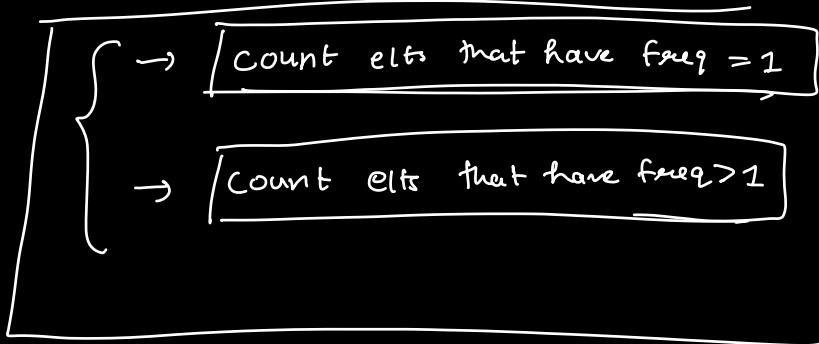
Pseudo-Code

0	1	2	3	4	5	6
1	2	3	4	5	6	7

↑ ↑
K

0	1	2	3	4	5	6
1	2	3	4	5	6	7

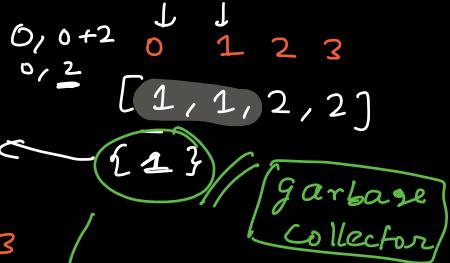
↑ ↑
K



$$N=4$$

$$K=2$$

$$\text{ans} = [1, 2, 1]$$



$$\text{ans} = []$$

```

for start in range(0, N-K+1):
    my_set = set() # starting from scratch.
    for idx in range(start, start+K):
        my_set.add(A[idx])
    ans.append(len(my_set))
    # set.clear() →
return ans;
  
```

start	idx	# iterations.
0	[0, K)	$K - 0 = K$
1	[1, K+1)	$K + 1 - 1 = K$
2	[2, K+2)	$K + 2 - K = K$
:	:	
$N-K$	$[N-K, N)$	$\frac{N - (N-K) = K}{(N-K+1) * K}$

$$\begin{matrix}
 0 & 1 & 2 & 3 \\
 \boxed{1, 1, 2, 2} \\
 \uparrow & \uparrow \\
 \{2\} \rightarrow 1
 \end{matrix}$$

$$(a, b) \Rightarrow b - a$$

$$\text{Total iterations} = \underbrace{(N-K+1) * K}_{\downarrow}$$

$N-K+1 \uparrow$

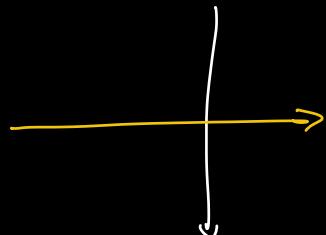
$K \uparrow$

Worst case

$$\rightarrow K=1$$

$$(N-K+1) * 1 = N \quad (N \text{ subarray})$$

$$\begin{matrix} \text{mid} \\ K = N/2 \end{matrix}$$



$$\left(N - \frac{N}{2} + 1 \right) * \frac{N}{2} = \frac{2N-N+2}{2} * \frac{N}{2} = \frac{(N+2)N}{4} = \frac{N^2+2N}{4}$$

$$\rightarrow K=N$$

$$(N-N+1) * N$$

$$= N$$

(1 subarray)

$$\begin{matrix} N \\ N-1 \\ N-2 \\ \vdots \\ 1 \end{matrix}$$

$$\boxed{\text{TC: } O(N^2)}$$

$$\text{SC: } O(K + (N-K+1))$$

\uparrow
set

\uparrow
ans array

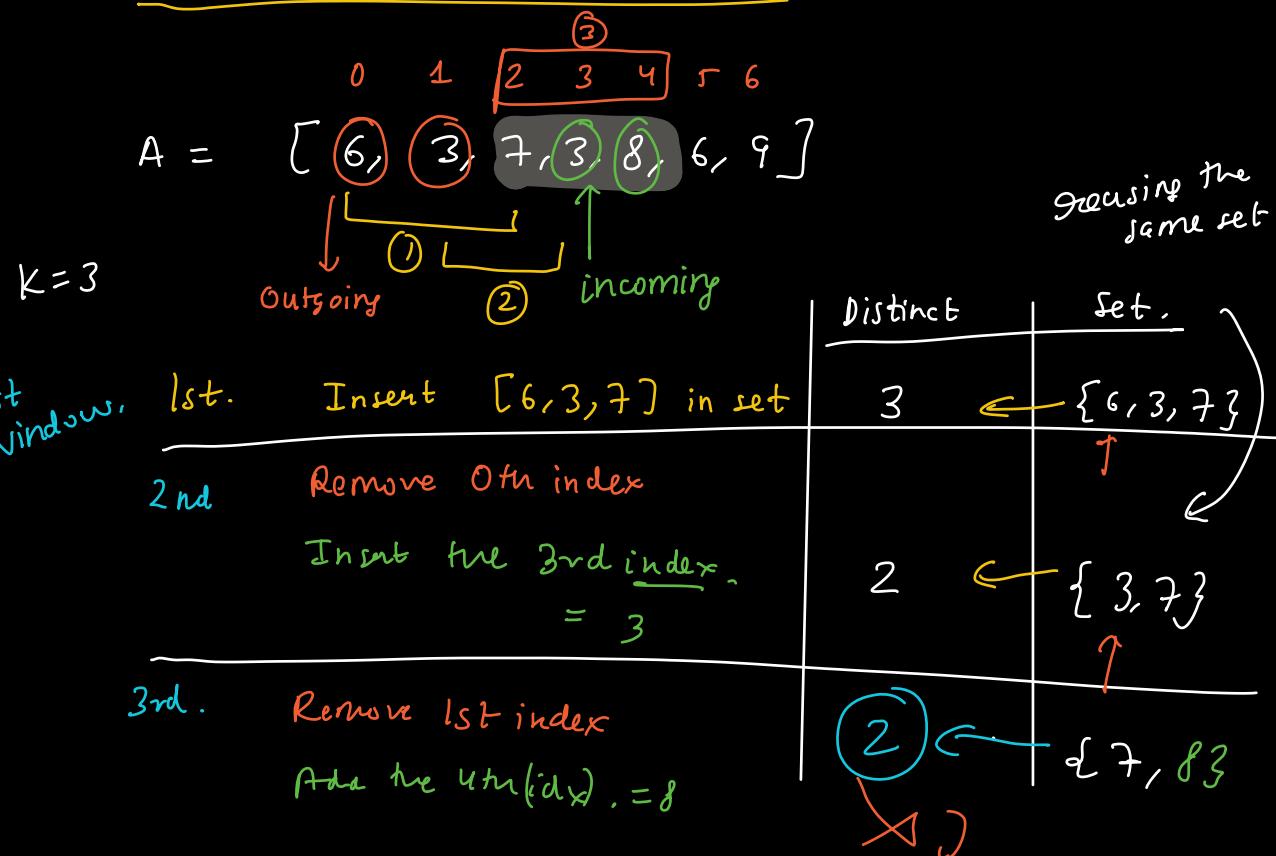
$$\Rightarrow O(N+1)$$

$$\curvearrowleft \boxed{\text{SC} = O(N)}$$

$$\begin{matrix} \text{SC:} \\ (N-K+1) * K \end{matrix}$$

if set is
not deleted
from memory
after every
iteration.

Optimisation using Sliding Window



Q) Why HashSet with sliding window does not work?

→ When we delete an elt, we indirectly delete

all its occurrences

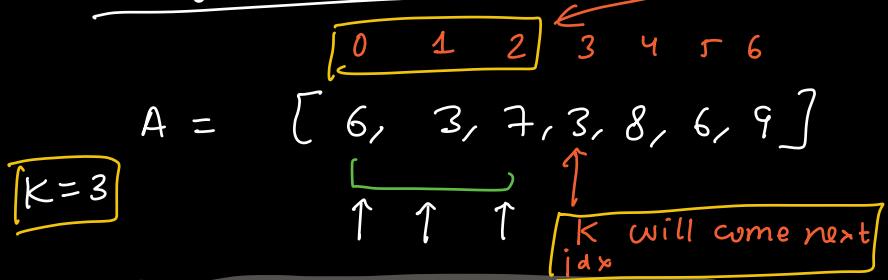
→ It does not maintain count/freq.

⇒ Use a HashMap = dict in Python.

Reverse of HashSet ↴

Sliding Window Technique

0 to $(K-1)$ already inserted.



① Insert first K elts in dict: freq_dict

default dict
default zero
. Check might
not be needed.

```

freq_dict = {}
ans = []
for i in range(0, K):
    freq_dict[A[i]] += 1
    if A[i] in freq_dict:
        freq_dict[A[i]] += 1
    else:
        freq_dict[A[i]] = 1

```

freq_dict

update.

dict {key: val}

freq_dict

Ans. append (len(freq_dict))

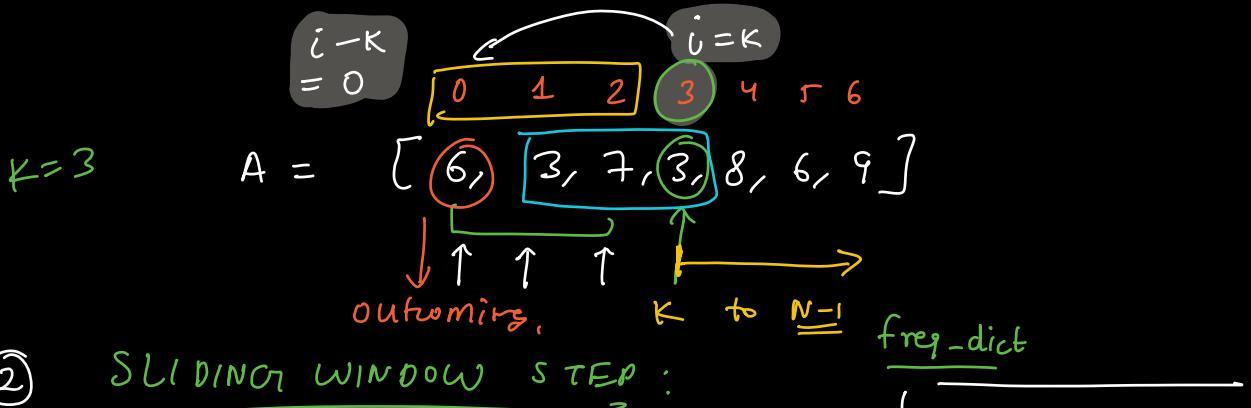
no. of keys

6, 7, 7

6: 1
7: 2

[6, 7]

len() → # keys



② SLIDING WINDOW STEP :

incoming index : $i = K$ index

Outgoing index : $(i-K)$ index.

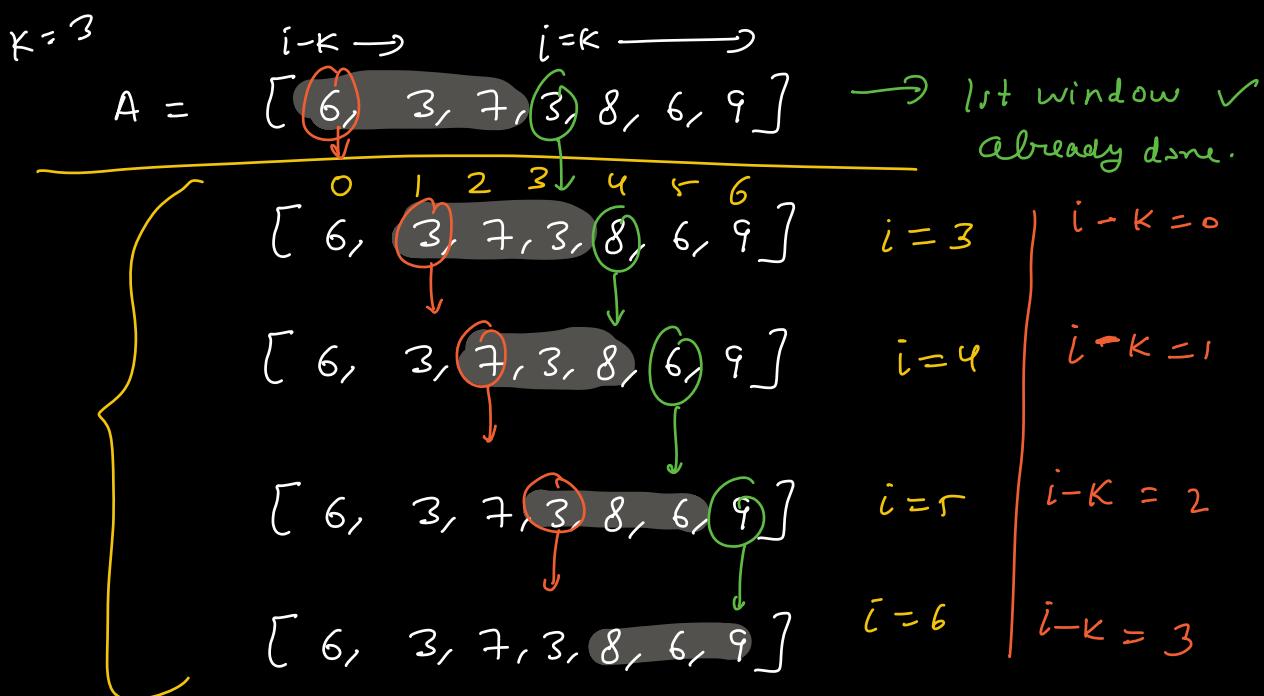
```

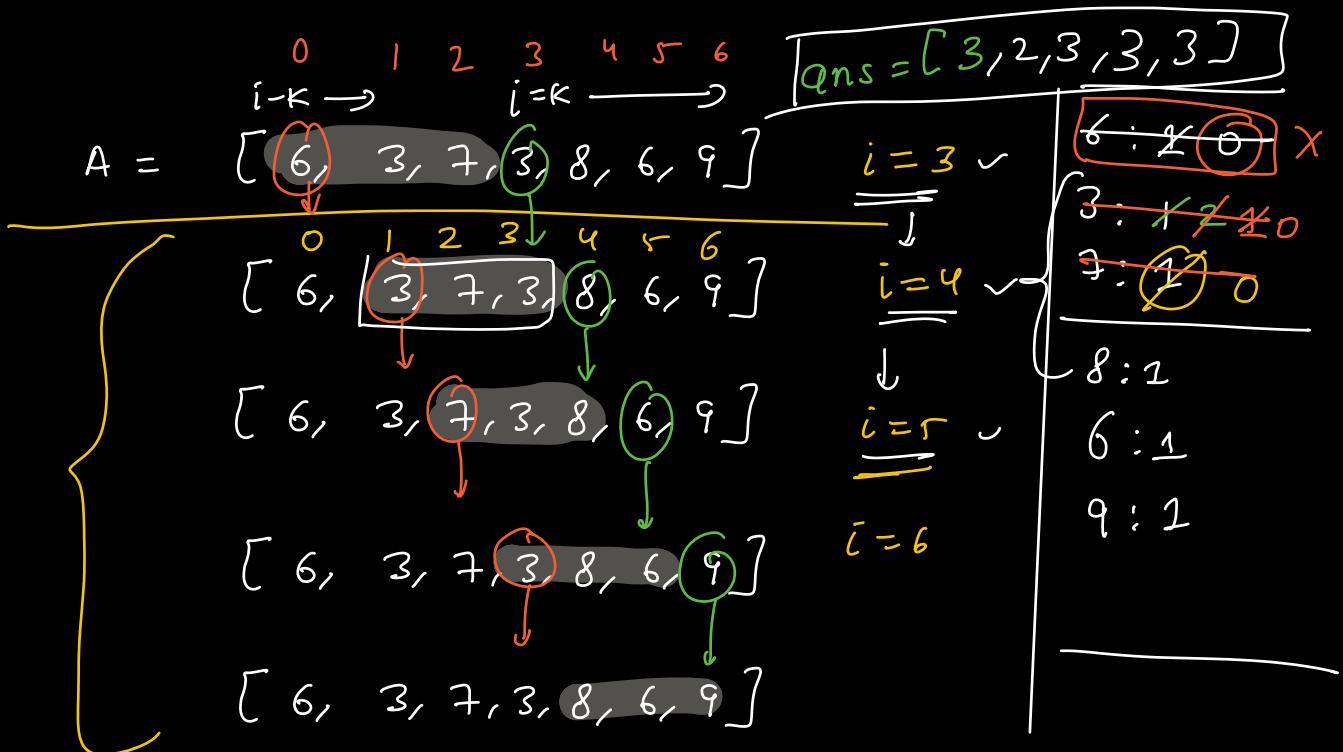
for i in range(K, N) :
    # insert A[i] in HashMap.
    # removing A[i-K] in HashMap.

```

6 : 1
3 : 2
7 : 1

freq-dict





for i in range(K, N):

insert $A[i]$ in hashMap.

if $A[i]$ in freq-dict:

 freq-dict [$A[i]$] += 1

else :

 freq-dict [$A[i]$] = 1

removing $A[i-K]$ in hashMap.

freq-dict [$\underline{A[i-K]}$] - = 1

if freq-dict [$\underline{A[i-K]}$] == 0 : # elts with

 # delete the key

$\cancel{\text{freq} > 0}$
 $\cancel{(\text{val})}$

 del freq-dict [$\underline{A[i-K]}$]

ans.append ($\underline{\text{len(freq-dict)}}$)

to set correct
no. of freqs

①

```

freq_dict = {} (0, K)
ans = []
for i in range(0, K):
    freq_dict[A[i]] += 1
    if A[i] in freq_dict:
        freq_dict[A[i]] += 1
    else:
        freq_dict[A[i]] = 1

```

ans.append(len(freq_dict))

Initialize

Wind. w

iterations

$k_0 = K$

* $O(1)$

= $O(K)$

② for i in range(K, N):

insert A[i] in HashMap.

if A[i] in freq_dict:

freq_dict[A[i]] += 1

else:

freq_dict[A[i]] = 1

removing A[i-K] in HashMap.

freq_dict[A[i-K]] -= 1

if freq_dict[A[i-K]] == 0:

delete the key

del freq_dict[A[i-K]]

ans.append(len(freq_dict))

Sliding
window
step.

iterations

[K, N)

$N - K$

* $O(1)$

= $O(N - K)$

Total Time = $O(K) + O(N - K) = O(N)$

Total space = dict + ans

at any time not exceed K .

dict $\left\{ \begin{array}{l} K - r + 1 \\ K - r + 1 \\ K - r + 1 \end{array} \right.$ does

+

ans $\left\{ \begin{array}{l} (N - K + 1) \end{array} \right.$

$$K + (N - K + 1) = O(N+1) = O(N)$$

Only dict space = $O(K)$

Q2

True | False.
↑
Given N array elements, check if there exists a pair (i, j) such that

Google
Amazon
Microsoft

$\text{arr}[i] + \text{arr}[j] = K$
C target

① $i \neq j$

Ex. 1 $A = [2, 7, 11, 13]$

$K=18$	i, j	3 y.	$K=18$	$7+11$	$\checkmark \text{ans} = \text{True}$
$K=9$	$(1, 2)$		$K=9$	$2+7$	$\checkmark \text{ans} = \text{True}$
$K=17$	$(0, 1)$		$K=17$	\times	$\text{ans} = \text{False.}$
	$K=17$	\times			

Ex. 2 $A = [8, 9, 1, -2, 4, 5, 11, -6, 7, 5]$

$K=6$	$(0, 3)$	$8-2=6$
\downarrow	$(2, 5)$	$1+5=6$
True	$(2, 9)$	$1+5=6$

$K=100 \rightarrow \text{False.}$

Soln 1 → Loop over all the pairs.

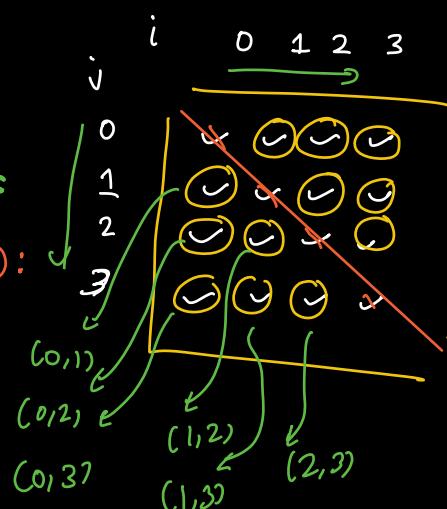
$N=4$

```

 $N = \text{len}(A)$ 
for i in range(0, N):
    for j in range(i+1, N):
        if  $(A[i] + A[j]) == K$ :
            return True
return False
  
```

TC: $O(N^2)$

SC: $O(1)$



Quiz - 4 * $A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 7, & \underline{1}, & 10, & \underline{-3}, & \underline{9}, & \underline{7} \end{bmatrix}$

-2 → (1, 3)

10 → (1, 4)

18 ×

14 → (0, 5)

$K=11$

$i \downarrow$ Issue
 $A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8, & 1, & -2, & 4, & 5, & 11, & -6, & 7, & 5 \end{bmatrix}$

$\left(\begin{array}{c} 8+9 \\ 8+1 \\ 8-2 \\ 8+4 \\ 8+5 \\ 8+11 \\ 8-6 \\ 8+7 \\ 8+5 \end{array} \right)$

Issue

Trying to find an ele j for i

$A[i] + A[j] = K$

\nwarrow fixed

$\Rightarrow A[j] = (K - A[i])$

f : search for 3

(Q) For given $i \rightarrow A[i]$

Check if $\underline{j} \rightarrow A[j] = \underline{K - A[i]}$

Hashset → allows to check known in $O(1)$ time. \Rightarrow doesn't track duplicates

① Add all elements in set. $\Rightarrow O(N)$

$K=11$

$A = [8, 9, 1, -2, 4, 5, 11, -6, 7, 5]$

set: $8, 9, 1, -2, 4, 5, 11, -6, 7$

i	$A[i]$	$j \rightarrow A[j] = K - A[i]$
0	8	$11 - 8 = 3$ $O(1)$
1	9	$11 - 9 = 2$
2	1	$11 - 1 = 10$
3	-2	$11 - (-2) = 13$
4	4	$11 - 4 = 7$

Reverse of HashSet!

$$K=22$$

$$A = \left[\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 9, 1, -2, 4, 5, 11, 6, 7, 5 \end{matrix} \right] \quad \begin{cases} (6, 6) \quad i! = j \\ 11 + 11 = 22 \end{cases}$$

set

i	$A[i]$	$j \rightarrow A[j] = K - A[i]$
0	8	$22 - 8 = 14 \times$
1	9	$22 - 9 = 13 \times$
2	1	$22 - 1 = 21 \times$
3	-2	$22 - (-2) = 24 \times$
4	4	$22 - 4 = 18 \times$
5	5	$22 - 5 = 17 \times$
6	11	$22 - 11 = 11$ Issue

8,
9, 1,
-2, 4,
5, 11,
-6, 7,

(Q) Can we handle this case using a freq-map/freq-dict

When 2 elts equal

$$A[i] == A[j]$$

$$\text{freq}[A[i]] \geq 2$$

Pseudo Code

explain {

① Build the freq map → similar to Q1.

② for i in range (0, N) :

a = A[i]

$\bar{b} = k - A[i] \# \underbrace{(A[i])}_{\sum_j}$

if $a == \bar{b}$ and freq[a] >= 2 :

return True

else if $a \neq \bar{b}$ and $\underbrace{b \text{ in freq}}_{\text{search}} :$ $O(1)$

return True.

return False.

$K = 2$
 explain
 {
 ① $K - A[i]$ [2, 3, 1, 4, 5, 6, 3, 6]
 Build the freq map → similar to Q1.
 [O(N)]
 }
 ② for i in range (0, N) : O(N)
 {
 $a = A[i]$
 $\rightarrow b = K - A[i] \# (A[j])$
 if $a == b$ and $\text{freq}[a] \geq 2$:
 return True
 else if $a \neq b$ and $b \in \text{freq}$:
 return True.
 search O(1)
 return False.

Time = O(N)

Space = O(N)

To do

1. It can be done using Hashset.

→ While iterating add to set.
 → Try to think

2. $(i, j) \Rightarrow A[i] - A[j] = K$

Single Pass

* [Optional] Given N array elements, find the length of the longest sequence which can be rearranged in a Strictly Increasing Order where each element is one more than the prev. element.

Q 3 might not contiguous \neq subarray.
 Tricky Advance level problem

I/P: $A = [100, 4, 200, 1, 3, 2]$ Max length = 4

~~4, 1, 3, 2, 00~~
 Seg. \Rightarrow deleting any # els from anywhere in the array.

$4, 1, 3, 2 \Rightarrow [1, 2, 3, 4]$
 Sort

$[1, 2, 3, 4] \quad [100, 200]$

Idea 1 : Sorting. $O(N \log N)$

Idea 2 : Hashmap / Hashset $O(N)$ Think f/w

Discuss in Next.

Doubts

$$\begin{aligned} 2 &\rightarrow 2 & (2 \ 3) \\ 3 &\rightarrow 3 & \\ 2, 3 &\rightarrow 6 & 2 * 3 = 6 \end{aligned}$$

$$\begin{array}{r} 2 \ 3 \ 6 \\ \boxed{2} \\ \downarrow \end{array} \quad 2^3 - 1$$

$$\begin{array}{r} 2 \\ 3 \\ (6) \\ \boxed{2, 3 \rightarrow 6} \end{array}$$

Idea subarray

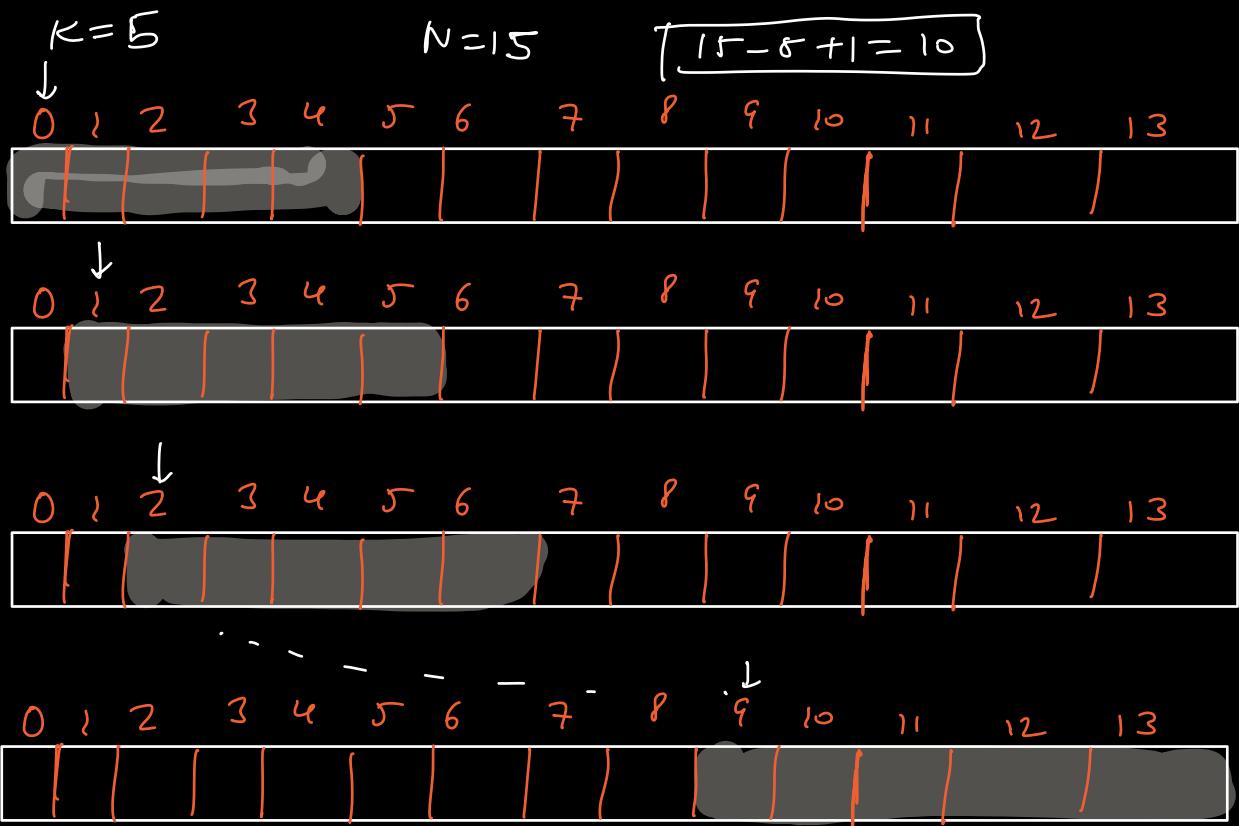
$$\begin{aligned} O(N) \\ O(\log N) \end{aligned}$$

$$\begin{array}{c} 2, 2 \\ \uparrow \uparrow \\ 2 \\ 2 \end{array}$$

$$\begin{array}{l} 6, 3, 2 \\ \boxed{6} \\ 3 \\ 2 \\ 6, 3 \\ \boxed{3, 2 \rightarrow 6} \\ \hline 6 \\ 32 \rightarrow 6 \end{array}$$

$$\begin{array}{c} 3 \ 2 \ 4 \ 5 \\ \downarrow \\ 3 \\ 2 \\ 4 \\ 5 \\ 32 \rightarrow 6 \\ 24 \rightarrow 8 \\ 45 \rightarrow 20 \\ 324 \rightarrow 2^4 \\ 245 \rightarrow \underline{20} \end{array}$$

$$\begin{array}{c} (2, 6, 3) \\ 6/3 = 2 \\ 2 \\ 6 \\ 3 \end{array}$$



Count of duplicates ($\text{freq} \geq 2$) for all windows of size K .

$\underline{N = 10}$ $\underline{K = 4}$	$2, 3, 2, 1, 4, 2, 3, 4, 5, 6$ $\text{ans} = 1$	$2: \cancel{2}, \cancel{2}$ $3: \cancel{2}, \cancel{1}$ $1: \cancel{1}, 0$ $4: \cancel{2}, \cancel{2}$ \uparrow $\text{cnt_dup} = 1$ $= x$ b
	$2, 3, 2, 1, 4, 2, 3, 4, 5, 6$ \downarrow $\text{ans} = 0$	$\text{cnt_dup} = 1$
	$2, 3, 2, 1, 4, 2, 3, 4, 5, 6$ \downarrow $\text{ans} = 1$	$\text{cnt_dup} = 1$
	$2, 3, 2, 1, 4, 2, 3, 4, 5, 6$ \downarrow $\text{ans} = 0$	$\text{cnt_dup} = 1$
	$2, 3, 2, 1, 4, 2, 3, 4, 5, 6$ \downarrow $\text{ans} = 2$	$\text{cnt_dup} = 2$

2, 3, 2, 1, 4, 2, 3, 4, 5, 6

2, 3, 2, 1, 4, 2, 3, 4, 5, 6

$K=6$

[2] \leftarrow

0 1 2 3 4 5 6 7 8 9 10 11 12
2, 2, 3, 4, 2, 3, 4, 3, 2, 2, 3, 4, 3

[3] \leftarrow

0 1 2 3 4 5 6 7 8 9 10 11 12
2, 2, 3, 4, 2, 3, 4, 3, 2, 2, 3, 4, 3

0 1 2 3 4 5 6 7 8 9 10 11 12
2, 2, 3, 4, 2, 3, 4, 3, 2, 2, 3, 4, 3

0 1 2 3 4 5 6 7 8 9 10 11 12
2, 2, 3, 4, 2, 3, 4, 3, 2, 2, 3, 4, 3

Cnt = 2 3

2: 2 3

3: 2

4: 2

1 \rightarrow 2
2 \rightarrow 1

Ans changing.