Naive Bayes :)

Lecture Notes

Q.1 Guess the word ?

Objectives

- Laplace Smoothing
- Hyperparameter
- Zero Conditional Probability
- Impact of Conditional Independence
- Underflow Problem
- Bottom Superclass
- Impact of Priors
- Multinomial NB

Naive Bayes

Bayes' rule: $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$

$P(Y=1|X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

$P(X) = P(X|Y=1)\,P(Y=1) + P(X|Y=0)\,P(Y=0)$

as $P(X|Y=1) \to 0$ or $P(Y=1) \to 0$

$P(Y=1|X) \to 0$

$P(Y=0|X) = \frac{P(X|Y=0)\,P(Y=0)}{P(X)}$

$P(Y=1|X) + P(Y=0|X) = \frac{P(X|Y=1)\,P(Y=1) + P(X|Y=0)\,P(Y=0)}{P(X)} = 1$

$P(Y=1|X) = 1$

Proof: Impact of Zero Priors on $P(Y=1|X=1)$ & $P(Y=0|X=1)$

Laplace Smoothing

$P(w^{i}|Y=1) = \frac{n_{i}}{n}$

$P(w^{i}|Y=1) = \frac{n_{i}+\alpha}{n+\alpha\,|V|}$

α = Laplace smoothing

α = hyperparameter

α = 1 Laplace smoothing

α = 0 MLE

α = 1 Laplace smoothing

α = 0 MLE

Impact of α

$\alpha \to \infty$ $\quad P(w^{i}|Y=1) \to \frac{1}{|V|}$

$\alpha \to 0$ $\quad P(w^{i}|Y=1) \to \frac{n_{i}}{n}$

Impact of Zero Conditional Probability

$P(w^{i}|Y=1) = 0$

$P(Y=1|X) = 0$

$P(Y=0|X) = 1$

Impact of Conditional Independence

$P(Y=1|X) = 0.9$

$P(Y=0|X) = 0.1$

$P(Y=1|X) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.34$

$P(Y=0|X) = 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 \times 0.1 = 0.0000000001$

Underflow

$\log(P(Y=1|X)) = \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9)$

$\log(P(Y=0|X)) = \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1)$

$\log(P(Y=1|X)) = -0.0457$

$\log(P(Y=0|X)) = -10.0$

Bottom Superclass

Bottom Superclass

Impact of Priors

$P(Y=1|X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

$P(Y=0|X) = \frac{P(X|Y=0)\,P(Y=0)}{P(X)}$

$P(Y=1|X) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.34$

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Multinomial NB

$P(w^{i}|Y=1) = \frac{n_{i}}{n}$

$P(w^{i}|Y=1) = \frac{n_{i}+\alpha}{n+\alpha\,|V|}$

α = Laplace smoothing

α = hyperparameter

α = 1 Laplace smoothing

α = 0 MLE

$\alpha \to \infty$ $\quad P(w^{i}|Y=1) \to \frac{1}{|V|}$

$\alpha \to 0$ $\quad P(w^{i}|Y=1) \to \frac{n_{i}}{n}$

Impact of Zero Conditional Probability

$P(w^{i}|Y=1) = 0$

$P(Y=1|X) = 0$

$P(Y=0|X) = 1$

Impact of Conditional Independence

$P(Y=1|X) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.34$

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Underflow

$\log(P(Y=1|X)) = \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9)$

$\log(P(Y=0|X)) = \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1) + \log(0.1)$

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$\log(P(Y=0|X)) = -10.0$

Bottom Superclass

Bottom Superclass

Impact of Priors

$P(Y=1|X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

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Conclusion

- Laplace Smoothing $\to$ Hyperparameter
- Zero Conditional Probability $\to$ Laplace Smoothing
- Underflow $\to$ Log
- Bottom Superclass $\to$ Impact of Priors
- Multinomial NB

Impact of Priors

Bayes' rule: $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$

Prior $\to$ Posterior

$P(A|B) \propto P(B|A)\,P(A)$

Posterior $\propto$ Likelihood $\times$ Prior

Bernoulli NB

$P(Y=1|X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

$P(Y=0|X) = \frac{P(X|Y=0)\,P(Y=0)}{P(X)}$

$P(Y=1|X) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.34$

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Multinomial NB

$P(Y=1|X) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

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Conclusion

- Prior $\to$ Posterior
- Bernoulli NB $\to$ Multinomial NB
- Multinomial NB $\to$ Bernoulli NB
- Bernoulli NB $\to$ Multinomial NB

Multinomial Naive Bayes

Bernoulli NB $\to$ Multinomial NB

$P(w^{i}|Y=1) = \frac{n_{i}}{n}$

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Impact of Zero Conditional Probability

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$P(Y=0|X) = 1$

Impact of Conditional Independence

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Underflow

$\log(P(Y=1|X)) = \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9) + \log(0.9)$

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Conclusion

- Prior $\to$ Posterior
- Bernoulli NB $\to$ Multinomial NB
- Multinomial NB $\to$ Bernoulli NB
- Bernoulli NB $\to$ Multinomial NB

Bernoulli NB

$P(X|Y=1) = \frac{P(X|Y=1)\,P(Y=1)}{P(X)}$

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