

Last ML2 session

## Time Series Forecasting - 5

→ ARIMA

→ CI

→ ~~fb~~ Prophet

→ Exogenous vars

# ARIMA

→ AR : Auto Regressor AR(p)

$$\hat{y}_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} \dots \alpha_p y_{t-p} + \alpha_0$$

Regression

feature			target
$y_{t-3}$	$y_{t-2}$	$y_{t-1}$	$y_t$
$t-4$	$t-3$	$y_{t-2}$	$y_{t+1}$
$t-5$	$t-4$	$t-3$	$y_{t+2}$
← 2			$y$
X			

past 3 values  
feature

past P values are feat

AR(p)

p+1 params

$$\hat{y}_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + \underline{\alpha_0}$$

Key: idea  $\rightarrow$  future depends on past values

MA(q)

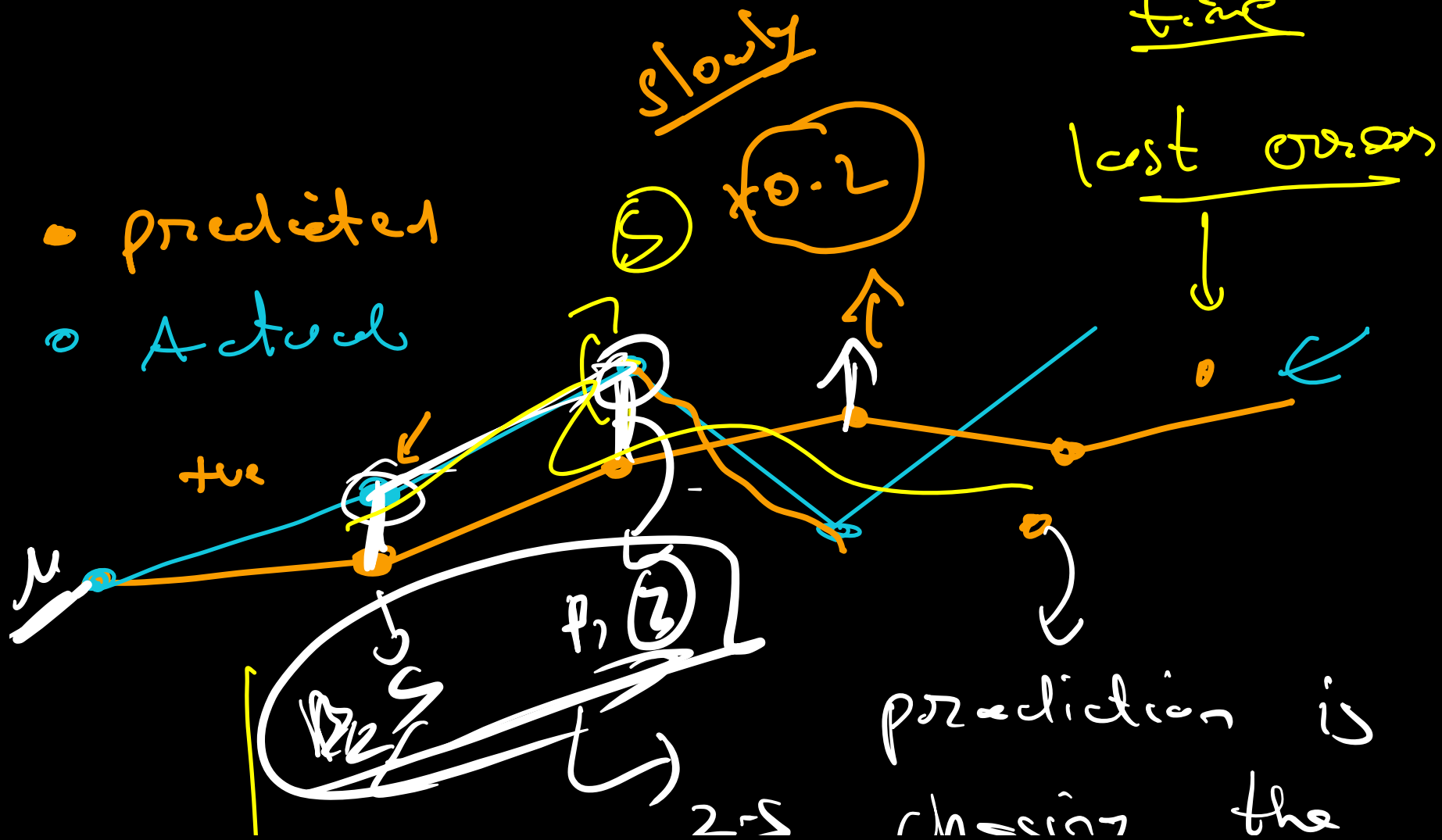
$\nearrow$  global mean

$$\hat{y}_t = \mu + \beta_1 e_{t-1} + \beta_2 e_{t-2} \dots \beta_q e_{t-q} \leftarrow$$

Key: future value depends on the  
past errors

Stationary TS : Mean does not change with time

- predicted
- Actual





moving  
true value

$$\hat{y}_t = \mu \oplus \beta_1 e_{t-1} \oplus \beta_2 e_{t-2} \oplus \dots \oplus \beta_q \underline{e_{t-q}}$$

Annotations:  
- A blue checkmark is next to  $e_{t-1}$ .  
- A blue arrow points from  $e_{t-1}$  to  $\underline{e_{t-q}}$ .  
- A blue arrow points from  $\beta_1$  to 0.2.  
- A blue arrow points from  $\beta_q$  to  $e_{t-q}$ .

ARMA(p, q)

$$\hat{y}_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p}$$

$$+ \beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q e_{t-q}$$

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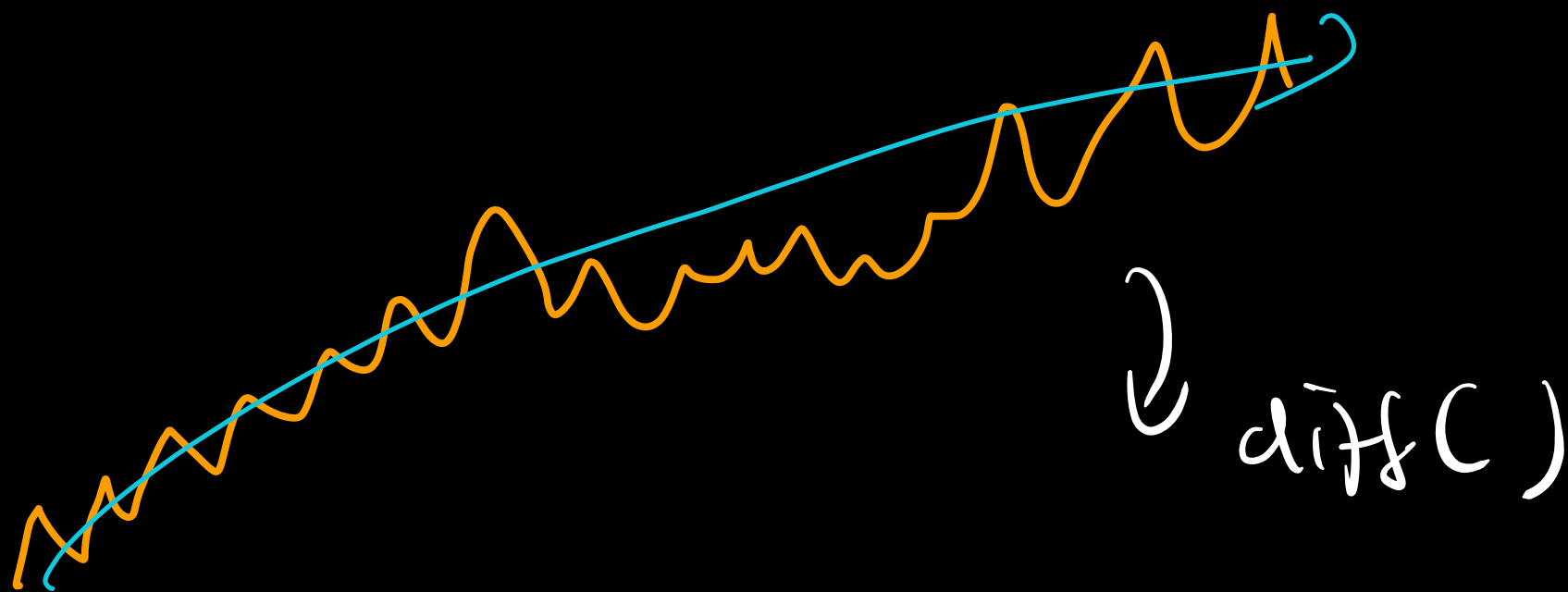
these models assume  
stationarity

inp = mobile\_sales.diff() ←

pred = SARIMAX( )

... season() + sales[-1]

$$\text{out} = \text{pred} \cdot \text{in}$$



not



Jan

↓

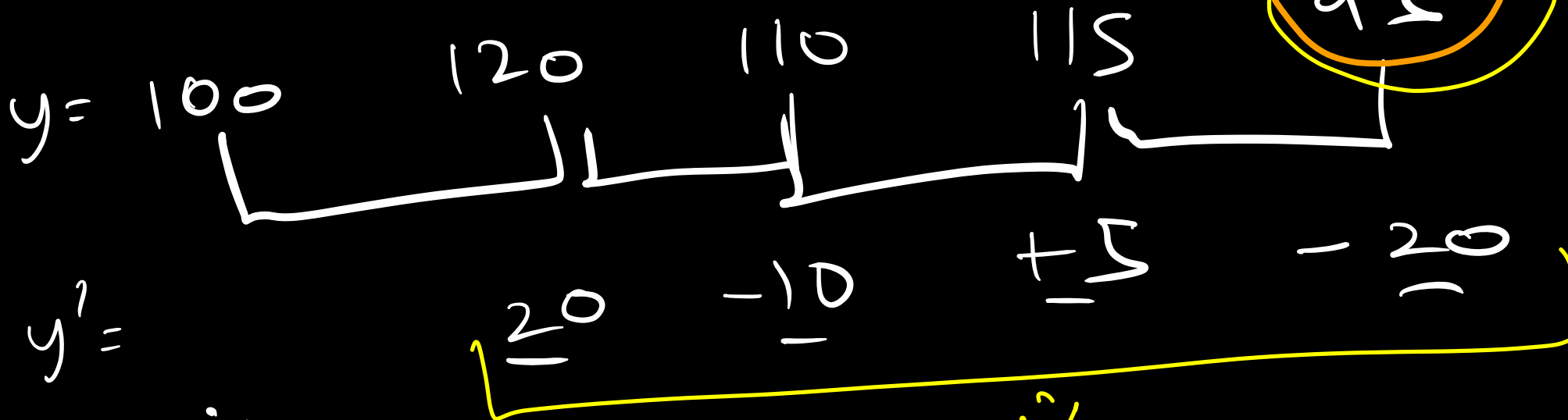
J

F

M

A

May

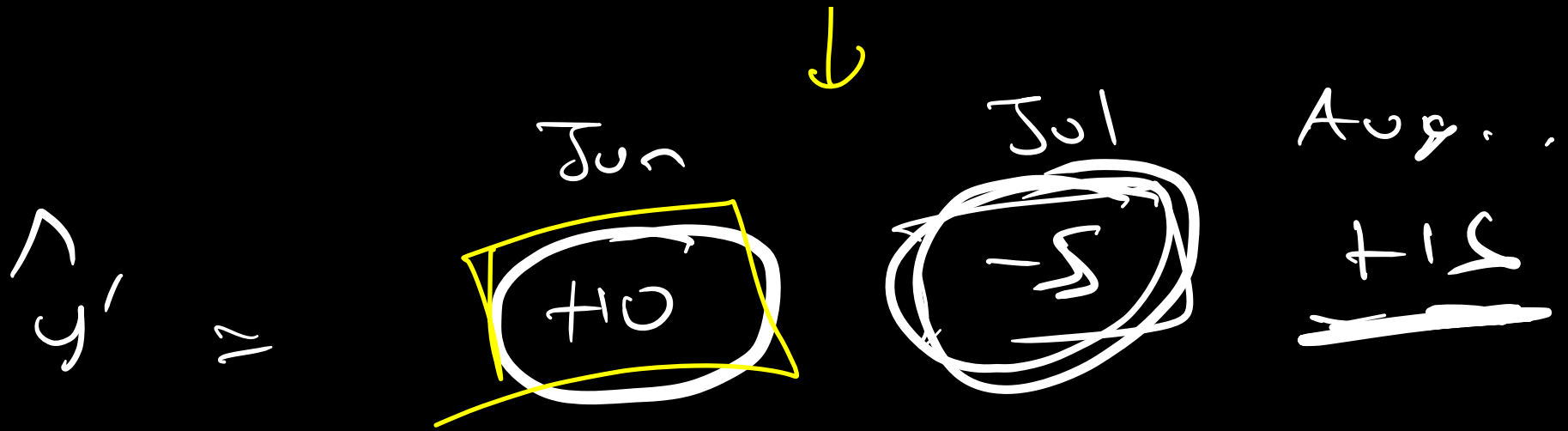


$y' =$

diff

1/4  
↓

AR / MA / ARMA



inter

$$y[-1] = 95$$

$$\hat{y}_t = y_{t-1} + \hat{y}_{t+1}$$

$\downarrow$   $\downarrow$

y

Jun  
105

Jul  
100

Aug  
115

qs + y' cumsum()

+10	-5	+15
<hr/>		
+10	<u>+5</u>	<u>+20</u>

7 10

105

100

115

ARIMA (p, d, q)  
↑  
Integration

$d \Rightarrow 1$   
(3)  
0  
~~2~~  
(3)

AR(p)

MA(q)

I (d)

$$\begin{array}{l}
 y = [y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5] \\
 y' = [y'_1 \quad y'_2 \quad y'_3 \quad y'_4] \\
 \textcircled{y''} [y''_1 \quad y''_2 \quad y''_3]
 \end{array}$$

$$\frac{d}{dt} \left( \frac{d(c)}{dt} \right) \rightarrow \frac{d^2(c)}{dt^2}$$

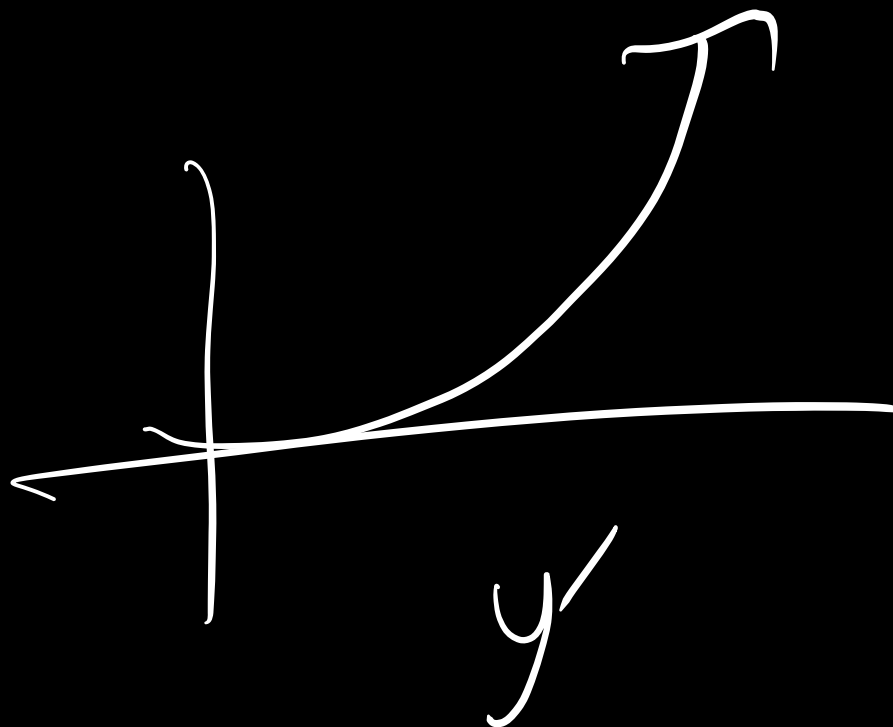
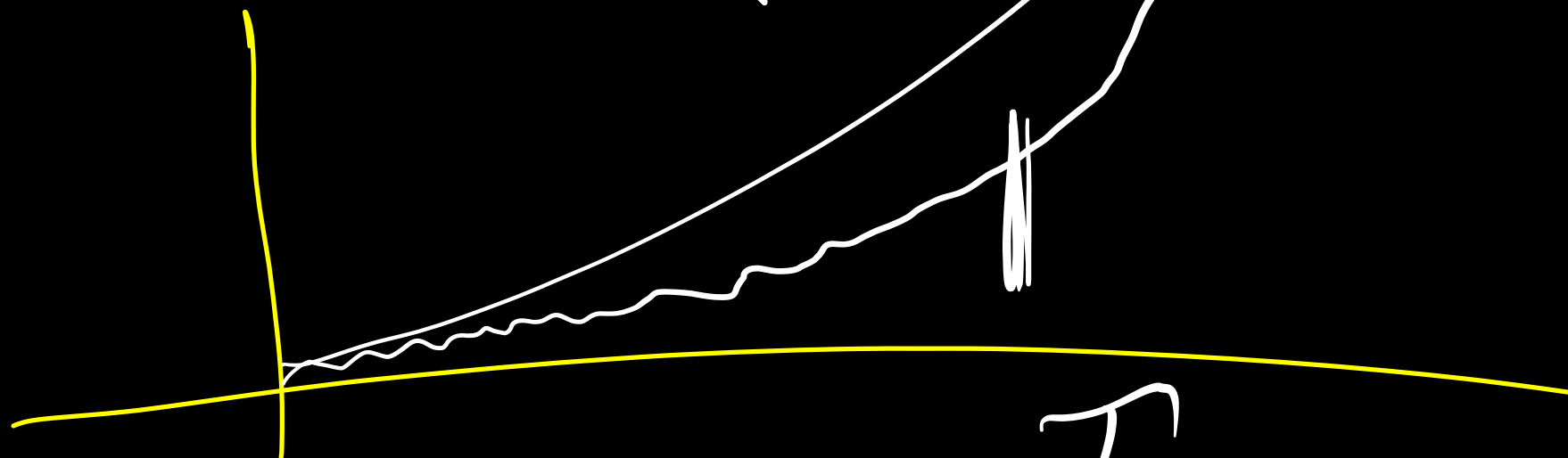
" 0^0 ..id x 0

# views

$\propto v$

non-linear

$t^2$



SARIMA (p, d, q, [P, D, Q, s])

$$\hat{y}_t = c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p}$$

$$\beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q e_{t-q}$$

$$\gamma_1 y_{t-s} + \gamma_2 y_{t-2s} + \dots + \gamma_P y_{t-Ps}$$

$$\delta_1 e_{t-s} + \delta_2 e_{t-2s} + \dots + \delta_Q e_{t-Qs}$$

SARIMA (p, d, q, P, D, Q, S)

init

diff (D)

1

2

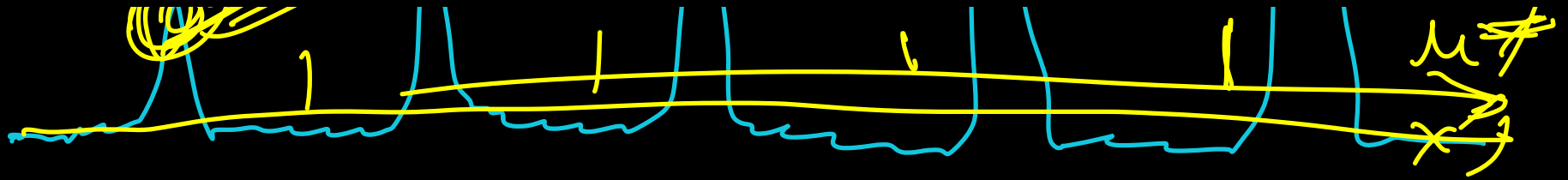
3

4

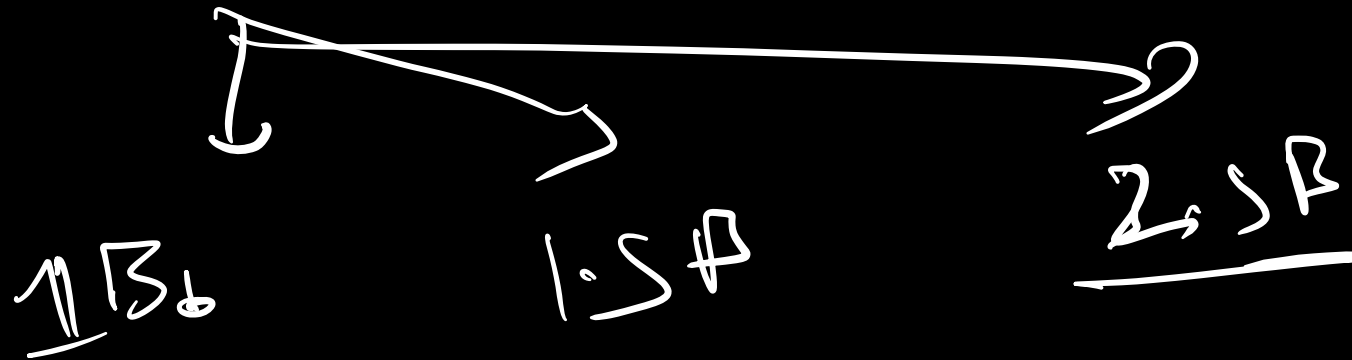
5

detrend

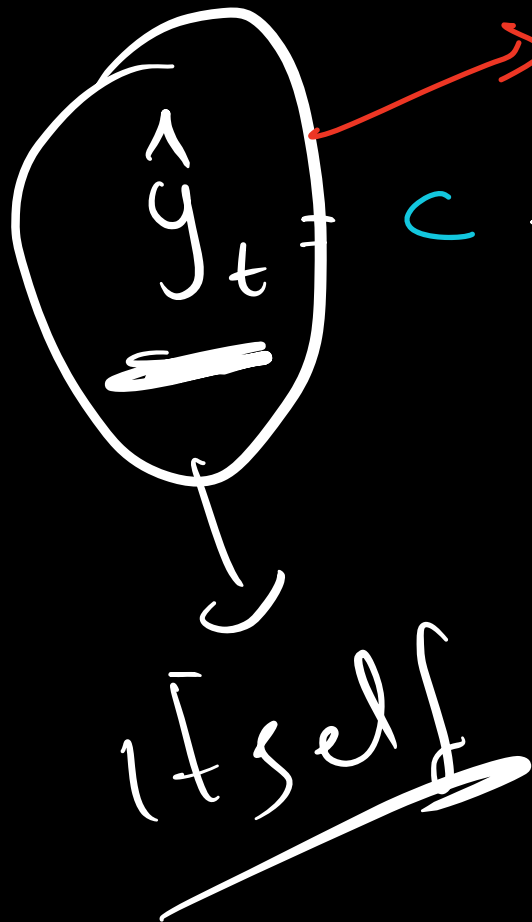




FLPKT  $\rightarrow$  Big Billion Sale



# SARIMAX



$$\begin{aligned}
 & c + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} \\
 & + \beta_1 e_{t-1} + \beta_2 e_{t-2} + \dots + \beta_q e_{t-q} \\
 & \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \dots + \gamma_p y_{t-p} \\
 & \delta_1 e_{t-1} + \delta_2 e_{t-2} + \dots + \delta_q e_{t-q} \\
 & + \varepsilon_1 (\text{holiday}) + \varepsilon_2 (\text{GDP}) \\
 & + \varepsilon_3 (\text{LG penetration})
 \end{aligned}$$

SARIMAX

which model to use !!

if complete random

↳ Sample methods

mean  
naïve  
ARMA  
SES  
↳ trend

DES

ARIMA  
          

↳ Seasonality

↳ TES

↳ SARIMA

↳ Exog

custom / manual

SARIMAX