# Time Series Forecasting -3

- → MA for forecasting (code)
- → Smoothing Methods
- → Stationarity
- → Autocorrelation / PACF

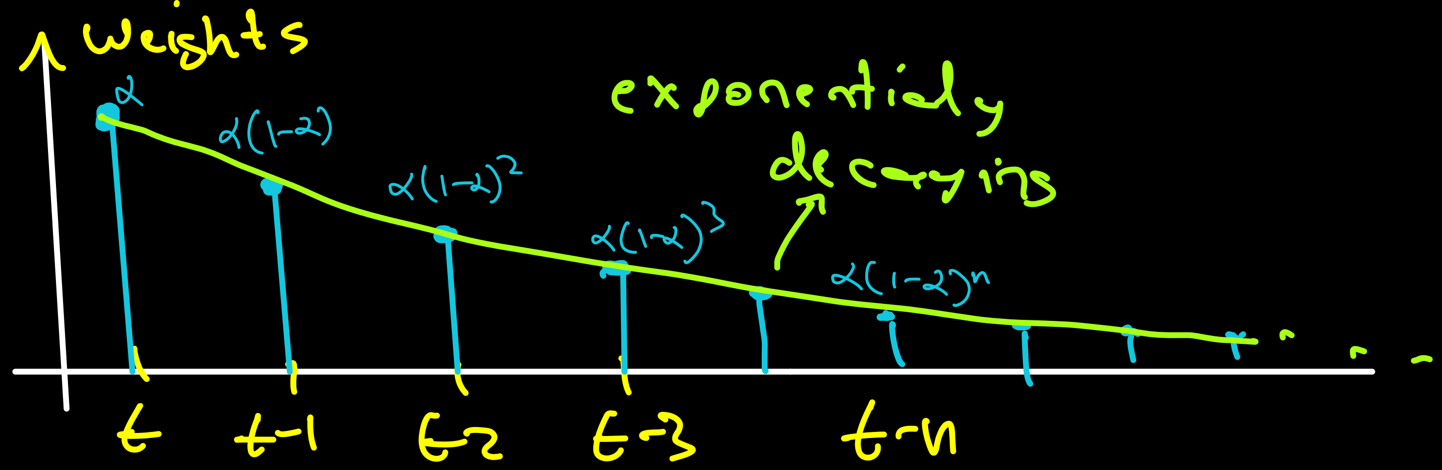
# Simple exponential smoothing (SES)

$$\hat{y}_{t+h} = \alpha y_t + (1-\alpha)\hat{y}_t$$

$$\hat{y}_{t+h} = \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}\right]$$

$$= \underbrace{\alpha}_{\text{weigh}} \underbrace{y_t}_{\text{last value}} + \underbrace{(1-\alpha)\alpha}_{\text{less weigh}} . \underbrace{y_{t-1}}_{\text{old value}} + \underbrace{(1-\alpha)^2}_{\text{lesser weight..}} \underbrace{\left[\alpha y_{t-i} \cdots\right]}_{\text{older values}}$$

It will slowly "forget" older values




Simple way to remember:

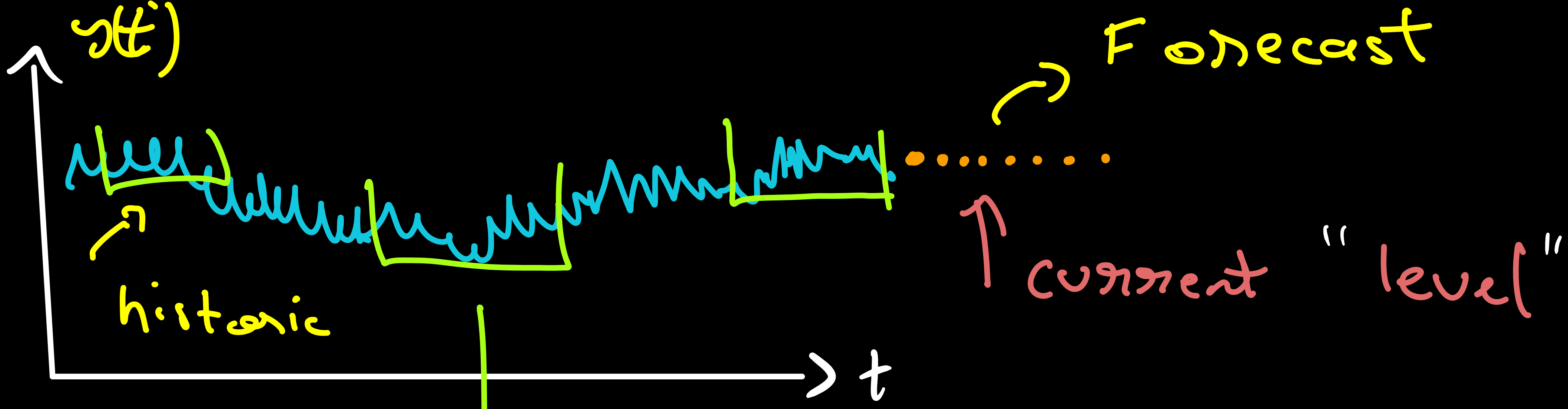
$$\hat{y}_{t+1} = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \dots + \underbrace{\alpha(1-\alpha)^t y_0}_{\text{coeff is too small} \sim \underline{\underline{0}}}$$

if $\alpha = 0.8$

$$\hat{y}_{t+1} = 0.8 y_t + 0.16 y_{t-1} + 0.032 y_{t-2} + 0.0064 y_{t-3} \sim 0$$

↗ weighted average!

Note : All future values have the same forecast




level of TS is different at diff t

SES is good for predicting level of TS

$\alpha \rightarrow$ low $\rightarrow$ global mean

$\alpha \rightarrow$ high $\rightarrow$ naive

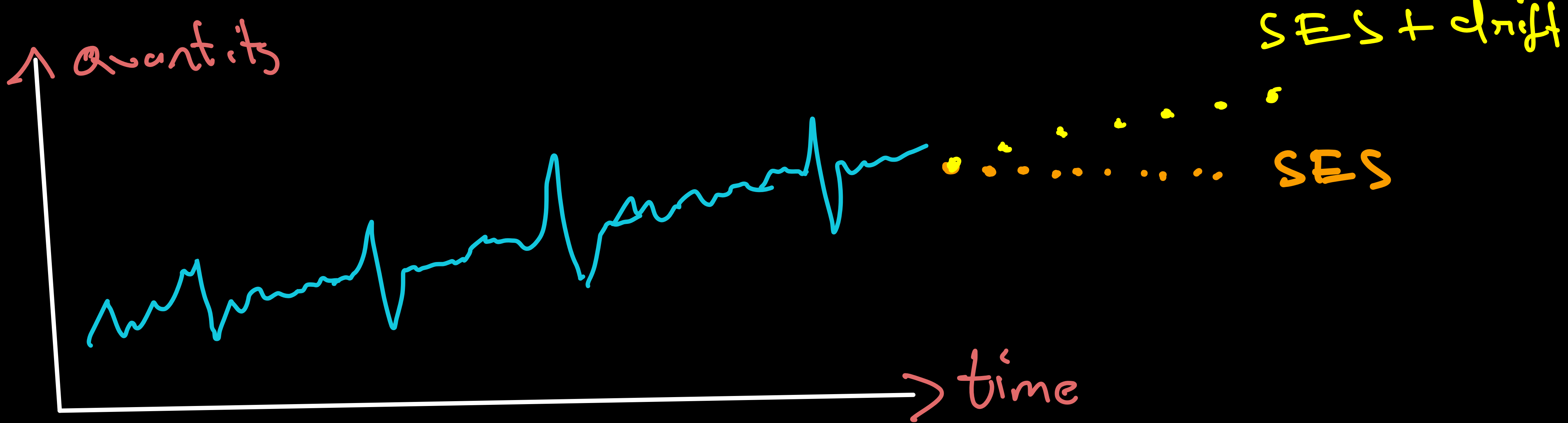
# Double exponential smoothing DES

↳ The above method only captures level.

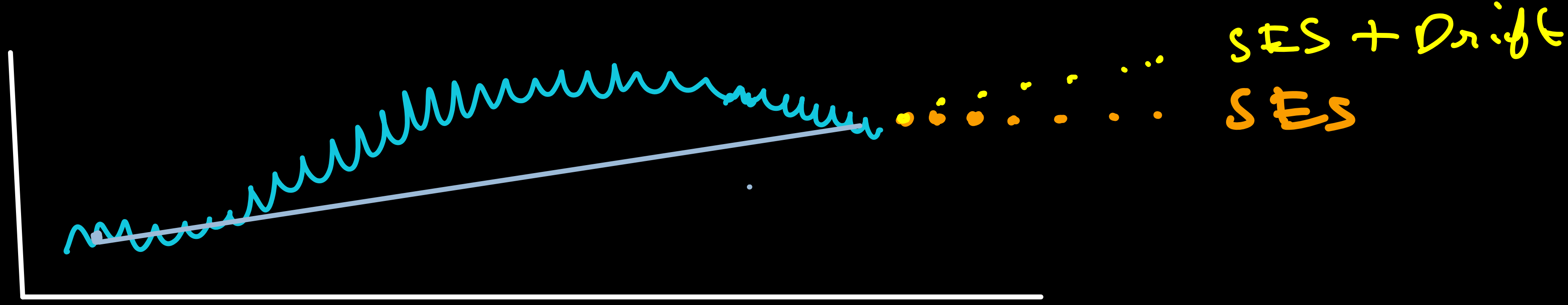
we need trend.

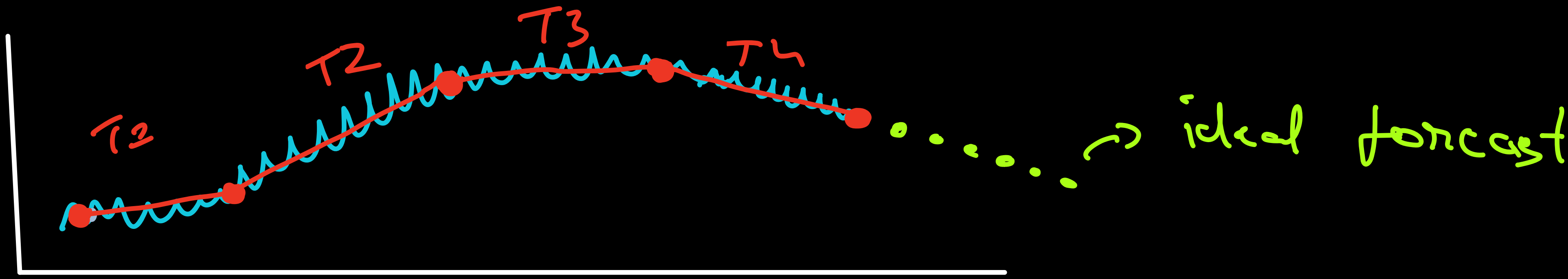
Assume: SES + drift method → DES

naive + mean



But, is the trend always constant?

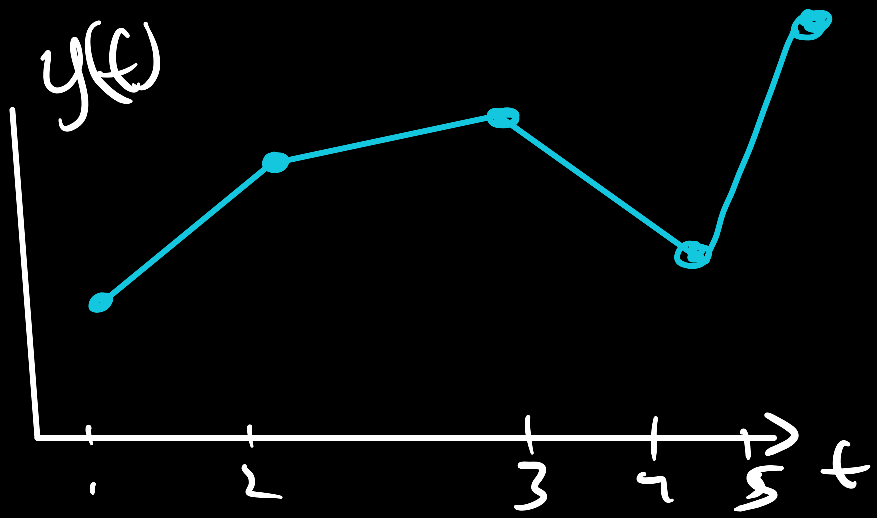


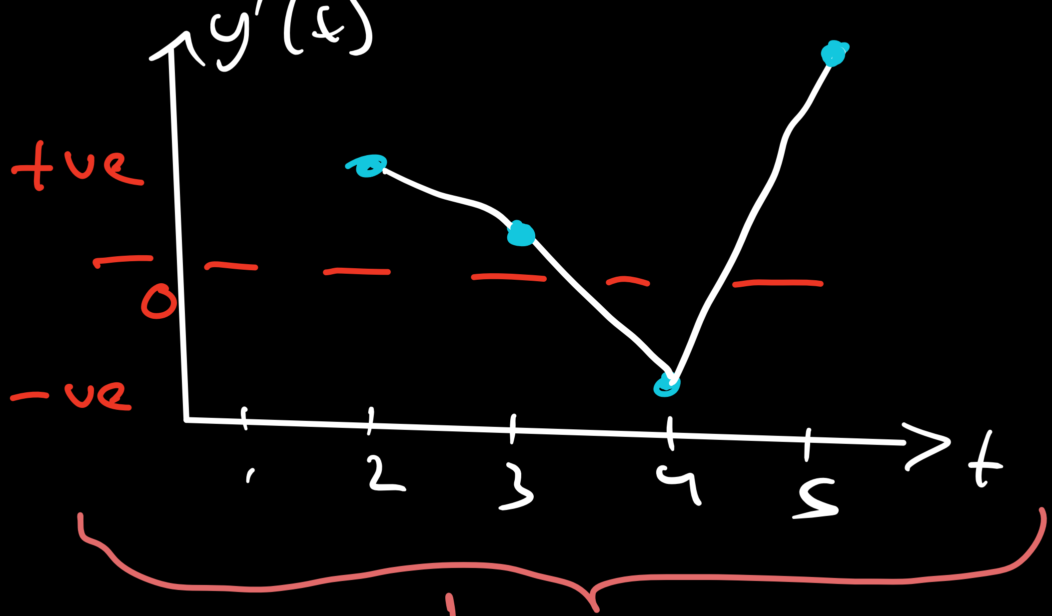

Key idea: Just as we too decaying weighted avg for "level", we should take exponential smoothing on trend too

# Trend Revisit

Can I say, trend = slope = rate of change?




→ Zoom in, slope is diff for every point.




$$y'(t) = \text{slope}(t)$$

$$= \frac{\Delta y}{\Delta t} = \frac{y_t - y_{t-1}}{1}$$

$$= \text{y.diff()}$$

Trend Signal!

Exponentially smooth this signal!

$$\hat{b}_{t+1} = \beta\, b_t + (1-\beta)\cdot \hat{b}_t$$

predicted rate of change

new hyper param

current rate of change

previous predicted

But we have to do simultaneous recursive calculation for level and trend!

DES = Level + Trend.

current level

# steps in future

current trend

$$\hat{y}_{t+h} = l_t + h b_t$$

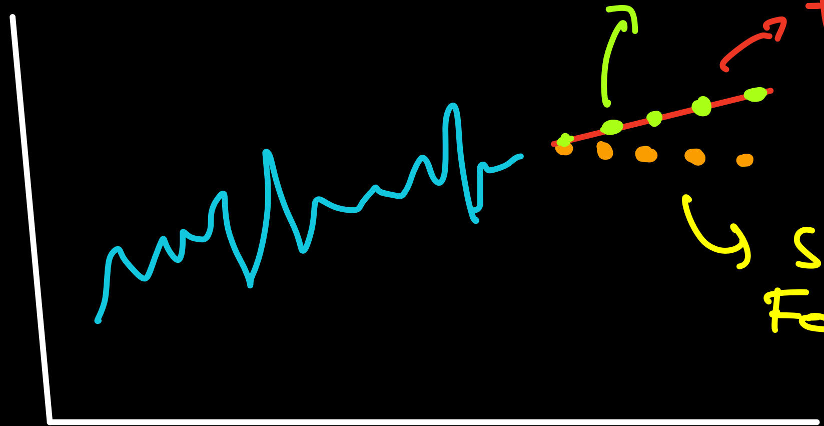
Add increase due to trend

$$l_t = \alpha y_t + (1-\alpha)[\underbrace{l_{t-1} + b_{t-1}}]$$

$$b_t = \beta[\underbrace{l_t - l_{t-1}}_{\text{current actual slope}}] + (1-\beta)\underbrace{b_{t-1}}_{\text{previous smoothed slope}}$$

DES Forecast

trend line

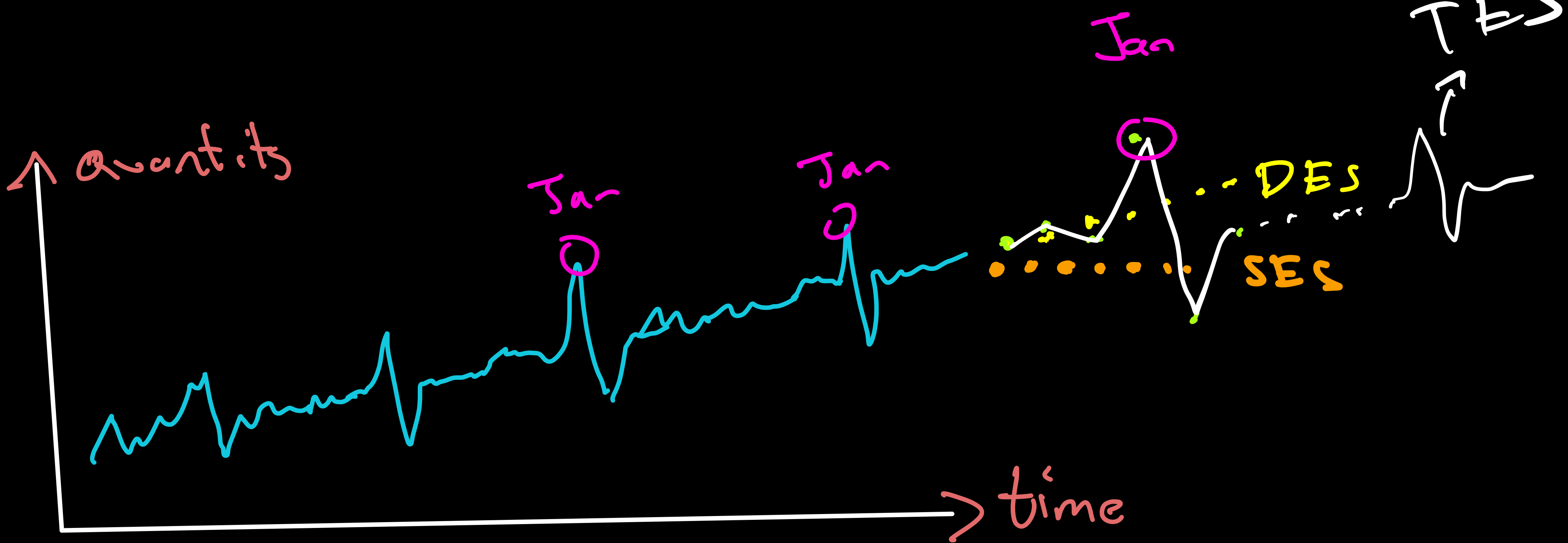


SES Forecast

# Tripple Exponential Smoothing (TES)

Holt-Winter's method

Q: What's left? → Seasonality




$$Jan(2023) = \gamma\, Jan\,22 + \gamma(1-\gamma)\, Jan\,21 + \gamma(1-\gamma)^2\, Jan\,20.$$

→ decaying weighted avg of seasonal value

Simultaneous recursive formulation:

1) $\hat{y}_{t+h} = l_t + h b_t + S_{t+h-m}$ $\longrightarrow$ m = seasonality
Eg: monthly data, m = 12

(assume, $h < m$)

$S_{t+h-m}$ :

$t$ = Jan 23
$t+h$ = Apr 23
$t+h-m$ = Apr 22

| $t = t$
| $h = 3$
| $m = 12$

if you want to predict Apr 23, what is the decaying weighted avg until Apr 22?

$$l_t = \alpha\left[y_t - \underbrace{S_{t-m}}_{\text{Subtract effect of seasonal variation}}\right] + (1-\alpha)\left[l_{t-1} + \underbrace{b_{t-1}}_{\text{Add effect of trend}}\right]$$

$$b_t = \beta\left[l_t - l_{t-1}\right] + (1-\beta)\, b_{t-1}$$

$$S_t = \gamma\left[y_t - \underbrace{l_{t-1} - b_{t-1}}_{\text{Subtract level and trend from } y\text{; you get current season value}}\right] + (1-\gamma)\, \underbrace{S_{t-m}}_{\text{past season value}}$$

Multiplicative formulations [Not imp]

$$
\begin{aligned}
\hat{y}_{t+h|t} &= (\ell_t + hb_t)s_{t+h-m(k+1)} \\
\ell_t &= \alpha \frac{y_t}{s_{t-m}} + (1-\alpha)(\ell_{t-1} + b_{t-1}) \\
b_t &= \beta^*(\ell_t - \ell_{t-1}) + (1-\beta^*)b_{t-1} \\
s_t &= \gamma \frac{y_t}{(\ell_{t-1} + b_{t-1})} + (1-\gamma)s_{t-m}
\end{aligned}
$$