Agenda:
1. Quadratic Solution
2. Hard Margin Classifier
3. Soft Margin Classifier
4. Hinge Loss
5. Comparison of loss fns.
6. Primal vs Dual

Background

- Lagrange Mult. for $\min_{\mathbf{w}} f(\mathbf{w})$ s.t. $g(\mathbf{w}) = 0$
- Dual function: $\min_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \lambda)$
- Dual problem: $\max_{\lambda} \min_{\mathbf{w}} \mathcal{L}(\mathbf{w}, \lambda)$
- Lagrange's trick

Geometric Solution

Goal: Find that π that maximizes the margin.

Hard Margin Classifier

Assumptions: Linear Separable

(1) Geometry Representation

$\pi^+ \parallel \pi \parallel \pi^-$

Goal: "Max margin"

S.t. → all point are correctly classified

$\pi^+: w^T x + b = +1$

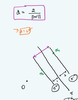
$\pi: w^T x + b = 0$

$\pi^-: w^T x + b = -1$

$\boxed{\text{Data}}$

$\boxed{\frac{\mathbf{x} - \mathbf{x}_0}{\|\mathbf{w}\|}}$

$\rightarrow \frac{\mathbf{w}^T \mathbf{x}}{\|\mathbf{w}\|}$



$\pi^+ \Rightarrow w^T x + b = +1$

$\pi^- \Rightarrow w^T x + b = -1$

$\pi^+ \Rightarrow w^T x + b = 0$

$\pi^- \Rightarrow w^T x + b = 0$

$\pi^+ \Rightarrow w^T x + b = 0$

$\pi^- \Rightarrow w^T x + b = 0$

$\pi^+: w^T x + b = +1$

$\pi: w^T x + b = 0$

$\pi^-: w^T x + b = -1$

$\pi^+: w^T x + b = +1$

$\pi^-: w^T x + b = -1$

$d = \frac{|b_1 - b_2|}{\|w\|}$

$\partial(\pi^+, \pi^-) = d(x, \pi^+) - d(x, \pi^-)$

$= \frac{|b+1|}{\|w\|} - \frac{|b-1|}{\|w\|}$

$\Rightarrow \frac{b+1-b+1}{\|w\|}$

$\Rightarrow \boxed{\frac{2}{\|w\|}}$

(1) $\pi^+: w^T x + b = +1$

$\pi: w^T x + b = 0$

$\pi^-: w^T x + b = -1$

$\pi^+: w^T x + b = +1$

$\pi^-: w^T x + b = -1$

When $K = 2$

$\boxed{\frac{\|w_1 - w_2\|}{\|w\|}} = \boxed{\frac{\|w_1 - w_2\|}{\|w_1 - w_2\|}}$

$\Rightarrow 1$

$\Rightarrow \frac{2\|w\|}{\|w\|} = \frac{2}{\|w\|}$

When $K > 2$

$\boxed{\max_{i,j} \frac{\|w_i - w_j\|}{\|w\|}}$

$\Rightarrow \frac{2}{\|w\|}$

Logistic Regression

$\pi^+ \parallel \pi \parallel \pi^-$

$\pi^+ \parallel \pi \parallel \pi^-$

$\pi^+ \parallel \pi \parallel \pi^-$

Non-linear Classifier

Sigmoid $\frac{1}{1+e^{-z}}$

S.t. $\boxed{(w^T x + b) \geq 1}$

$\xi_i \geq 0$

$D = \{(x, y)\}$

$\{(w^T x + b) \geq 1\}$

$\{w^T x + b \geq 1\}$

$\{(w^T x + b) \geq 1\}$

$\{(w^T x + b) \geq 1\}$

$\{(w^T x + b)