Assignment-3 → Arijit Maity
ONAP

(7) We consider the radix-2 FFT algorithm, then the algorithm actually divides the whole series of DFT into halves containing even and odd terms and we work our way back.

This algorithm of radix 2 works only for step size $= 2^m$ ($m \in N$) and we work out the complexity for them only say $\tilde{\omega}_q$ represent the DFT for the frequency point q.

$$\tilde{\omega}_q = \frac{1}{\sqrt{n}} \sum_{p=0}^{2^m - 1} \exp\left\{-i \frac{2\pi q p}{n}\right\} \omega_p$$

where $\{\omega_p\}$ are the data points.

we can divide the series as

$$\frac{1}{\sqrt{n}} \left[\sum_{p=0}^{n/2-1} \omega_{2p} \exp\left\{-\frac{i \cdot 2\pi q p}{n/2}\right\} + \sum_{p=0}^{n/2-1} \omega_{2p+1} \exp\left\{-i \cdot \frac{2\pi q}{n}\right\} \exp\left\{-i \frac{2\pi q p}{n/2}\right\} \right]$$

we can write this as —

$$\frac{1}{\sqrt{2}} \left\{ \text{DFT}\left(\tfrac{n}{2} \text{ points (even points)}\right) + \phi^q \text{DFT}\left(\tfrac{n}{2} \text{ points (odd points)}\right) \right.$$

where $\phi = \exp\left(-\frac{i2\pi}{n}\right)$

Again we can divide the $n/2$ points into $n/4$ points each so, the chain goes as —

$$\text{DFT}_n = \text{DFT}^{e}_{n/2} + \phi^q \text{DFT}^{o}_{n/2}$$

$$= \text{DFT}^{ee}_{n/2} + \text{DFT}^{eo}_{n/4}(\phi)^{(1)} + \left(\text{DFT}^{oe}_{n/4} + \text{DFT}^{oo}_{n/4}\right)(\phi)^{(1)}$$

i.e we keep multiplying phases and computing DFT's.

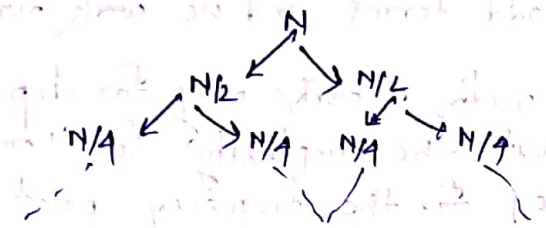
How long this split go on?

until we reach only single number

But DFT of single number is that number itself. what truely remains is multiplying the pase factor and adding them up.

What we need to do is do this recursively multiply phases and added up.

So,



At each step of the triangle what matters are the number of operations we are doing at each step. So at each step we have $O(N)$ operational complexity. and there are $m = \log_2 N$ steps to compute them at. So, the net complexity of the algorithm is given by $O(N \log_2 N)$ (By product of complexities.)