# CLL231 MINI PROJECT

## HOW DOES A GAS BUBBLE MOVE IN A VISCOELASTIC FLUID

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## PEER EVALUATION FORM:

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## ABSTRACT:

In this report we discussed the rise of gas bubble in the viscoelastic fluid. We have discussed the acoustic vibration of the gas bubble in the viscoelastic fluid using UCM model. Along with this another aspect of gas bubble is that how the radius is changing with the rise. We have shown the governing mathematical equations of these both situations. Without getting into analytical solution, we have used the given numerical computed solutions in different papers and draw conclusions from there graphs and qualitatively discuss these effects in detail. Another remarkable phenomenon that occurs when gas bubble rises in the viscoelastic fluid that is the velocity of the bubble suddenly rises after a certain radius value. We have discussed this behaviour and what modern researchers' explanation regarding this phenomenon.


## INTRODUCTION:

RELEVENCE OF THE SUBJECT:

One of the most important subjects in non-Newtonian fluid dynamics is the motion of gas bubbles in viscoelastic materials like polymer solutions. The jump discontinuity in the rise velocity of gas bubbles in viscoelastic liquids is a long-standing puzzle among specialists and something that is particularly relevant in many industrial operations.

This topic is important from both a technological and a research standpoint. From a technical standpoint, it's interesting since basic ideas are used to build and run gas-liquid contact mass transfer equipment. Medical ultrasonography applications rely on the forced oscillations of bubbles in biological fluid. Two urgent challenges are determining the dynamics of ultrasonic contrast ants in biological fluids and evaluating cavitation bioeffects from diagnostic and therapeutic ultrasound. For some biological applications, such as ultrasound-enhanced medicine administration, mediating cavitation-induced transport across viscoelastic tissue layers of the skin may be helpful. Experiments and theoretical studies are currently being carried out in order to establish safety standards for the prevention of potential cavitation bioeffects. The impact of tissue rheology on bubble dynamics is an important factor that has received a lot of attention recently. The current accepted exposure criteria and understanding of bubble dynamics in the body came from a study of bubble dynamics in Newtonian fluids. Cavitation collapse and rebound behaviour may be influenced by the viscoelastic characteristics of biological fluids and tissue. The use of an adequate constitutive equation is recommended since enormous bubble expansions have been linked to cavitation damage in general.

PREVIOUS RESEARCH:

Numerous investigations on the growth and collapse of spherical gas bubbles followed the creation of the Rayleigh-Plesset equation. For sufficiently high-pressure amplitudes of driving, some of this research has focused on the oscillatory behaviour of a single bubble in an acoustic field1, where the bubble oscillates nonlinearly about its equilibrium radius. The majority of these forced oscillations studies focused on the behaviour of bubbles in Newtonian fluids.

Some researchers employed empirical formulas that characterised viscosity as a function of shear stress to account for the non-Newtonian behaviour. Foggler and Goddard published ground-breaking research in this topic in 1970, which became the foundation for many subsequent investigations on viscoelastic bubble dynamics. They obtained an integro-differential equation describing the radial bubble dynamics using the Maxwell model for the constitutive relation for the fluid. The bubble was treated as an empty void, and the equation was examined at various asymptotic limits in order to draw some broad conclusions about the role of elasticity on bubble collapse. However, due to numerical problems in solving the integro-differential equation, the results were limited.

After that, the Fogler and Goddard integrodifferential formulation was merged with a three-constant Oldroyd constitutive equation, which allows for fluid strain relaxation and is better suited to bigger deformations. Ting was only able to partially fix a problem. A solitary collapse happened without a rebound due to numerical difficulties. Tanaswa and Yang were able to complete their integration for a few oscillations, but they ran into numerical issues. They found some unexpected results, but their groundbreaking research raised a lot of questions. The lack of radius-versus-time R-T! graphs in their paper, for example, makes the differences between viscoelastic and Newtonian bubble dynamics difficult to grasp.

Other publications have attempted to handle the more difficult problem of solving the fluid's continuity, momentum, and constitutive equations all at the same time. Zana and Leal numerically solved the mass and momentum conservation equations, as well as a gas diffusion equation, for a single bubble collapse.

Kim has solved the continuity and momentum equations in a Lagrangian frame to investigate the free oscillations of a bubble in an Upper-Convective Maxwell (UCM)fluid. Kim employed a sophisticated finite element approach to solve these equations in spherical coordinates and compared his results to those of Fogler and Goddard. As a result of this, he arrived to certain conclusions about the differences between the linear Maxwell and the UCM models.

WHAT ARE THE SCOPES LEFT?

In viscoelastic liquids, there are still unanswered questions about bubble dynamics. Following studies applied Foglller and Goddard's formulation based on a traceless stress tensor consistent with a linear viscoelastic constitutive equation for objective constitutive relations without any extensive explanations. Recent research, such as Kim's, has revealed that a traceless formulation is not always acceptable; nevertheless, there has been little discussion as to why.

WHAT HAVE YOU OUTLINED?

In this report we have tried to outline 3 basic topics related to bubble dynamics in viscoelastic liquids. These are:

1. Acoustic Oscillation of Bubble in Viscoelastic Fluid.

2. Determining the equations related to change in size of gas bubble during motion

3. Why Does Speed Of Gas Bubble Suddenly Rise In Viscoelastic Fluid?

## DISCUSSION:

### Acoustic Oscillation of Bubble in Viscoelastic Fluid:

The concept of non-linear oscillation is basically very simple. A system will oscillate when it is moved from its equilibrium position. For the simple case of a spring mass system, we all know that the movement is such a way that the acceleration of the mass is exactly proportional to the displacement from the mean position. We call it linear. Now, if there is some source like someone is vibrating the spring from outside periodically, then we can get a non-linear differential equation and a non-linear system. A single disturbance impulse or continuous driving can cause a bubble to oscillate in a certain mode. The bubble will oscillate due to a sound wave, which is a periodic mechanical disturbance of the surrounding medium.

Two scientists, Fogler and Goddard's worked out the dynamics of this oscillation. Here we are going to show the basic governing equations and the conclusions we can derive from them.

First of all, due to the spherical symmetry we can have the following equality[1],

$$\tau_{\theta\theta} = \tau_{\phi\phi},$$

Now, we have to borrow another equation from work of Rayleigh-Plesset[4]. They provided the equation that governs the oscillation dynamics of bubble in the fluid.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[\rho_{go}\left(\frac{R_o}{R}\right)^{3\kappa} - (p_o + p_A \sin(\omega t)) - \frac{2\sigma}{R} - 2\int_R^\infty \left(\frac{\tau_{rr} - \tau_{\theta\theta}}{r}\right) dr\right].$$

This equation describes the motion of a spherical bubble of radius R(t) in incompressible liquid of density ρ, surface tension σ, an initial internal gas pressure $\rho_{go}$, and ambient pressure $\rho_o$. The acoustic pressure amplitude is $P_A$ and the frequency with which it is oscillating is ω.

Here we are using UCM model (under-convected Maxwell, which is a generalised version of Maxwell model). It can be written as[1],

$$\tau + \lambda_1 \tau_{(1)} = -\eta_o \dot{\gamma}.$$

Where,

$$\tau_{(1)} = \frac{D}{Dt}\tau - ((\nabla u)^{\perp} \cdot \tau + \tau \cdot (\nabla u))$$

It represents the first contravariant (upper) convective time derivative.

In the above equations, $\lambda_1$ refers to the relaxation time, ϒ is the deformation strain tensor, u is the velocity field and $\dot{\eta}_o$ is zero shear-rate viscosity. The challenge is to solve the above equation coupled

$$\mathrm{De} = \lambda_1 \omega.$$

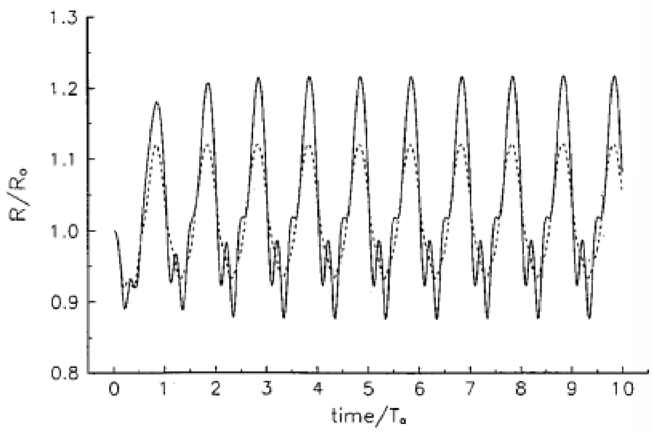
with Rayleigh equation which is to be done numerically. We can do nondimensionalizing and can get some relevant constants.

This parameter is known as Deborah number which describes the ration of the characteristic time scale of the fluid. Along with this we have another parameter called weber number that is defined as the ratio of surface tension to inertial forces,

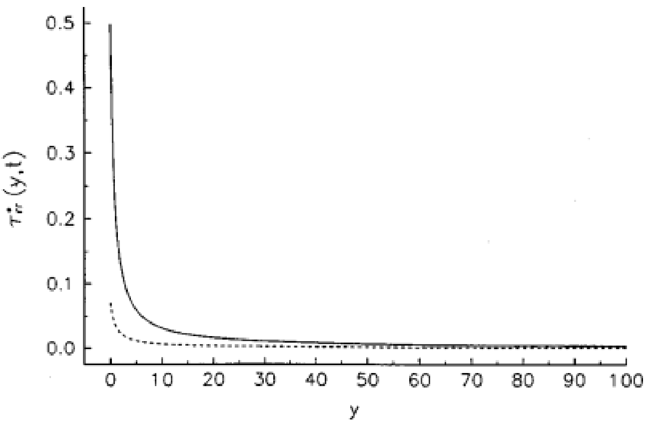
We are not discussing the numerical techniques for solving the above equations, rather we are going to

$$\mathrm{We} = \frac{2\sigma}{p_o R_o}.$$

discuss about the conclusions we can draw from various graphs[1] plotted via numerical computation.

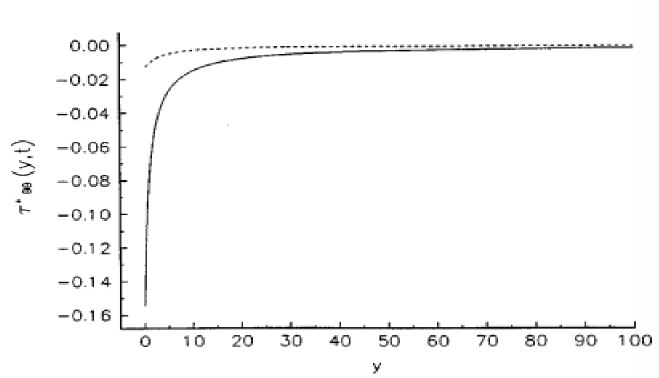



First, the above graph is between the radius and time. We are not going into details of experimental conditions. But immediately we can see that the radius is an oscillatory function and anther information to be mentioned is that the solid line corresponds to Reynolds number 0.6 and the dashed line to Reynolds number 0.45. We can see that the dashed line is below the solid line. Why? Reynolds number is nothing but the ratio of inertial force and viscosity. A low Reynolds number suggests a high viscous fluid and due to this the damping increases in this case.



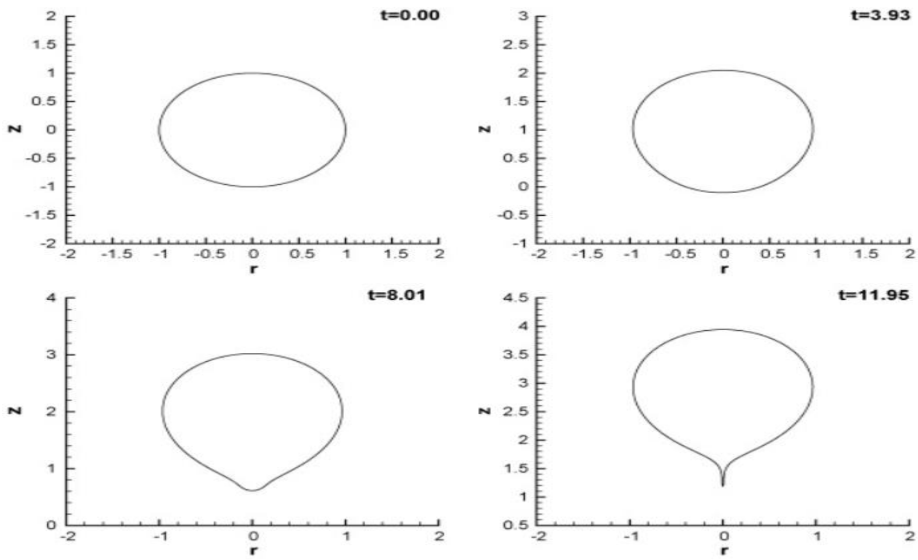

Second, this graph is between the radial stress of the bubble and the nondimensional y variable which corresponds to the distance from the bubble wall (actually y is given by the following equation which is a transformation such that $y = r^3 - R^3(t)$). We can see that the stresses fall off in a great rate and asymptoticly approach a value as going further out in the liquid. Here, we can see the solid line means Reynolds number = 0.6 has higher stress than the more viscous fluid case.

Similarly, we can plot the graph[1] between the shear stress on the wall of the bubble with the y variable.




There is also comparison between linear maxwell model and UCM model and what are the differences and issues with numerical computation in various research paper. But so far, we can understand that a gas bubble in the viscoelastic fluid oscillates due to the acoustic effects (sound wave or pressure variation) and it happens in non-linear manner. This bubble oscillation phenomena can also be described in Newtonian fluid model as the main source of this oscillation is some kind of pressure variation.

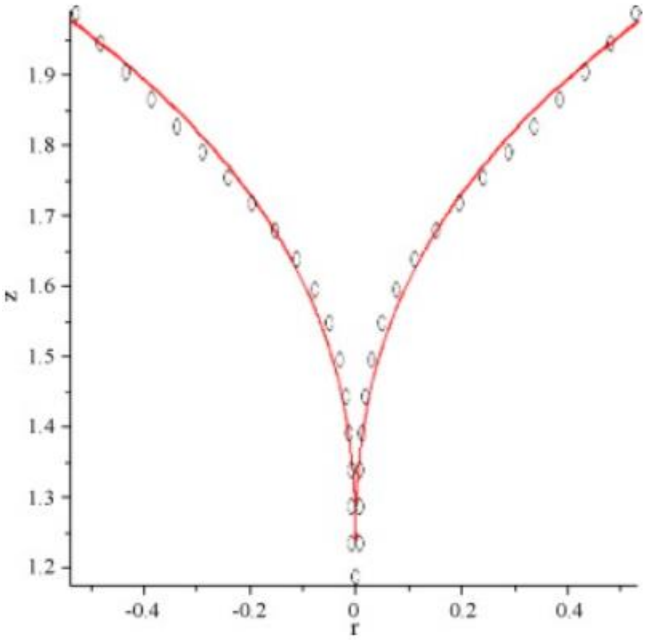
## Size of Gas Bubble in Viscoelastic Fluid:



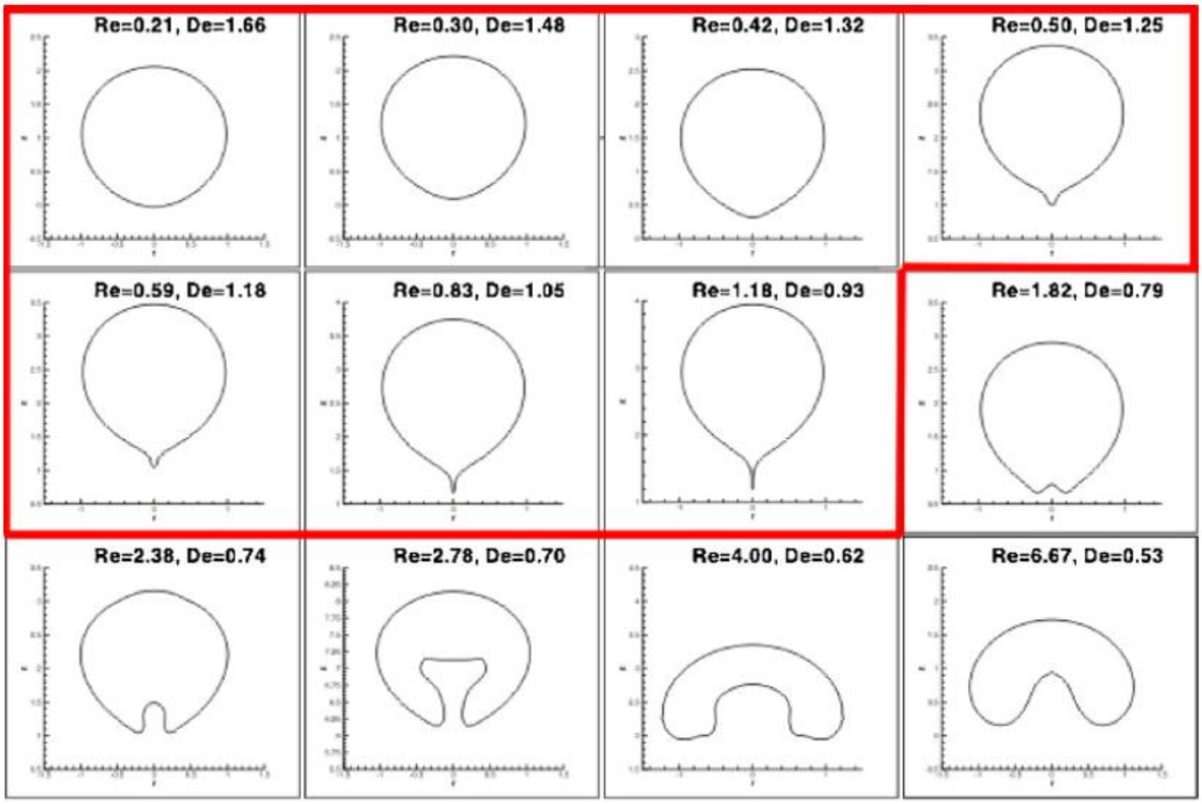

The bubble profile during ascension for a viscoelastic fluid with De=0.93 and Re=1.18 is shown in the diagram above. As the bubble rises, it takes on a more prolate shape before the underside draws out and forms a trailing edge cusp, as shown in the last frame.

Some numerical analyses have generated similar cusp-like forms, but none have reproduced such a sharp interface.

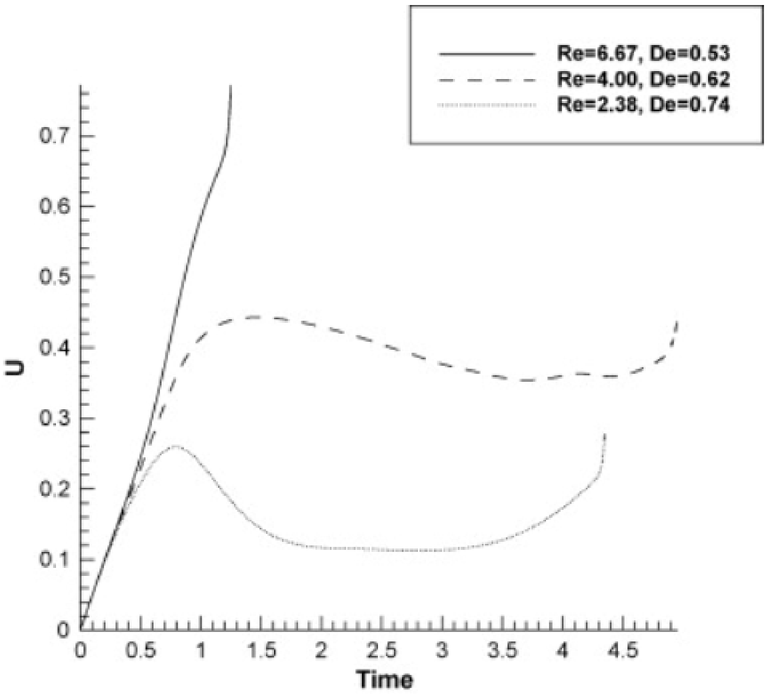
The results of the trials imply that rising bubbles create 2D-cusps with a universal asymptotic shape of z=a|r|2/3. Despite the fact that the cusps are axisymmetric rather than 2D, it's fascinating to see if they can match a comparable analytical equation. The cusp created for De=0.934, Re=1.180 is shown in detail in the figure below. The curve z=1.01|r|0.381 (shown in red) is determined to offer a fit with a residual mean square of 1.46103 using the least squares method. This shows that analytical formulas of the kind z=a|r|n, at least to leading order, could be used to define more general cusps.




Because of the velocity jump discontinuity that occurs once the bubble exceeds a certain critical volume, the variation in bubble volume is a major subject of experiment investigations. The fluid's material properties are stable, but the appropriate Reynolds and Deborah numbers are given throughout each frame due to the change in volume. The shape transition from Re=0.21 to Re=1.18 is qualitatively similar to the bubble shape shift witnessed experimentally when increasing volume. Because viscous effects dominate and hinder deformation at low Reynolds numbers, such as Re=0.21,0.30, terminal bubble morphologies depart little from sphericity. As the bubble gets more prolate with signs of a more pointed underside at Re=0.42, viscous effects fade significantly and elastic effects take over. Between Re=0.42 and Re=0.50, the shape changes dramatically from a completely convex interface to a concave part with a pseudo-cusp on the underside. The increased influence of elasticity leads this pseudo-cusp to sharpen and lengthen when the volume is increased further, culminating in the terminal cusped bubble shape observed for Re=1.18, De=0.93. Beyond this (those outside of the red box), the shapes do not appear to attain a stable state. The shapes depicted are those that exist before the computation encounters obstacles. Yet, once again, a general change in bubble shape may be seen. Instead of a cusp, a liquid jet appears to form on the bubble's bottom. Viscous and elastic effects are evidently muted by the growing influence of buoyancy and inertia due to the increase in volume. The size of the jets in subsequent bubbles grows greater, and the shape shifts from prolate to oblate.

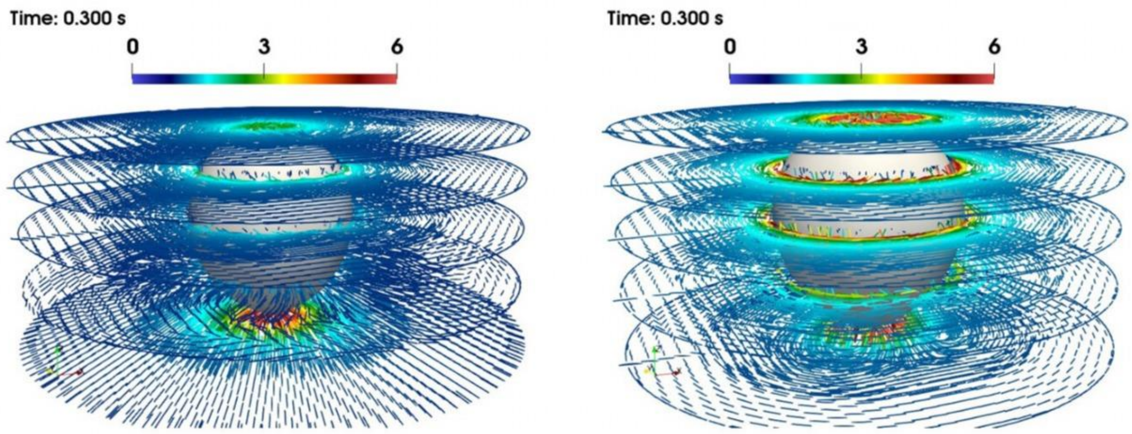



The most prominent aspect is the damped oscillation in velocity, which is common in viscoelastic phenomena. The buoyancy force's impulse initiates the velocity's overshoots and undershoots, which diminish in amplitude until U achieves a steady-state value of UT0.01. Pillapakkam et al. observed oscillations in the rise velocity in their research of a bubble rising in an Oldroyd B fluid. Their oscillations are often smaller, although this is due to the Oldroyd model's higher viscosity compared to the more elastic Maxwell model utilised here.



## Why Does Speed Of Gas Bubble Suddenly Rise In Viscoelastic Fluid?[3]

Why do huge gas bubbles rise so much faster than predicted in viscoelastic liquids (like polymer and protein solutions)? An unanswered question having significant implications for industrial manufacturing processes. Researchers from the Technical University of Graz and the Technical University of Darmstadt have now discovered an explanation.



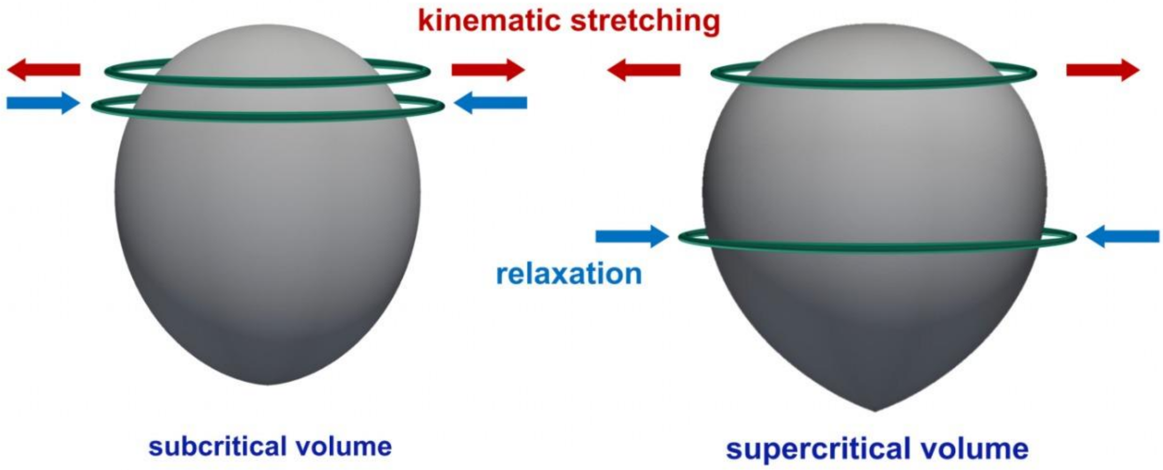

The alignment and deformation of polymer molecules in the viscoelastic fluid flow around the bubble are visualised using simulation results. Polymer molecules align themselves circumferentially to the contour of the bubble interface in the bubble flow around the bubble. In the upper region of the bubble, the molecules deform at the same moment. Below the bubble equator, the polymer molecules are already relaxed back to their relaxed state in the subcritical state (left). The relaxation essentially occurs below the bubble equator in the supercritical condition (right)[3].

Large gas bubbles in viscoelastic fluids (which mix the characteristics of liquid and elastic material, such as polymer and protein solutions) rise quicker than predicted, resulting in a jump discontinuity.

When we turn a clear, nearly totally filled shampoo upside down, the enclosed air rises as a bubble with a unique shape. For more than 60 years, scientists have known that the rise velocity of gas bubbles in viscoelastic liquids jumps at a key bubble diameter. The bubble's speed can then suddenly increase by up to ten times.

The odd velocity behaviour of the gas bubbles is due to the interaction of the polymer molecules with the flow surrounding them. With this knowledge, the oxygen input into these solutions can now be predicted more precisely, allowing for better equipment design in biotechnology, process engineering, and the pharmaceutical industry, for example.

The dissolved polymer molecule stretches due to the flow around the bubble. The molecules are highly resistant to this state, preferring to return to their original state as quickly as possible. If this return to the relaxed state is faster than the molecules' travel to the bubble's equator, the bubble will remain sluggish. If, on the other hand, returning to a relaxed state takes longer than the voyage to the equator of the bubble, tension in the fluid that pushes the bubble is released. As a result, succeeding polymer molecules position themselves below the equator and relax, emptying their elastic energy and producing a propulsive force, causing self-amplification.




In the above image two rising bubbles in a viscoelastic fluid, on the left in the subcritical state and on the right in the supercritical state, are depicted schematically [3].

Another striking aspect of the flow field of these solutions, the so-called 'negative wake'[3] of the gas bubble, was shown to be due to the molecular mechanism we demonstrated. The fluid generally "follows" the bubble at a modest velocity in this portion of the flow field below the bubble. However, with polymeric liquids, the flow of the liquid is in the opposite direction of the movement of the bubble. The same stress that "pushes" the bubble causes this fluid flow. This knowledge can lead to new ways of controlling flow processes.

## CONCLUSIONS:

The major conclusion we can derive from the discussion part are the following:

1. In the viscoelastic fluid due to any acoustic disturbance or pressure change gas bubble can vibrate and this vibration frequency certainly depends of the viscosity.

2. With the development of mathematics we have seen that how shear stress and radial stress on the wall of the bubble varies with viscosity and up to what range from the wall of the bubble the effect exists.

3. When gas bubble rises in the viscoelastic fluid, after reaching a certain radius value called critical radius, it suddenly increases its velocity.

4. The interaction of the polymer molecules which is the main nature of viscoelastic fluid with the flow around the bubbles is the main reason of this strange behaviour.

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