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1) Given point in projective space is $p(4, 18, 2)$

so, the point corresponding to p in real 2D plane can be given as,

$$\begin{pmatrix} 4/2 \\ 18/2 \end{pmatrix} \equiv \begin{pmatrix} 2 \\ 9 \end{pmatrix}$$

2) The possible equivalent representations in $\mathbb{P}^2$ of the point $(3, 8)$ in $\mathbb{R}^2$ can be written as,

$$\begin{pmatrix} 3k \\ 8k \\ k \end{pmatrix} \text{ for any } k \in \mathbb{R} \setminus \{0\}.$$

3) The lines described by $(5, 2, 1)$ & $(5, 2, 9)$ in $\mathbb{P}^2$ are the lines in $\mathbb{R}^2$ whose eq^n s are,

$$5x + 2y + 1 = 0$$

and $5x + 2y + 9 = 0$ respectively.

So, since the coefficient of x & y co-ordinate are equal for both the eq^n s, we can conclude that the lines are parallel. So, they doesn't intersect or they are intersecting at infinity.

Also, from the result that, intersection of two lines can be given as, $x = L \times L'$

then, here,

$$L \times L' = (5, 2, 1) \times (5, 2, 9)$$

$$= \begin{vmatrix} x & y & z \\ 5 & 2 & 1 \\ 5 & 2 & 9 \end{vmatrix}$$

$$= 16x - 40y + 0 \cdot z$$

i.e., the point in $\mathbb{P}^2$ is $\begin{pmatrix} 16 \\ -40 \\ 0 \end{pmatrix}$

so, the point in 2D plane is $\begin{pmatrix} 16/0 \\ -40/0 \end{pmatrix} \sim$ infinity

Hence, the lines are parallel.

5) Let us consider the homography matrix H between two images I_1 & I_2 of the same scene.

$$H = \begin{pmatrix} 1 & 3 & 5 \\ 4 & 3 & 7 \\ 5 & -4 & 2 \end{pmatrix}$$

The given point in I_1 is $p(5, 6)$

So, the ~~pack~~ equivalent representation of $p(5,6)$ in $\mathbb{P}^2$ can be given as,

$$\begin{pmatrix} 5k \\ 6k \\ k \end{pmatrix} \quad \text{for, any } k \in \mathbb{R} \setminus \{0\}$$

Now, the transformation can be given as,

$$\begin{pmatrix} 1 & 3 & 5 \\ 4 & 3 & 7 \\ 5 & -4 & 2 \end{pmatrix} \begin{pmatrix} 5k \\ 6k \\ k \end{pmatrix}$$

$$= k \begin{pmatrix} 5 + 18 + 5 \\ 20 + 18 + 7 \\ 25 - 24 + 2 \end{pmatrix}$$

$$= k \begin{pmatrix} 28 \\ 45 \\ 3 \end{pmatrix}$$

So, the corresponding point in I_2 will be,

$$\begin{pmatrix} 28/3 \\ 45/3 \end{pmatrix} \approx \begin{pmatrix} 9.33 \\ 15 \end{pmatrix}$$

Q4) Let us consider the projective transformation.

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

And, let C be the conic given by $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$

So, the transformation of C under H will be,

$$(H^{-1})^T C (H^{-1})$$

Now, $$H^{-1} = \frac{\begin{pmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}}{(-1-1)} = \begin{pmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \end{pmatrix}$$

So, $(H^{-1})^T C (H^{-1})$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

6) The line at infinity containing ideal points is given by

$L_\infty = (0, 0, 1)$

Let us consider the transformation as $H = \begin{pmatrix} 2 & 3 & -4 \\ -4 & 3 & 2 \\ 3 & 1 & 1 \end{pmatrix}$

Then, the vanishing line in the transformed space can be computed as,

$$l' = (H^{-1})^T L_\infty$$

Now,

$$H^{-1} = \begin{pmatrix} \frac{1}{84} & -\frac{1}{12} & \frac{3}{14} \\ \frac{5}{42} & \frac{1}{6} & \frac{1}{7} \\ -\frac{13}{84} & \frac{1}{12} & \frac{3}{14} \end{pmatrix}$$

So, $$l' = \begin{pmatrix} \frac{1}{84} & \frac{5}{42} & -\frac{13}{84} \\ -\frac{1}{12} & \frac{1}{6} & \frac{1}{12} \\ \frac{3}{14} & \frac{1}{7} & \frac{3}{14} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \left(-\frac{13}{84}, \frac{1}{12}, \frac{3}{14}\right)$$

(Ans.)