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CS181-S16

Assignment #3
Due: 5:00pm March 25, 2016

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Homework 3: SVM

There is a mathematical component and a programming component to this homework. Please submit ONLY your PDF to Canvas, and push all of your work to your Github repository. If a question requires you to make any plots, like Problem 3, please include those in the writeup.

Problem 1 (Fitting an SVM by hand, 8pts)
Consider a dataset with the following 6 points in $1D$:

$$\{(x_1, y_1)\} = \{(-3, +1), (-2, +1), (-1, -1), (1, -1), (2, +1), (3, +1)\}$$

Consider mapping these points to 2 dimensions using the feature vector $\phi : x \mapsto (x, x^2)$. The max-margin classifier objective is given by:

$$\min_{\mathbf{w}, w_0} \|w\|_2^2 \quad \text{s.t.} \quad y_i(w^T \phi(x_i) + w_0) \geq 1, \; \forall i \tag{1}$$

Note: the purpose of this exercise is to solve the SVM without the help of a computer, relying instead on principled rules and properties of these classifiers. The exercise has been broken down into a series of questions, each providing a part of the solution. Make sure to follow the logical structure of the exercise when composing your answer and to justify each step.

1. Write down a vector that is parallel to the optimal vector w. Justify your answer.
2. What is the value of the margin achieved by w? Justify your answer.
3. Solve for w using your answers to the two previous questions.
4. Solve for w_0. Justify your answer.
5. Write down the discriminant as an explicit function of x.

Solution

Problem 2 (Composing Kernel Functions, 7pts)
Prove that

$$K(\boldsymbol{x}, \boldsymbol{x}') = \exp\{-||\boldsymbol{x} - \boldsymbol{x}'||_2^2\}\,,$$

where $\boldsymbol{x}, \boldsymbol{x}' \in \mathbb{R}^D$ is a valid kernel, using only the following properties. If $K_1(\cdot,\cdot)$ and $K_2(\cdot,\cdot)$ are valid kernels, then the following are also valid kernels:

$$\begin{aligned}
&K(\boldsymbol{x}, \boldsymbol{x}') = c\,K_1(\boldsymbol{x}, \boldsymbol{x}') \quad \text{for } c > 0 \\
&K(\boldsymbol{x}, \boldsymbol{x}') = K_1(\boldsymbol{x}, \boldsymbol{x}') + K_2(\boldsymbol{x}, \boldsymbol{x}') \\
&K(\boldsymbol{x}, \boldsymbol{x}') = K_1(\boldsymbol{x}, \boldsymbol{x}')\,K_2(\boldsymbol{x}, \boldsymbol{x}') \\
&K(\boldsymbol{x}, \boldsymbol{x}') = \exp\{K_1(\boldsymbol{x}, \boldsymbol{x}')\} \\
&K(\boldsymbol{x}, \boldsymbol{x}') = f(\boldsymbol{x})\,K_1(\boldsymbol{x}, \boldsymbol{x}')\,f(\boldsymbol{x}') \quad \text{where } f \text{ is any function from } \mathbb{R}^D \text{ to } \mathbb{R}
\end{aligned}$$

Solution

Problem 3 (Scaling up your SVM solver, 10pts)
We will release Problem 3 shortly!

Solution

Calibration [1pt]

Approximately how long did this homework take you to complete?