

## Homework 3: SVM

There is a mathematical component and a programming component to this homework. Please submit ONLY your PDF to Canvas, and push all of your work to your Github repository. If a question requires you to make any plots, like Problem 3, please include those in the writeup.

### Problem 1 (Fitting an SVM by hand, 8pts)

Consider a dataset with the following 6 points in 1D:

$$\{(x_1, y_1)\} = \{(-3, +1), (-2, +1), (-1, -1), (1, -1), (2, +1), (3, +1)\}$$

Consider mapping these points to 2 dimensions using the feature vector  $\phi : x \mapsto (x, x^2)$ . The max-margin classifier objective is given by:

$$\min_{\mathbf{w}, w_0} \|\mathbf{w}\|_2^2 \quad \text{s.t.} \quad y_i(\mathbf{w}^T \phi(x_i) + w_0) \geq 1, \quad \forall i \quad (1)$$

Note: the purpose of this exercise is to solve the SVM without the help of a computer, relying instead on principled rules and properties of these classifiers. The exercise has been broken down into a series of questions, each providing a part of the solution. Make sure to follow the logical structure of the exercise when composing your answer and to justify each step.

1. Write down a vector that is parallel to the optimal vector  $\mathbf{w}$ . Justify your answer.
2. What is the value of the margin achieved by  $\mathbf{w}$ ? Justify your answer.
3. Solve for  $\mathbf{w}$  using your answers to the two previous questions.
4. Solve for  $w_0$ . Justify your answer.
5. Write down the discriminant as an explicit function of  $x$ .

## Solution

**Problem 2** (Composing Kernel Functions, 7pts)

Prove that

$$K(\mathbf{x}, \mathbf{x}') = \exp\{-\|\mathbf{x} - \mathbf{x}'\|_2^2\},$$

where  $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^D$  is a valid kernel, using only the following properties. If  $K_1(\cdot, \cdot)$  and  $K_2(\cdot, \cdot)$  are valid kernels, then the following are also valid kernels:

$$K(\mathbf{x}, \mathbf{x}') = c K_1(\mathbf{x}, \mathbf{x}') \quad \text{for } c > 0$$

$$K(\mathbf{x}, \mathbf{x}') = K_1(\mathbf{x}, \mathbf{x}') + K_2(\mathbf{x}, \mathbf{x}')$$

$$K(\mathbf{x}, \mathbf{x}') = K_1(\mathbf{x}, \mathbf{x}') K_2(\mathbf{x}, \mathbf{x}')$$

$$K(\mathbf{x}, \mathbf{x}') = \exp\{K_1(\mathbf{x}, \mathbf{x}')\}$$

$$K(\mathbf{x}, \mathbf{x}') = f(\mathbf{x}) K_1(\mathbf{x}, \mathbf{x}') f(\mathbf{x}') \quad \text{where } f \text{ is any function from } \mathbb{R}^D \text{ to } \mathbb{R}$$

**Solution**

**Problem 3** (Scaling up your SVM solver, 10pts)  
We will release Problem 3 shortly!

**Solution**

**Calibration [1pt]**

Approximately how long did this homework take you to complete?