Exercise 4. Tikhonov-regularization and parameter selection

Consider again the integral operator $A : L^2(0,1) \to L^2(0,1)$,

$$Af(x) = \int_0^x f(t)\,\mathrm{d}t\,.$$

and its discretization (see exercise 6.4). Moreover, let the functions

$$f_1(x) = \operatorname{sign}\left(x - \frac{1}{2}\right) \qquad \text{and } f_2(x) = \sin(\pi x)$$

be given.

a) Create for the function f_2 and $N = 300$ noisy data g^δ such that the difference to the real data $g = Af_2$ is 5 %, i.e. it should hold that

$$\frac{\|g - g^\delta\|}{\|g\|} = 0.05\,.$$

b) Implement the Tikhonov-regularization. Determine the regularization parameter α visually.

c) Determine now a good regularization parameter with help of the discrepancy principle of Morozov under the assumption that the noise level is bounded by $\delta = 0.4$. Plot both solutions and compare them to the original function.

d) Repeat all steps for the function f_1.