$Amortized\ Analysis -$
$Accounting\ Method$

In the accounting method, an artificial cost is assigned to every operation. Like banks maintain accounts of the customers, one must keep track of the operations in this case. Some of the operations are charged extra, and this extra cost would later used to offset the cost of some expensive operations. Let c_i be the actual cost of the operation and a_i *be the amortized cost of the ith operation. Then the credit is defined as the difference between the actual and amortized cost*:

$$Credit = a_i - c_i$$

For a sequence of `m` operations, the costs are given as follows:

$$c_i = \sum_{i=1}^{m} c_i \ \ and\ a_i = \sum_{i=1}^{m} a_i$$

The relation between the actual costs and amortized costs:

$$\sum_{i=1}^{m} c_i \leq \sum_{i=1}^{m} a_i$$

The credit is now given as the difference between the actual and amortized costs:

$$\boldsymbol{Credit = c_i = \sum_{i=1}^{m} c_i - \sum_{i=1}^{m} a_i \geq 0}$$

Therefore, the actual cost is bounded by the total amortized cost $\boldsymbol{O(a)}$.

Let us take example of Dynamic table:

$\boldsymbol{Item\ No.\ (i)}$	$\boldsymbol{Table\ Size}$	$\boldsymbol{Total\ Cost_i(C_i)}$	$\boldsymbol{Amortized\ Cost\ (a_i)}$	Check $\sum_{i=1}^{m} c_i \leq \sum_{i=1}^{m} a_i$	$\boldsymbol{Credit\ (a_i - c_i)}$	$\boldsymbol{Balance\ in\ Bank\ (previous\ balance\ +new\ balance\ /Credit)}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\boldsymbol{2[Extra\ 1\ for\ credit\ in\ bank]}$	$\mathbf{1 \leq 2}$	$\mathbf{2 - 1 = 1}$	$\mathbf{1}$(Initial balance/ initial credit)
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2(1+1)}$	$\mathbf{2+1=3}$ $\boldsymbol{[To\ keep\ \sum_{i=1}^{m} c_i \leq \sum_{i=1}^{m} a_i\ ]}$	$\mathbf{3 \leq 5}$	$\mathbf{3 - 2 = 1}$	$\mathbf{1+1=2}$
$\mathbf{3}$	$\mathbf{4}$	$\mathbf{3(2+1)}$	$\mathbf{3}$	$\mathbf{6 \leq 8}$	$\mathbf{3 - 3 = 0}$	$\mathbf{2+0=2}$
$\mathbf{4}$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{7 \leq 11}$	$\mathbf{3 - 1 = 2}$	$\mathbf{2+2=4}$
$\mathbf{5}$	$\mathbf{8}$	$\mathbf{5(4+1)}$	$\mathbf{3}$	$\mathbf{12 \leq 14}$	$\boldsymbol{3 - 5 = -2(hence\ 2\ will\ be\ taken\ from\ the\ bank\ balance)}$	$\mathbf{4+(-2)=2}$
$\mathbf{6}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{13 \leq 17}$	$\mathbf{3 - 1 = 2}$	$\mathbf{2+2=4}$

7	8	1	3	$14 \leq 20$	$3-1=2$	$4+2=6$
8	8	1	3	$15 \leq 23$	$3-1=2$	$6+2=8$
9	16	$9(8+1)$	3	$24 \leq 26$	$3-9 = -6(\textit{hence 6 will be taken from the bank balance})$	$8+(-6)=2$
10	16	1	3	$25 \leq 29$	$3-1=2$	$2+2=4$

Hence $\boldsymbol{we\ get\ `a`\ as\ 3\ in\ most\ cases\ hence\ if\ it\ run\ at\ n\ times\ we\ get\ O(3n) = O(n)}$.

$$\boldsymbol{And\ the\ actual\ cost\ \ bounded\ by\ total\ amortized\ cost}$$

$$\Rightarrow \boldsymbol{O(a) = O(3) = O(1)}$$

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