Water Jug Problem

Problem:

You are given two jugs a 4-gallon one and a 3 gallon one. Neither has any measuring markers on it. There is a pump that can be used to fill the jugs with water. How can you get exactly 2 gallons of water into the 4-gallon jug?

Ans:

Representation

1. **State of problem or state space for this problem can be described as the set of ordered pairs of integers(x,y).**

2. **x represents amount of water in the 4 gallons of jug.**

3. **y represents amount of water in 3 gallons of jug.**

4. **The start state is (0, 0) as initial state for x is 0 and y is 0.**

5. **Limitations:** $0 \leq x \leq 4 \text{ and } 0 \leq y \leq 3$.

6. **Goal State = (2,y) where** $0 \leq y \leq 3$.

Solution 1

Note: There is pump outside which is used to fill x and y respectively i.e. pump is an external source which have abundance of water.

1. First x and y in initial state , x = 0 and y = 0 i.e. x and y are empty.

2. Pour 3 gallons of water in y and make it full as $y < 3$. i.e. new state become : x=0,y=3 where $0 \le x \le 4$ *and* $0 \le y \le 3$.

3. Pour all the 3 gallons of water from y to x i.e. new state obtained x = 3 , y = 0 $as\ 0 < x + y \ge 3\ and\ x > 0$,where $0 \le x \le 4\ and\ 0 \le y \le 3$.

4. Fill up y with 3 gallons of water by the pump used as an external source $as\ y < 3$ where $0 \le x \le 4\ and\ 0 \le y \le 3$, now we have new state : x= 3, y=3.

5. Pour 1 gallon of water to 'x' from 'y' and limit the 'x' to 4 $as\ 0 < x + y \ge 4\ and\ y > 0$ where $0 \le y \le 3\ and\ 0 \le x \le 4$, i.e. new state obtained: x = 4 and y = 2.

6. Now pour all the water out from x to ground $as\ x > 0$ where $0 \le y \le 3\ and\ 0 \le x \le 4$ so that x becomes 0. Therefore, new state obtained is x = 0, y =2.

7. Pour all the water from y to x $as\ 0 < x + y \le 4\ and\ y \ge 0$ where $0 \le y \le 3\ and\ 0 \le x \le 4$ so that we get x = 2 i.e. new state obtained :
x= 2 , y = 0.

We reached the goal state i.e. (2, y).

Representation of the above solution through table.

Gallons in 4-gallon jug	Gallons in 3-gallon jug
0	0
0	3
3	0
3	3
4	2
0	2
2	0

Solution 2

Note: There is pump outside which is used to fill x and y respectively i.e. pump is an external source which have abundance of water.

1. **First x and y in initial state , x = 0 and y = 0 i.e. x and y are empty.**

2. **Pour 4 gallons of water in x and make it full $as, x < 4$. i.e. new state obtained x=4,y=0 where $0 \leq y \leq 3$ and $0 \leq x \leq 4$.**

3. **Pour all the 3 gallons out of 4 gallons of water from x to y as $0 < x + y \geq 3$ and $x > 0$ i.e. new state obtained x = 1 , y = 3 where $0 \leq y \leq 3$ and $0 \leq x \leq 4$.**

4. **Pour all the water out from the jug 'y' to ground as $y > 0$ $where$ $0 \leq y \leq 3$ and $0 \leq x \leq 4$ now we have new state: x = 1, y=0.**

5. **Pour all water (1 gallon of water) to 'y' from 'x' as $0 < x + y \leq 3$ and $x \geq 0$ $where$ $0 \leq y \leq 3$ and $0 \leq x \leq 4$ and now we have new state obtained: x = 0 and y = 1.**

6. **Now pour 4 gallons of water x from pump as $x < 4$, $where$ $0 \leq y \leq 3$ and $0 \leq x \leq 4$ so that we get new state :x = 4, y =1.**

7. **Pour 2 gallons of water from x to y as $0 < x + y \geq 3$ and $x > 0$ where $0 \leq y \leq 3$ and $0 \leq x \leq 4$ such that : x= 2 , y = 3.**

We reached the goal state i.e. (2, y).

Representation of the above solution through table.

Gallons in 4-gallon jug	Gallons in 3-gallon jug
0	0
4	0
1	3
1	0
0	1
4	1
2	3

Therefore we can get any type of solutions to achieve our goal state (2,y).

From the above solutions, we get some set of rules:

S. No.	Actions Performed	Constraint on the state	Successor State or New State.
1.	Fill 4-gallon jug.	(x,y), $0 \leq x \leq 4$	(4,y)
2.	Fill 3-gallon jug.	(x,y), $0 \leq y \leq 3$	(x,3)
3.	Empty the 4-gallon jug on the ground.	(x,y),if(x>0)	(0,y)
4.	Empty the 3-gallon jug on the ground.	(x,y),if(y>0)	(x,0)
5.	Pour water from the 3 gallon jug into the 4 gallon jug until the 4 gallon jug is full.	(x,y), if $(0 < x + y \geq 4 \; and \; y > 0)$	(4,(y-(4-x))
6.	Pour water from the 4 gallon jug into the 3 gallon jug until the 3 gallon jug is full.	(x,y), if $(0 < x + y \geq 3 \; and \; x > 0)$	((x-(3-y),3)
7.	Pour all the water from the 3 gallon jug into the 4 gallon jug .	(x,y), if $(0 < x + y \leq 4 \; and \; y \geq 0)$	(x+y,0)
8.	Pour all the water from the 4 gallon jug into the 3 gallon jug .	(x,y), if $(0 < x + y \leq 3 \; and \; x \geq 0)$	(0,x+y)