20.ASYMPTOTIC ANALYSIS FOR LOOP.

Approach:

Finding $Big\ (O)\ i.e. upto\ n\ times\ run\ of\ the\ particular\ code\ or\ we\ can\ tell\ traverse\ to\ the\ last$.

When we write for loop, we write it as:

$for(i = 1\ ;\ \ i \leq n;\ \ i++)\ \{$

c=c+1;

$\}$

$runs\ at\ constant\ time = O(1)$

$lower\ bound\ of\ for\ loop.$

$upper\ bound\ of\ for\ loop.$

$Hence\ `n`\ is\ upper\ bound\ and\ lower\ bound\ is\ 1$, $input\ size\ is\ 1\ to\ n\ for\ `for\ loop`$.

$Rule\ 1: How\ much\ time\ the\ inner\ most\ loop's\ statement\ run = Time\ complexity\ of\ the\ loop.$

SINGLE LOOP STATEMENT

$for(i = 1; i \leq n; i++)\{$

$c = c + 1;$

$\}$

SOLUTION:

$1. Inner\ most\ loop's\ statement \Longrightarrow c = c + 1\ which\ runs\ at\ O(1)\ time.$

$2. No.\ of\ inputs\ in\ for\ loop\ takes\ 1\ to\ n\ times.$

$Hence\ we\ can\ say:$

$lower\ bound\ or\ c_1 g(n) = 1, hence\ g(n) = 1$

$upper\ bound\ or\ c_2 g(n) = n, hence\ g(n) = n$

$$we\ can\ write\ it\ as: c_1 g(n) \leq f(n) \leq c_2\big(g(n)\big)$$

$$or, c_1 \times 1 \leq f(1,2,3,\dots,n) \leq c_2 \times n$$

$As\ we\ are\ focused\ on\ upper\ bound\ we\ need$:

$$f(1,2,3,\dots,n) \leq c_2 \times n\ or\ f(1,2,3,\dots,n) \leq c \times n$$

$Hence$:

$f(1) \leq c \times n \Rightarrow when\ i = 1$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$f(2) \leq c \times n \Rightarrow when\ i = 2$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$f(3) \leq c \times n \Rightarrow when\ i = 3$

$$c = c + 1\ runs\ \ in\ 1\ unit\ of\ time\ i.e. 1\ time.$$

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$f(n) \leq c \times n \Rightarrow when\ i = n$

$$c = c + 1\ runs\ in\ 1\ unit\ of\ time\ i.e. 1\ time.$$

$Rule\ 2$: $Add\ up\ all\ the\ units\ of\ time\ taken\ by\ innermost\ loop\ statement.$

HENCE, WE ALL KNOW THAT ADDING UP $1\ to\ n$ TIMES,

GIVES RESULT $1+1+1+1+\cdots+n=n$

$Eg: n = 5$, $add\ 1\ to\ 5\ times\ gives$, $1+1+1+1+1=5$

$Eg: n = 6$, $add\ 1\ to\ 6\ times\ gives$, $1+1+1+1+1+1=6$

$$T(n) = \sum_{i=1}^{n} 1 = 1+1+1+1+\cdots+n = n\ times$$

Hence , we can tell that :

$Time\ complexity\ of\ the\ \ loop = O\big(g(n)\big) = O(n), where$

$g(n) is\ the\ upper\ bound\ of\ the\ loop.$

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