21.B. SOME MORE EXAMPLES OF FOR LOOP TIME COMPLEXITIES

$$
\begin{aligned}
&for(i = 1; i \leq n; i++)\{ \\
&\quad for(j = 1; j \leq n; j = j + i)\{ \\
&\quad\quad print(\text{Hello}); \\
&\quad \} \\
&\}
\end{aligned}
$$

SOLUTION

$\boldsymbol{i = 1, j = 1}$, $\boldsymbol{print}$ Hello $\boldsymbol{runs\ 1\ time}$.

$\boldsymbol{j = (1 + 1) = 2}$, $\boldsymbol{print}$ Hello $\boldsymbol{runs\ 1\ time}$.

$\boldsymbol{j = (2 + 1) = 3}$, $\boldsymbol{print}$ Hello $\boldsymbol{runs\ 1\ time}$.

… ..

$\boldsymbol{j = n}$, $\boldsymbol{print}$ Hello $\boldsymbol{runs\ 1\ time}$.

$\boldsymbol{print}$ Hello $\boldsymbol{executed\ \frac{n}{1}\ time}$.

$i = 2, j = 1$, $print$ Hello $runs$ 1 $time$.

$j = (2 + 2) = 4$, $print$ Hello $runs$ 1 $time$.

$j = (4 + 2) = 6$, $print$ Hello $runs$ 1 $time$.

$\dots..$

$j = n$, $print$ Hello $runs$ 1 $time$.

$print$ Hello $executed \frac{n}{2}$ $time$.

$i = 3, j = 1$, $print$ Hello $runs$ 1 $time$.

$j = (1 + 3) = 4$, $print$ Hello $runs$ 1 $time$.

$j = (4 + 3) = 7$, $print$ Hello $runs$ 1 $time$.

$\dots..$

$j = n$, $print$ Hello $runs$ 1 $time$.

$print$ Hello $executed \frac{n}{3}$ $time$.

$Therefore\ no.\ of\ time\ print$ Hello $executed$:

$$\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \cdots + n\ times$$

$$\sum_{i=1}^{n} \frac{n}{i}\ times$$

we can write it as:

$$n \times \sum_{i=1}^{n} \frac{1}{i}$$

NOW THIS IS A HARMONIC SERIES

$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \approx logn$$

$$Hence\ complexity\ is\ O(nlogn)$$

Deduction

$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = \int_{0}^{1} \frac{(1 - x^n)}{1 - x}\, dx$$

$$= \psi(n) + \Upsilon\ (Here\ \Upsilon\ is\ is\ the\ Euler - Mascheroni\ constant)$$

$$= ln(n) + \Upsilon$$

$$= ln(n) + 0.57721 \ldots\ (\Upsilon \approx 0.57721 \ldots)$$

$$= \boldsymbol{n} \times \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \frac{\mathbf{1}}{\boldsymbol{i}} = \boldsymbol{n}(\,\boldsymbol{ln}(\boldsymbol{n}) + \mathbf{0.57721}\ \ldots)$$

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