20.9. TIME COMPLEXITY CALCULATION
FOR LOOP (EG-8).

STUDYING SOME FOR-LOOP EXECUTION:

$\boldsymbol{for(int\ i = 1; i - n \leq n\ ; i++)\{}$

$\boldsymbol{k = k + 1;}$

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SOLUTION

THE UPPER BOUND BECOME = $\boldsymbol{i \leq n + n = i \leq 2n}$

$\boldsymbol{k = k + 1\ runs\ 1 + 1 + 1 + ..2n\ times}$

$\boldsymbol{Therefore\ complexity\ become : O(2n) = O(n)}$

$$for\left(int\ i = 1; i * \frac{n}{2} \leq n\ ; i++\right)\{$$

$$k = k + 1;$$

$$\}$$

SOLUTION

THE UPPER BOUND BECOME = $i * \frac{n}{2} \leq n = i \leq \frac{2n}{n} = 2$

$k = k + 1\ runs\ 2\ times$

$Therefore\ complexity\ become : O(1)\ constant\ .$

$$for\left(int\ i = 1; i + \frac{n}{2} \leq n\ ; i++\right)\{$$

$$k = k + 1;$$

$$\}$$

SOLUTION

THE UPPER BOUND BECOME = $i + \frac{n}{2} \leq n = i \leq n - \frac{n}{2}$

$$= i \leq \frac{2n - n}{2} = i \leq \frac{n}{2}$$

$$k = k + 1\ runs\ 1 + 1 + 1 + ..\frac{n}{2}\ times = \frac{n}{2}$$

$$Therefore\ complexity\ become : O\left(\frac{n}{2}\right) = O(n)$$

$$\boldsymbol{for\left(int\ i=\frac{n}{2};i\leq n\,;i++\right)\{}$$

$$\boldsymbol{k=k+1;}$$

$$\}$$

SOLUTION

AT FIRST RUN THE INCREMENT I'S INCREMENT WILL BE

$$\boldsymbol{Iteration\ 1:\frac{n}{2}+0=\frac{n}{2}\ ,increment\ i=i+1}$$

$$\boldsymbol{Iteration\ 2:\frac{n}{2}+1\ ,\qquad increment\ \ i=i+1}$$

$$\boldsymbol{Iteration\ 3:\frac{n}{2}+2\ ,\qquad increment\ \ i=i+1}$$

$$\boldsymbol{Iteration\ 4:\frac{n}{2}+3\ ,\qquad increment\ \ i=i+1}$$

....

As we do not know how many iterations have taken place, lets consider the last iteration is $\boldsymbol{k}$

Rewriting the iterations:

$$\boldsymbol{Iteration\ 1 : \frac{n}{2} + (1 - 1)\ , increment\ i = i + 1} \quad \longrightarrow \boxed{\boldsymbol{Iteration\ 1}}$$

$$\boldsymbol{Iteration\ 2 : \frac{n}{2} + (2 - 1)\ , \quad increment\ \ i = i + 1} \quad \longrightarrow \boxed{\boldsymbol{Iteration\ 2}}$$

$$\boldsymbol{Iteration\ 3 : \frac{n}{2} + (3 - 1)\ , \quad increment\ \ i = i + 1} \quad \longrightarrow \boxed{\boldsymbol{Iteration\ 3}}$$

$$\boldsymbol{Iteration\ 4 : \frac{n}{2} + (4 - 1)\ , \quad increment\ \ i = i + 1} \quad \longrightarrow \boxed{\boldsymbol{Iteration\ 4}}$$

....

$$\boldsymbol{Iteration\ k : \frac{n}{2} + (k - 1)\ , \quad increment\ \ i = i + 1} \quad \longrightarrow \boxed{\boldsymbol{Iteration\ k}}$$

$$\boldsymbol{And, \frac{n}{2} + (k - 1) = n}$$

$\boldsymbol{, as\ n\ is\ the\ upper\ bound\ upto\ which\ loop\ will\ run}$

$$\boldsymbol{\frac{n}{2} + (k - 1) = n}$$

$$\boldsymbol{\Longrightarrow \frac{n + 2k - 2}{2} = n}$$

$$\boldsymbol{\Longrightarrow n + 2k - 2 = 2n}$$

$$\boldsymbol{\Longrightarrow 2k - 2 = n}$$

$$\boldsymbol{\Longrightarrow 2k = n + 2}$$

$$\Rightarrow \boldsymbol{k} = \frac{(\boldsymbol{n}+\boldsymbol{2})}{\boldsymbol{2}}$$

$$\Rightarrow \boldsymbol{k} = \frac{\boldsymbol{n}}{\boldsymbol{2}} + \frac{\boldsymbol{2}}{\boldsymbol{2}}$$

$$\Rightarrow \boldsymbol{k} = \frac{\boldsymbol{n}}{\boldsymbol{2}} + \boldsymbol{1}$$

$\boldsymbol{i.e. number\ of\ iteration} = \frac{\boldsymbol{n}}{\boldsymbol{2}} + \boldsymbol{1}$, $\boldsymbol{and\ k = k + 1\ prints}$

$\boldsymbol{1 + 1 + 1 + \cdots + \left(\frac{n}{2} + 1\right) \Rightarrow \frac{n}{2} + 1\ \ times}$

$\boldsymbol{O\left(\frac{n}{2} + 1\right) = O\left(\frac{n}{2}\right) = O(n)\ Ans.}$

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