20.4. TIME COMPLEXITY CALCULATION FOR LOOP (EG-3).

$//outer\ loop\ executed\ \ n\ times$

$for(i = 1; i \leq n; i + 2)\{$

$\qquad k = k + 1\ ;\ //\ constant\ time.$

$\}$

SOLUTION

Here if we notice $i + 2\ is\ increment\ factor\ , where\ 1\ is\ lower\ bound\ and\ n\ is\ lower\ bound.$

$We\ can\ write\ it\ as : c_1\ \times 1 \leq f(n) \leq c \times n\ ,$
$Focusing\ on\ upper\ bound\ or\ Big\ O\ time\ complexity:$
$f(n) \leq c \times n\ , where\ n\ is\ g(n).$

$Note : At\ every\ incement\ of\ 2\ , k = k + 1\ prints.$

Hence when:

$f(1) \le c \times n \Rightarrow when\ i = 1$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 1 + 2 = 3$

$f(3) \le c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 3 + 2 = 5$

$f(5) \le c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 5 + 2 = 7$

....

$Now\ we\ have\ to\ divide\ it\ into\ even\ or\ odd\ when\ it\ comes\ running\ upto\ n\ times.$

$We\ know: 2 + 2 = 4\ i.e. Even + Even = Even$

$1 + 1 = 2\ i.e. Odd + Odd = Even$

$1 + 2 = 3\ i.e. odd + Even = Odd$

$And\ \ here\ i's\ \ \ \ lower\ bound\ started\ with\ 1\ and\ hence\ at\ each\ time\ it\ gets\ added$
$with\ even\ number\ 2\ \ to\ produce\ an\ odd\ output.$

$\underline{Assuming\ n\ is\ odd\ then:}$

$Then\ the\ whole\ iteration\ will\ run\ upto\ 'n'\ times.$

$f(n) \leq c \times n$

$k = k + 1$ *executes in* 1 *unit of time*

i *increments* $n + 2$

At every $i + 2$ *times* $k = k + 1$ *prints at* 1 *unit of time upto* n,

common difference at every iteration is 2, *therefore we can directly tell that*

$k = k + 1$ *prints at* 1 *unit of time upto* $\frac{n}{2}$ *times* $(1 \times \frac{n}{2})$, *when* n *is odd.*

$$k = k + 1 \text{ prints } \left\lfloor \frac{n}{2} \right\rfloor \text{ times when } n \text{ is odd .}$$

$$\text{HENCE } O\left(\left\lfloor \frac{n}{2} \right\rfloor\right) = O\left(\frac{1}{2} \times n\right) = O(n)$$

Here we will use $\lfloor\ \rfloor$ *which represents floor value*:

. *i.e.* $\lfloor 1.23 \rfloor = 1, \lfloor 1 \rfloor = 1, \lfloor 2.26 \rfloor = 2$

Assuming n *is even then* :

Then the whole iteration will run upto '$n - 1$' *times.*

As from 1 *to* n *we have odd, even, odd, even …… etc. in count if we visualize*

from $1, 2, 3, 4, \ldots, $ *upto* n.

Suppose if n *is* 10, *then the iteration will go upto* 9 *i.e.* $n - 1$ *times.*

Hence,

$f(n - 1) \leq c \times n$

$k = k + 1$ *executes in* 1 *unit of time*

i *increments* $n + 2$

$\boldsymbol{If\ we\ notice,}$

$for\ n = 1\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{1-1}{2} \right\rfloor + 1\right) = 0 + 1 = 1$

$for\ n = 2\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{2-1}{2} \right\rfloor + 1\right) = 0 + 1 = 1$

$for\ n = 3\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{3-1}{2} \right\rfloor + 1\right) = 1 + 1 = 2$

$for\ n = 4\ , k = k + 1\ will\ print\ at\ i = 1, 3 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{4-1}{2} \right\rfloor + 1\right) = 1 + 1 = 2$

$for\ n = 5\ , k = k + 1\ will\ print\ at\ i = 1, 3, 5 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{5-1}{2} \right\rfloor + 1\right) = 2 + 1 = 3$

$for\ n = 6\ , k = k + 1\ will\ print\ at\ i = 1, 3, 5 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{6-1}{2} \right\rfloor + 1\right) = 2 + 1 = 2$

$for\ n = 7\ , k = k + 1\ will\ print\ at\ i = 1, 3, 5, 7 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$

$$\Rightarrow \left(\left\lfloor \frac{7-1}{2} \right\rfloor + 1\right) = 3 + 1 = 4$$

$$for\ n = 8\ , k = k + 1\ will\ print\ \ at\ i = 1, 3, 5, 7 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{8-1}{2} \right\rfloor + 1\right) = 3 + 1 = 4$$

$$for\ n = 9\ , k = k + 1\ will\ print\ \ at\ i = 1, 3, 5, 7, 9 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{9-1}{2} \right\rfloor + 1\right) = 4 + 1 = 5$$

$$for\ n = 10\ , k = k + 1\ will\ print\ \ at\ i = 1, 3, 5, 7, 9 = \left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{9-1}{2} \right\rfloor + 1\right) = 4 + 1 = 5$$

HENCE $O\left(\left(\left\lfloor \frac{n-1}{2} \right\rfloor\right) + 1\right) = O(n)$

BUT HERE IS THE TWIST $\left\lfloor \frac{n}{2} \right\rfloor$ **DOES NOT STAND TRUE FOR ALL INPUTS**

$1.\ if\ n = 1\ , \left\lfloor \frac{1}{2} \right\rfloor = 0\ [False],$

$2. if\ n = 2\ , \left\lfloor \frac{2}{2} \right\rfloor = 1\ [True]$

$3. if\ n = 3\ , \left\lfloor \frac{3}{2} \right\rfloor = 1\ [False],$

as inner most statement will print 2 times.

HENCE THE ACCEPTED ANSWER FOR 'N' TIMES: $\left\lfloor \frac{n}{2} \right\rfloor + 1$

and $O\left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right) = O(n)$

****