21.1 . SOME MORE EXAMPLES OF FOR LOOP TIME COMPLEXITIES

$$for(k = 1, i = 1; k \leq n; i++)\{$$

$$c = c + 1; \; ---\rightarrow Statement$$

$$k = k + i;$$

$$\}$$

SOLUTION

THEN WHAT IS THE UPPER BOUND HERE?

k	i	n	$i++$	$k+i$
1	1	n	2	$1+1$
2	2	n	3	1+1+2
4	3	n	4	1+1+2+3
….	…	…	….	…
$1+1+2+3+\cdots+k-1$	k	n		1+1+2+3+…+k-1+k

$$k = 1 + (1 + 2 + 3 + \cdots + k - 1 + k)$$

if we exclude 1 *here* : *Then we get* $\Longrightarrow$

$$\Longrightarrow (1+2+3+\cdots+k-1+k) \leq n$$

$$\Longrightarrow \frac{k(k+1)}{2} \leq n$$

$$\Longrightarrow \frac{k^2+k}{2} \leq n$$

$$\Longrightarrow k^2+k \leq 2n$$

$$\Longrightarrow k^2+k-2n \leq 0$$

By quadratic equation:

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \frac{-1 \pm \sqrt{1^2-4\times 1\times -2n}}{2}$$

$$\Rightarrow \frac{-1 \pm \sqrt{1+8n}}{2}$$

We get:

$$\left(\frac{-1+\sqrt{1+8n}}{2}\right) \text{ and } \left(\frac{-1-\sqrt{1+8n}}{2}\right)$$

And we take the general term for number of times the inner most statement gets printed :

$$\text{ceil of } \left\lceil \frac{-1+\sqrt{1+8n}}{2} \right\rceil$$

And time complexity : $O(\sqrt{n})$