13. L' HOSPITAL THEOREM

If $\lim_{n\to\infty} f(n) = \infty$ and $\lim_{n\to\infty} g(n) = \infty$, that is both converges to zero , this rule can be used .

The rule is: $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$

Thus, the ratio of two complexity functions is the same as that of its derivatives.

Example

Prove that $\ln(n)\ \epsilon\ O(n^2)$ *using L'Hospital rule*.

Solution

Consider the following limit to find the asymptotic notation:

$$= \lim_{n\to\infty} \frac{\ln(n)}{n^2}$$

Here $\ln(n)$ *is* $f(n)$ *and* $g(n)$ *is* n^2 .

And

$\lim_{n\to\infty} ln(n) = \infty$ [*Limit inifity theorem*]

$\lim_{n\to\infty} n^2 = \lim_{n\to\infty} n \times n = \lim_{n\to\infty} n \times \lim_{n\to\infty} n = \infty \times \infty = \infty.$

Hence the above equation is in: $\frac{\infty}{\infty}$

Therefore, Applying L'Hospital Theorem:

$$\frac{d}{d(x)}(\ln(n)) = \frac{1}{n} \ \ and \ \ \frac{d}{d(x)}(n^2) = 2n$$

Hence:

$$\lim_{n\to\infty} \frac{\ln(n)}{n^2} = \lim_{n\to\infty} \frac{\frac{1}{n}}{2n} = \lim_{n\to\infty} \frac{1}{2n^2}$$

We can now do it in two ways:

1st way taking the constant out and applying Limit infinity theorem:

$$\Longrightarrow \frac{1}{2} \times \lim_{n\to\infty} \frac{1}{n^2} = \frac{1}{2} \times 0 = 0 \ as, \lim_{n\to\infty} \frac{c}{n^a} = 0$$

Another way is using $\lim_{n\to a} \frac{f(n)}{g(n)} = \frac{\lim_{n\to a} f(n)}{\lim_{n\to\infty} g(n)}$

$$= \lim_{n\to\infty} \frac{1}{2n^2} = \frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}(2n^2)} = \frac{1}{2 \times \lim_{n\to\infty}(n^2)} = \frac{1}{2 \times \infty} = \frac{1}{\infty} = 0.$$

$$\left[As, \lim_{n\to a} c \times f(n) = c \times \lim_{n\to a} f(n), where\ c\ is\ constant\right] - - - i$$

$$\left[As, \lim_{n\to\infty}(n) = \infty\right] - (ii)\ and\ \left[\frac{c}{\infty} = 0, where\ c\ is\ constant\right] - (iii)$$

As it converges to **0** , $\boldsymbol{therefore}$ $\mathbf{ln}(\boldsymbol{n})\ \boldsymbol{\epsilon}\ \mathbf{O}(\mathbf{n}^2)$.

****