9.A.2 BIG O NOTATION WITH LIMITS- BIG OH RATIO THEOREM.

INTRODUCTION:

Big -Oh of an algorithm can be obtained using limit also. The theory of limits is the basis of calculus. Limit is a notion that indicates whether a function diverges or converges. In other words, limits indicate the behaviour of a function.

As the time complexity of an algorithm is also a function, the limit theory can be used to study the Behaviour of the Algorithm

DEFINITION: If the limit $\lim_{i \to \infty} \frac{f(n)}{g(n)} \neq \infty$ holds, then $f(n) = O\left(g(n)\right)$. This is called the big-Oh ration theorem . As $n \to \infty$, the ratio will not be ∞ .

Example 1: Let $f(n) = 9n^2$. $Prove\ that\ this\ algorithm\ is\ of\ the\ order\ of\ O(n^2)$

Solution

The definition of the Big -Oh notation is that $f(n) \leq c \times g(n)$.

we can write $9n^2 \leq c \times n^2$.

Now form the ratio of $\lim_{i \to \infty} \frac{f(n)}{g(n)}$:

$$\Rightarrow \lim_{n \to \infty} \frac{9n^2}{n^2}$$

$$= \lim_{n \to \infty} 9$$

We know that

$\lim_{n \to a} c = c, where\ c\ is\ constant.$

Hence answer is : 9 $\neq \infty$

Therefore , $O(n^2)$ is correct.

Example 2: Let $f(n) = 3n^3 + 2n^2 + 3$. Prove that this algorithm is of $O(n^3)$.

Solution

The definition of the Big -Oh notation is that $f(n) \leq c \times g(n)$.

we can write: $3n^3 + 2n^2 + 3 \leq c \times n^3$.

Now form the ratio of $\lim_{i \to \infty} \frac{f(n)}{g(n)}$:

$$\lim_{n \to \infty} \left(\frac{3n^3 + 2n^2 + 3}{n^3} \right)$$

The above can be written as:

$$\lim_{n \to \infty} \left(\frac{3n^3}{n^3} + \frac{2n^2}{n^3} + \frac{3}{n^3} \right)$$

$$\Rightarrow \lim_{n \to \infty} \left(3 + \frac{2}{n} + \frac{3}{n^3} \right)$$

Distribute $\lim_{n \to \infty}$ in the equation:

$$\lim_{n \to \infty} (3) = 3 \text{ , } \lim_{n \to a} c = c, \textit{where } c \textit{ is constant.}$$

$$\lim_{n \to \infty} \left(\frac{2}{n} \right) = 0, \lim_{n \to \infty} \left(\frac{c}{x^a} \right) = 0 \longrightarrow \textit{Applying Infinity Property}$$

$$\lim_{n \to \infty} \left(\frac{3}{n^3} \right) = 0, \lim_{n \to \infty} \left(\frac{c}{x^a} \right) = 0 \longrightarrow \textit{Applying Infinity Property}$$

Therefore,

$$\Rightarrow \lim_{n\to\infty}\left(3+\frac{2}{n}+\frac{3}{n^3}\right) = 3+0+0$$

$$\Rightarrow 3 \neq \infty$$

Hence $O(n^3) is\ correct$.