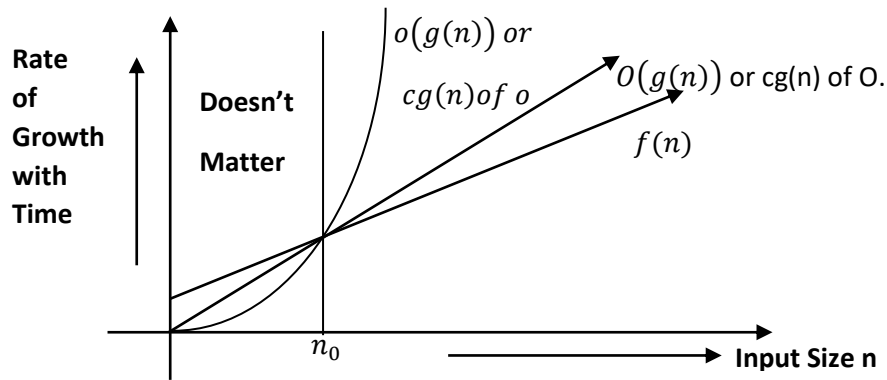


10. LITTLE – OH NOTATION



DEFINITION: Let f and g be two functions that map a set of natural numbers to a set of positive real numbers, $f: \mathbb{N} \rightarrow \mathbb{R}$.

Let $o(g)$ be the set of all functions with a similar rate of growth.

The relation $f(n) = o(g(n))$ holds good, if there exist two positive constants c and n_0 such that $f(n) < c \times g(n)$.

Some points over little-oh notion:

- The little-oh notion is used very rarely.
- Here the value of c is very small.
- The *little – oh* notation can be used instead of the big-Oh notation as the little-oh notation represents a **loose upper bound**.

LITTLE OH DEFINITION IN LIMITS - LITTLE OH RATIO THEOREM

DEFINITION: The function $f(n) = o(g(n))$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$,
which implies that $f(n) = o(g(n))$.

EXAMPLES OF LITTLE OH

Example 1

Let $f(n) = 7n + 6$. Show that $f(n)$ is in $o(n^2)$.

Solution

As we know, $f(n) = o(n^2)$ as $\lim_{n \rightarrow \infty} \frac{7n+2}{n^2}$

$$= \lim_{n \rightarrow \infty} \frac{7n + 2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{7n}{n^2} + \frac{2}{n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{7}{n} + \frac{2}{n^2}$$

As per infinity theory of limit $\lim_{n \rightarrow \infty} \frac{c}{x^a} = 0$

$$= \lim_{n \rightarrow \infty} \frac{7}{n} + \lim_{n \rightarrow \infty} \frac{2}{n^2}$$

$$= 0 + 0$$

$$= 0$$

Therefore, we can say that $7n + 6 \in o(n^2)$.

Example 2

*Let $f(n) = 7n + 6$. Show that $f(n)$ is in $o(n^2)$.
[Without Limit]*

Solution

$$7n + 6 < c \times n^2$$

One can write it as:

$$f(n) < 7n^2 + 6n^2$$

$$f(n) < 13n^2$$

Now $c = 13$

$$\Rightarrow 13n^2 - 7n - 6 < 0$$

$$\Rightarrow 13n^2 + 6n - 13n - 6 < 0$$

$$\Rightarrow 13n^2 + 6n - 13n - 6 < 0$$

$$\Rightarrow n(13n + 6) - 1(13n + 6) < 0$$

$$\Rightarrow (13n + 6)(n - 1) < 0$$

$$= 13n + 6 < 0$$

$$= 13n < -6$$

$$= n < -\frac{6}{13}$$

$$= n - 1 < 0$$

$$= -1 < -n$$

$$= n > 1$$

Hence, we got $n > 1$ and $c = 13$

Now if we see it through a table

N	$7n + 6$	$13n^2$
2	20	52
3	27	117
4	34	208
5	41	325

Hence, we can tell $c \times g(n)$ is loosely bound and $f(n) = o(n^2)$.

**NOW COMPARE IF IT IS TIGHTLY UPPER
BOUND $O(n)$ THEN:**

$$\Rightarrow f(n) \leq 7n + 6n = 13n$$

Therefore

$$\Rightarrow 7n + 6 \leq 13n$$

$$\Rightarrow 6 \leq 6n$$

$$\Rightarrow 1 \leq n$$

$$\Rightarrow n \geq 1$$

N	$f(n) = 7n + 6$	$g(n) = 13n$
1	13	13
2	20	26
3	27	39
4	34	52
5	41	65

Hence, we see $O(n)$ is tightly upper bound other than $o(n)$ which is loosely upper bound and graph to see the difference .
