20.11. TIME COMPLEXITY CALCULATION

NESTED FOR LOOP (EG-10).

$$for\left(i = \frac{n}{2}; i \leq n; i++\right)\{$$

$$for\left(j = 1; j + \frac{n}{2} \leq n; j++\right)\{$$

$$k = k + 1\ ; //\ constant\ time.$$

$$\}$$

SOLUTION

AT FIRST RUN THE INCREMENT I'S INCREMENT WILL BE

$$Iteration\ 1 : \frac{n}{2} + 0 = \frac{n}{2}\ , increment\ i = i + 1$$

$$Iteration\ 2 : \frac{n}{2} + 1\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 3 : \frac{n}{2} + 2\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 4 : \frac{n}{2} + 3\ , \quad increment\ \ i = i + 1$$

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As we do not know how many iterations have taken place, lets consider the last iteration is k.

Rewriting the iterations:

$$\boldsymbol{Iteration\ 1 : \frac{n}{2} + (1-1)\ , increment\ i = i + 1}$$

$\boxed{\boldsymbol{Iteration\ 1}}$

$$\boldsymbol{Iteration\ 2 : \frac{n}{2} + (2-1)\ , \quad increment\ \ i = i + 1}$$

$\boxed{\boldsymbol{Iteration\ 2}}$

$$\boldsymbol{Iteration\ 3 : \frac{n}{2} + (3-1)\ , \quad increment\ \ i = i + 1}$$

$\boxed{\boldsymbol{Iteration\ 3}}$

$$\boldsymbol{Iteration\ 4 : \frac{n}{2} + (4-1)\ , \quad increment\ \ i = i + 1}$$

$\boxed{\boldsymbol{Iteration\ 4}}$

....

$$\boldsymbol{Iteration\ k : \frac{n}{2} + (k-1)\ , \quad increment\ \ i = i + 1}$$

$\boxed{\boldsymbol{Iteration\ k}}$

$$\boldsymbol{And, \frac{n}{2} + (k-1) = n}$$

$\boldsymbol{, as\ n\ is\ the\ upper\ bound\ upto\ which\ loop\ will\ run}$

$$\boldsymbol{\frac{n}{2} + (k-1) = n}$$

$$\boldsymbol{\Longrightarrow \frac{n + 2k - 2}{2} = n}$$

$$\Longrightarrow n + 2k - 2 = 2n$$

$$\Longrightarrow 2k - 2 = n$$

$$\Longrightarrow 2k = n + 2$$

$$\Longrightarrow k = \frac{(n + 2)}{2}$$

$$\Longrightarrow k = \frac{n}{2} + \frac{2}{2}$$

$$\Longrightarrow k = \frac{n}{2} + 1$$

$$\textit{Outer loop i runs} = \frac{n}{2} + 1 \textit{ times.}$$

The upper bound of j become = $j + \frac{n}{2} \le n = j \le n - \frac{n}{2}$

$$= j \le \frac{2n - n}{2} = j \le \frac{n}{2}$$

Hence number of $k = k + 1$ *prints* $= \frac{n}{2}\left(\frac{n}{2} + 1\right)$

$$\frac{n^2}{4} + \frac{n}{2} = \frac{n^2 + 2n}{4} = O\left(\frac{n^2 + 2n}{4}\right) = O(n^2)$$

Hence the time complexity $= O(n^2)$

THE ABOVE ITERATION LOOKS LIKE:

$$f\left(\frac{n}{2}\right) \leq c \times n \Longrightarrow when\ i = \frac{n}{2}$$

$$f(1) \leq c \times n \Longrightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Longrightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Longrightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

… … ….

$$f\left(\frac{n}{2}\right) \leq c \times n \Longrightarrow when\ j = \frac{n}{2}$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = \frac{n}{2}\ , the\ inner\ most\ loop\ statement$$

$$run\ \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = n\ times$$

$$T(n) = \sum_{j=1}^{\frac{n}{2}} 1 = \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

$$f\left(\frac{n}{2} + 1\right) \leq c \times n \Longrightarrow when\ i = 2$$

$$f(1) \leq c \times n \Longrightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Longrightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Longrightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

… … ….

$$f\left(\frac{n}{2}\right) \le c \times n \Longrightarrow when\ j = \frac{n}{2}$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = \frac{n}{2} + 1\ , the\ inner\ most\ loop\ statement$$

$$run\ \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

$$T(n) = \sum_{j=1}^{\frac{n}{2}} 1 = \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

.....

$$f(n) \le c \times n \Longrightarrow when\ i = n$$

$$f(1) \le c \times n \Longrightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \le c \times n \Longrightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \le c \times n \Longrightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$\ldots \ldots ....$

$$f\left(\frac{n}{2}\right) \le c \times n \Longrightarrow when\ j = \frac{n}{2}$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = n\ , the\ inner\ most\ loop\ statement$$

$$run\ \left(1 + 1 + 1 + 1 + \cdots \frac{n}{2}\ \ times\right) = \frac{n}{2}\ times$$

$$T(n) = \sum_{j=1}^{\frac{n}{2}} 1 = \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\ times\right) = \frac{n}{2}$$

$$\boldsymbol{T(n) = \sum_{i=\frac{n}{2}}^{n} 1 \times \sum_{j=1}^{\frac{n}{2}} 1 = \sum_{i=\frac{n}{2}}^{\frac{n}{2}+1} i = \left(\frac{n}{2}\right) + \left(\frac{n}{2}\right) + \cdots \left(\frac{n}{2}+1\right)\ times = \frac{n}{2}\left(\frac{n}{2}+1\right)}$$

$$\boldsymbol{Therefore, we\ get := \frac{n}{2}\left(\frac{n}{2}+1\right) = \frac{n^2}{4} + \frac{n}{2} = \frac{n^2+2n}{4}}$$