

THEOREM BASED ON ASYMPTOTIC NOTATION

Theorem: *If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then $f(n) = \Theta(g(n))$.*

Proof: If $f(n) = O(g(n))$, then there exists c_1 such that:

$$f(n) \leq c_1(g(n))$$

Moreover, $f(n) = \Omega(g(n))$, therefore there exists c_2 such that:

$$f(n) \geq c_2 g(n)$$

Combining the above two result it may be stated that

$$c_1 \times g(n) \leq f(n) \leq c_2 \times g(n)$$

The above theorem can be understood with the help of the following example . Let $f(n) = c_1 x^n + c_2 x^{n-1} + \dots + c_n x^0$ then:

$$f(n) = O(x^n)$$

also,

$$f(n) = \Omega(g(n))$$

therefore,

$$f(n) = \Theta(g(n)).$$
