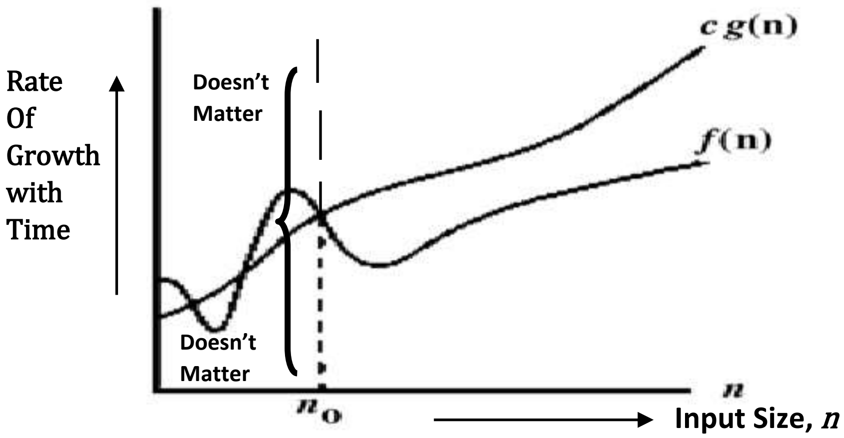
9.A. BIG -O NOTATION



DEFINITION: A function $\boldsymbol{f}(\boldsymbol{n})$ is said to be in $\boldsymbol{O}(\boldsymbol{g}(\boldsymbol{n}))$, denoted $\boldsymbol{f}(\boldsymbol{n}) \in \boldsymbol{O}(\boldsymbol{g}(\boldsymbol{n}))$, if $\boldsymbol{f}(\boldsymbol{n})$ *is bounded above by some constant multiple of* $\boldsymbol{g}(\boldsymbol{n})$ *for all large* $\boldsymbol{n}$, *i.e. if there exist some positive constant* $\boldsymbol{c}$ *and some nonnegative integer* $\boldsymbol{n_0}$ *such that*:

$$\boldsymbol{f}(\boldsymbol{n}) \le \boldsymbol{cg}(\boldsymbol{n}) \ \boldsymbol{for\ all}\ \boldsymbol{n} \ge \boldsymbol{n_0}$$

ON THE ABOVE DIAGRAM 'STARTING FROM n_0 AND BEYOND ONLY MATTERS', BUT THE PORTION LESSER THAN AND WITHOUT THE STARTING POINT OF n_0 DOESN'T MATTER.

NOTE: THE BIG OH NOTATION FINDS OUT THE UPPER BOUND OF A POLYNOMIAL. AND $f(n)$ AND $g(n)$ ARE THEREFORE TWO POLYNOMIAL FUNCTIONS.

MORE DESCRIPTIVE WAY OF ABOVE DEFINITION:

Let $\boldsymbol{f}$ *and* $\boldsymbol{g}$ *be two functions that map a set of natural numbers to a set of positive real numbers, that is,* $f: \mathbb{N} \rightarrow \mathbb{R}_{\geq 0}$.

(*It mean function* $\boldsymbol{f}$ *contains Set of Natural Numbers* $\mathbb{N}$ *which maps to real numbers* $\mathbb{R}$ *which are greater than and equal to* 0 *i.e.* ***Positive Real Numbers***).

THE THINGS MAY LOOK LIKE:

SAY, $f(n) = n^3 + n^2 + 3$ [POLYNOMIAL FUNCTION]

$if\ n = 0, f = 0^3 + 0^2 + 3 = 3$

$if\ n = 1, f = 1^3 + 1^2 + 3 = 2 + 3 = 5$

$if\ n = 2, f = 2^3 + 2^2 + 3 = 8 + 4 + 3 = 15$

THEN WE HAVE SET:

$\{(0,3), (1,5), (2,15)\}$

HENCE:

$Domain\ of\ Function\ (D_f)\ = \{0,1,2\}$

$Range\ of\ Function\ (R_f) = \{3,5,15\}$

$Now, Let\ O(g) be\ the\ set\ of\ all\ functions\ with\ similar\ rate\ of\ growth.$

$i.e. if\ f(n) = n^3 + n^2 + 3, and\ S\big(f(n)\big) = \{(0,3), (1,5), (2,15)\}.$

$Then\ O(g) will\ have\ same\ type\ of\ set\ of\ pairs\ may\ not\ be\ equal\ .$

$Then\ , the\ relation\ \boldsymbol{f(n) = O\big(g(n)\big)} holds\ true\ , if\ there\ exist$

$two\ positive\ constants\ \boldsymbol{c}\ and\ \boldsymbol{n_0}\ such\ that\ \boldsymbol{f(n) \le c \times g(n)}.$

- $\boldsymbol{The\ function\ f(n)\ is\ said\ to\ be\ in\ O\big(g(n)\big).}$
- $\boldsymbol{This\ is\ denoted\ as\ f(n) \in O\big(g(n)\big)}\ .$

$\boldsymbol{i.e. f(n)\ belongs\ to\ O\big(g(n)\big).}$

$\boldsymbol{i.e. the\ elements\ in\ f(n)\ contains\ in\ O\big(g(n)\big).}$

$\boldsymbol{or\ simply\ as:\ f(n) = O\big(g(n)\big)}$

- $\boldsymbol{This\ implies\ that\ f(n)\ never\ takes\ more\ than\ approximately\ g(n)\ operations.}$
- $\boldsymbol{This\ implies\ f(n) is\ in\ the\ order\ of\ g(n).}$
- $\boldsymbol{Which\ implies\ f(n)\ having\ a\ growth\ rate\ that\ is\ less\ than\ or\ equal\ to\ that\ of\ g(n).}$
- $\boldsymbol{This\ implies\ again\ that\ function\ f\ grows\ at\ a\ slower}$

$rate\ than\ a\ constant\ time\ \ g(n)\ for\ all\ the\ values\ of\ a\ larger\ input\ of\ size\ 'n'$.

MORE PRECISELY

We can distinguish $f\ (n)\ and\ g(n)\ like\ say$:

$f(n) = 4n^2 + 5n + 3\ and\ g(n) = 5n^2$, on a note we can see the rate of growth:

n	$4n^2 + 5n + 3$	$5n^2$
1	12	5
2	29	20
3	54	45
4	87	80
5	128	125
6	177	180

Comparison of $f(n)\ and\ g(n)$

Some Points on Big-Oh Notation:

1. $f(n) = O(g(n))$ *means at larger values of* n *, the* *the upper bound of* $f(n)$ *is* $g(n)$. *For example , if* $f(n) = n^4 + 100n^2 + 10n + 50$ *is the given algorithm,* *then* n^4 *is* $g(n)$.

2. $g(n)$ *gives the maximum rate of growth for* $f(n)$ *at* *at larger values of* n .

3. $O(g(n))$
   $= \{f$: *there exist positive constant* c *and* n_0 *such that* $0 \leq f(n) \leq cg(n)$ *for all* $n \geq n_0\}$.

4. $g(n)$ *is an asymptotic tight upper bound for* $f(n)$.

5. n_0 *is the point from which we need to consider the rate of growth for a given algorithm ,*
   hence n_0 *is called threshold of the given function.*

6. Generally, we discard lower values of n . That means the rate of growth at lower values of n is not important.