BIG OMEGA NOTATION WITH LIMITS

-BIG OMEGA RATIO THEOREM

Definition: $If\ the\ \lim_{i\to\infty}\frac{f(n)}{g(n)} \neq 0\ holds\ good, then\ f(n) = \Omega(g(n)). This\ is\ also\ called\ the\ Omega\ Ratio\ Theorem$.

Example: $Let\ us\ consider\ that\ f(n) = 7n + 4. Prove\ that\ this\ is\ of\ the\ order\ of\ \Omega(n)$.

Solution:

Here $f(n) = 7n + 4\ and\ g(n) = n$.

We know by definition of Big Omega Ω notation:

$f(n) = \Omega(g(n)), if\ 0 \leq c \times g(n) \leq f(n)\ for\ all\ n \geq n_0,$
$where\ c\ and\ n_0\ are$ constants.

$$\Rightarrow 0 \leq c \times n \leq 7n + 4$$

$Or\ , f(n) \geq c \times g(n)$

$$\Rightarrow 7n + 4 \geq c\ \times n$$

$$\Rightarrow \frac{7n + 4}{n} \geq c$$

By above definition, of $Big\ Omega\ Ratio\ Theorem$:

$$\lim_{i\to\infty} \frac{7n+4}{n}$$

$$\Rightarrow \lim_{i\to\infty} \frac{7n}{n} + \frac{4}{n}$$

$$\Rightarrow \lim_{i\to\infty} 7 + \frac{4}{n}$$

Distributing $\lim_{i\to\infty}$ in both the sides we get:

$$\Rightarrow \lim_{i\to\infty} 7 + \lim_{i\to\infty} \frac{4}{n}$$

Here $\lim_{x\to a} c = c$ and $\lim_{x\to\infty} \frac{c}{x^a} = 0 [Infinity\ property\ of\ Limit], where: 'c' is\ constant.$

$Hence: \lim_{i\to\infty} 7 = 7\ and\ \lim_{i\to\infty} \frac{4}{n} = 0$

$$\Rightarrow 7 + 0$$

$$\Rightarrow 7 \neq 0$$

Therefore, one can conclude that $f(n) = \Omega(n)$