THEOREM BASED ON ASYMPTOTIC NOTATION

Theorem: $\boldsymbol{If\ f(n) = O(g(n))\ and\ f(n) = \Omega\big(g(n)\big), then\ f(n) =\ \Theta\big(g(n)\big)}$.

$\boldsymbol{Proof}: If\ f(n) = \boldsymbol{O\big(g(n)\big), then\ there\ exists\ c_1\ such\ that}:$

$$\boldsymbol{f(n) \leq c_1\big(g(n)\big)}$$

$\boldsymbol{Moreover, f(n) = \Omega\big(g(n)\big), therefore\ there\ exists\ c_2\ such\ that}:$

$$\boldsymbol{f(n) \geq c_2 g(n)}$$

$\boldsymbol{Combining\ the\ above\ two\ result\ it\ may\ be\ stated\ that}$

$$\boldsymbol{c_1 \times g(n) \leq f(n) \leq c_2 \times g(n)}$$

$\boldsymbol{The\ above\ theorem\ can\ be\ understood\ with\ the\ help\ of\ the\ following\ example\ . Let\ f(n) = c_1 x^n + c_2 x^{n-1} + \cdots + c_n x^0\ then}:$

$$f(n) = O(x^n)$$

also,

$$f(n) = \Omega(g(n))$$

therefore,

$$f(n) = \Theta(g(n)).$$

The above theorem is only possible when $\Omega(g(n))$ *exists i.e.* $c_1 g(n) \leq f(n)$ and $f(n) \leq c_2 g(n)$. Hence it stands like:

$c_1 g(n) \leq f(n) \leq c_2 g(n)$, here $c_1 g(n) \leq f(n) = \Omega(g(n))$ and $f(n) \leq c_2 g(n) = O(g(n))$. Hence there exists:

$c_1 g(n) \leq f(n) \leq c_2 g(n) = \Theta(g(n))$.

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