20.9. TIME COMPLEXITY CALCULATION FOR LOOP (EG-8).

STUDYING SOME FOR-LOOP EXECUTION:

$$for(int\ i = 1; i - n \leq n\ ; i++)\{$$

$$k = k + 1;$$

}

SOLUTION

THE UPPER BOUND BECOME = $i \leq n + n = i \leq 2n$

$k = k + 1\ runs\ 1 + 1 + 1 + ..2n\ times$

$Therefore\ complexity\ become : O(2n) = O(n)$

$$for\left(int\ i = 1; i * \frac{n}{2} \leq n\ ; i++\right)\{$$

$$k = k + 1;$$

}

SOLUTION

THE UPPER BOUND BECOME = $i * \frac{n}{2} \leq n = i \leq 2n$

$k = k + 1\ runs\ 1 + 1 + 1 + ..2n\ times\ = 2n$

$Therefore\ complexity\ become: O(2n) = O(n)$

$$for\left(int\ i = 1; i + \frac{n}{2} \leq n\ ; i++\right)\{$$

$$k = k + 1;$$

}

SOLUTION

THE UPPER BOUND BECOME = $i + \frac{n}{2} \leq n = i \leq n - \frac{n}{2}$

$$= i \leq \frac{2n - n}{2} = i \leq \frac{n}{2}$$

$$k = k + 1\ runs\ 1 + 1 + 1 + .. \frac{n}{2}\ times = \frac{n}{2}$$

$$Therefore\ complexity\ become: O\left(\frac{n}{2}\right) = O(n)$$

$$for\left(int\ i = \frac{n}{2}; i \leq n\ ; i++\right)\{$$

$$k = k + 1;$$

}

SOLUTION

AT FIRST RUN THE INCREMENT I'S INCREMENT WILL BE

$$Iteration\ 1 : \frac{n}{2} + 0 = \frac{n}{2}\ , increment\ i = i + 1$$

$$Iteration\ 2 : \frac{n}{2} + 1\ ,\qquad increment\ \ i = i + 1$$

$$Iteration\ 3 : \frac{n}{2} + 2\ ,\qquad increment\ \ i = i + 1$$

$$Iteration\ 4 : \frac{n}{2} + 3\ ,\qquad increment\ \ i = i + 1$$

....

As we do not know how many iterations have taken place, lets consider the last iteration is k

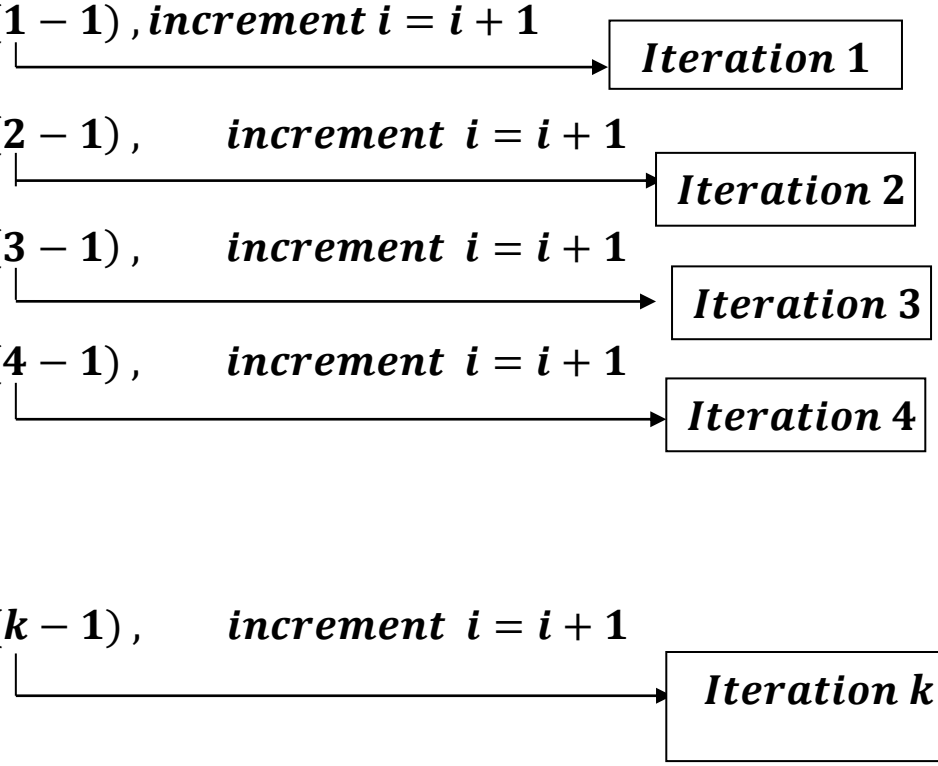
Rewriting the iterations:

$$Iteration\ 1 : \frac{n}{2} + (1-1)\ , increment\ i = i + 1$$

$$Iteration\ 2 : \frac{n}{2} + (2-1)\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 3 : \frac{n}{2} + (3-1)\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 4 : \frac{n}{2} + (4-1)\ , \quad increment\ \ i = i + 1$$



....

$$Iteration\ k : \frac{n}{2} + (k-1)\ , \quad increment\ \ i = i + 1$$

$$And, \frac{n}{2} + (k-1) = n$$

$$, as\ n\ is\ the\ upper\ bound\ upto\ which\ loop\ will\ run$$

$$\frac{n}{2} + (k-1) = n$$

$$\Longrightarrow \frac{n + 2k - 2}{2} = n$$

$$\Longrightarrow n + 2k - 2 = 2n$$

$$\Longrightarrow 2k - 2 = n$$

$$\Longrightarrow 2k = n + 2$$

$$\Rightarrow k = \frac{(n+2)}{2}$$

$$\Rightarrow k = \frac{n}{2} + \frac{2}{2}$$

$$\Rightarrow k = \frac{n}{2} + 1$$

$$i.e. number\ of\ iteration = \frac{n}{2} + 1\ , and\ k = k + 1\ prints$$

$$1 + 1 + 1 + \cdots + \left(\frac{n}{2} + 1\right) \Rightarrow \frac{n}{2} + 1\ \ times$$

$$O\left(\frac{n}{2} + 1\right) = O\left(\frac{n}{2}\right) = O(n)\ Ans.$$

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