

20.20. TIME COMPLEXITY CALCULATION NESTED FOR LOOP (MORE THAN TWO LOOP).

EXAMPLE 6

```
for(i = 1; i ≤ n; i++){  
    for(j = 1; j ≤ i; j++){  
        for(k = 1; k ≤ j; k++){  
            for(l = 1; l ≤ k; l++){  
                c = c + 1;  
            }  
        }  
    }  
}
```

ANSWER

*if we notice then when $i = 1, j = 1, k = 1, l = 1$,
 $c = c + 1$ prints 1 time only.*

*$i = 2$
 $j = 1$
 $k = 1$
 $l = 1$
 $c = c + 1$ (*prints 1 time*)*

*$j = 2$
 $k = 1$
 $l = 1$
 $c = c + 1$ (*prints 1 time*)*

*$k = 2$
 $l = 1$
 $c = c + 1$ (*prints 1 time*)
 $l = 2$
 $c = c + 1$ (*prints 1 time*)*

THE ITERATION GOES LIKE :

$1 + (1 + 3) + (1 + 3 + 5) + \dots$ from 1 to n times

AND IF WE NOTICE

$$\sum_{i=1}^n \left(\sum_{i=1}^n \frac{n(n+1)}{2} \right)$$

AND WE KNOW

$$\sum_{i=1}^n \frac{n(n+1)}{2} = \frac{(n^3 + 3n^2 + 2n)}{6}$$

THEREFORE,

$$\sum_{i=1}^n \frac{(n^3 + 3n^2 + 2n)}{6}$$

$$\Rightarrow \sum_{i=1}^n \frac{n^3}{6} + \sum_{i=1}^n \frac{3n^2}{6} + \sum_{i=1}^n \frac{2n}{6}$$

LETS TAKE $\sum_{i=1}^n \frac{n^3}{6}$

$$\Rightarrow \sum_{i=1}^n \frac{n^3}{6}$$

$$\Rightarrow \frac{1}{6} \sum_{i=1}^n n^3$$

$$\Rightarrow \frac{1}{6} \left(\frac{n^4 + 2n^3 + n^2}{4} \right)$$

$$\Rightarrow \frac{n^4 + 2n^3 + n^2}{24}$$

LETS TAKE $\sum_{i=1}^n \frac{3n^2}{6}$

$$\Rightarrow \sum_{i=1}^n \frac{3n^2}{6}$$

$$\Rightarrow \sum_{i=1}^n \frac{n^2}{2}$$

$$\Rightarrow \frac{1}{2} \sum_{i=1}^n n^2$$

$$\Rightarrow \frac{1}{2} \left(\frac{2n^3 + 3n^2 + n}{6} \right)$$

$$\Rightarrow \frac{2n^3 + 3n^2 + n}{12}$$

LETS TAKE $\sum_{i=1}^n \frac{2n}{6}$

$$\Rightarrow \sum_{i=1}^n \frac{2n}{6}$$

$$\Rightarrow \sum_{i=1}^n \frac{n}{3}$$

$$\Rightarrow \frac{1}{3} \sum_{i=1}^n n$$

$$\Rightarrow \frac{1}{3} \left(\frac{n(n+1)}{2} \right)$$

$$\Rightarrow \frac{(n^2 + n)}{6}$$

THEREFORE

$$\Rightarrow \sum_{i=1}^n \frac{n^3}{6} + \sum_{i=1}^n \frac{3n^2}{6} + \sum_{i=1}^n 2n$$

$$\Rightarrow \frac{n^4 + 2n^3 + n^2}{24} + \frac{2n^3 + 3n^2 + n}{12} + \frac{(n^2 + n)}{6}$$

$$\Rightarrow \frac{n^4 + 2n^3 + n^2 + 4n^3 + 6n^2 + 2n + 4n^2 + 4n}{24}$$

$$\Rightarrow \frac{n^4 + 6n^3 + 11n^2 + 6n}{24}$$

THEREFORE

$$O\left(\frac{n^4 + 6n^3 + 11n^2 + 6n}{24}\right) = O(n^4)$$