# 20.5. TIME COMPLEXITY CALCULATION FOR LOOP (EG-4).

$//outer\ loop\ executed\ \ n\ times$

$for(i = 1; i \leq n; i + 3)\{$

$\qquad k = k + 1\ ;\ //\ constant\ time.$

$\}$

## SOLUTION

Here if we notice $i + 3\ is\ increment\ factor\ , where\ 1\ is\ lower\ bound\ and\ n\ is\ lower\ bound.$

$We\ can\ write\ it\ as : c_1\ \times 1 \leq f(n) \leq c \times n\ ,$

$Focusing\ on\ upper\ bound\ or\ Big\ O\ time\ complexity:$

$f(n) \leq c \times n\ , where\ n\ is\ g(n).$

$Note : At\ every\ incement\ of\ i + 3\ , k = k + 1\ prints.$

Hence when:

$f(1) \leq c \times n \Longrightarrow when\ i = 1$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 1 + 3 = 4$

$f(4) \leq c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 4 + 3 = 7$

$f(7) \leq c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 7 + 3 = 10$

.......

$Here\ the\ loop\ runs\ either: 1 + (1 + 3) + (4 + 3) + \cdots + n$

$or: 1 + (1 + 3) + (4 + 3) + \cdots + n - 1$

$Here\ we\ will\ use\ \lfloor\ \rfloor\ which\ represents\ floor\ value$:

$.\,i.e.\ \lfloor 1.23 \rfloor = 1, \lfloor 1 \rfloor = 1, \lfloor 2.26 \rfloor = 2$

## WHEN IT RUN UP TO $n\ time.$

### 1. FOR MULTIPLES OF N WHICH ARE MULTIPLES OF 3:

*The innermost statement will be printed* $\frac{n}{3}$ *times*: $O\left(\frac{n}{3}\right) = O(n)$

$for\ n = 3\ , k = k + 1\ will\ print\ at\ i = \ 1\ \left(\frac{n}{3}\right)$

$\Rightarrow \left(\frac{3}{3}\right) = 1$

$for\ n = 6\ , k = k + 1\ will\ print\ at\ i = \ 1, 4\ \left(\frac{n}{3}\right)$

$\Rightarrow \left(\frac{6}{3}\right) = 2$

$for\ n = 9\ , k = k + 1\ will\ print\ at\ i = \ 1, 4, 7\ \left(\frac{n}{3}\right)$

$\Rightarrow \left(\frac{9}{3}\right) = 3$

## 2. FOR MULTIPLES OF N WHICH ARE NOT MULTIPLES OF 3:

*The innermost statement will be printed* $\left(\left\lfloor \frac{n}{3} \right\rfloor + 1\right)$ *times*

$for\ n = 1\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{1}{3} \right\rfloor + 1\right) = 0 + 1 = 1$

$for\ n = 2\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{2}{3} \right\rfloor + 1\right) = 0 + 1 = 1$

$for\ n = 4\ , k = k + 1\ will\ print\ at\ i = \ 1, 4 \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{4}{3} \right\rfloor + 1\right) = 1 + 1 = 2$

$for\ n = 5\ , k = k + 1\ will\ print\ at\ i = \ 1, 4 \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{5}{3} \right\rfloor + 1\right) = 1 + 1 = 2$

$for\ n = 7\ , k = k + 1\ will\ print\ at\ i = \ 1, 4,7 \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$\Rightarrow \left(\left\lfloor \frac{7}{3} \right\rfloor + 1\right) = 2 + 1 = 3$

$for\ n = 7\ , k = k + 1\ will\ print\ at\ i = \ 1, 4,7 \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$

$$\Rightarrow \left( \left\lfloor \frac{7}{3} \right\rfloor + 1 \right) = 2 + 1 = 3$$

$$for\ n = 8\ , k = k + 1\ will\ print\ at\ i =\ 1, 4{,}7 \left( \left\lfloor \frac{n}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{8}{3} \right\rfloor + 1 \right) = 2 + 1 = 3$$

$$for\ n = 10\ , k = k + 1\ will\ print\ at\ i =\ 1, 4{,}7{,}10 \left( \left\lfloor \frac{n}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{10}{3} \right\rfloor + 1 \right) = 3 + 1 = 4$$

## WHEN IT RUN UP TO $n - 1\ time$.

$If\ we\ notice\ for\ (n-1)time,$

$$for\ n = 1\ , k = k + 1\ will\ print\ at\ i =\ 1, \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{1-1}{3} \right\rfloor + 1 \right) = 0 + 1 = 1$$

$$for\ n = 2\ , k = k + 1\ will\ print\ at\ i =\ 1, \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{2-1}{3} \right\rfloor + 1 \right) = 0 + 1 = 1$$

$$for\ n = 3\ ,k = k + 1\ will\ print\ at\ i = \ 1, \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{3-1}{3} \right\rfloor + 1 \right) = 0 + 1 = 1$$

$$for\ n = 4\ ,k = k + 1\ will\ print\ at\ i = \ 1, 4 \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{4-1}{3} \right\rfloor + 1 \right) = 1 + 1 = 2$$

$$for\ n = 5\ ,k = k + 1\ will\ print\ at\ i = \ 1, 4 \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{5-1}{3} \right\rfloor + 1 \right) = 1 + 1 = 2$$

$$for\ n = 6\ ,k = k + 1\ will\ print\ at\ i = \ 1, 4 \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{6-1}{3} \right\rfloor + 1 \right) = 1 + 1 = 2$$

$$for\ n = 7\ ,k = k + 1\ will\ print\ at\ i = \ 1, 4, 7 \left( \left\lfloor \frac{n-1}{3} \right\rfloor \right) + 1$$

$$\Rightarrow \left( \left\lfloor \frac{7-1}{3} \right\rfloor + 1 \right) = 2 + 1 = 3$$

$$\textbf{\textit{The innermost statement will be printed}} : O\left(\left(\left\lfloor \frac{n-1}{3} \right\rfloor\right) + 1\right) = O(n)$$

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