20.11. TIME COMPLEXITY CALCULATION

NESTED FOR LOOP (EG-10).

$for\left(i = \frac{n}{2}; i \leq n; i++\right)\{$

$for\left(j = 1; j + \frac{n}{2} \leq n; j++\right)\{$

$k = k + 1$; // constant time.

}

SOLUTION

AT FIRST RUN THE INCREMENT I'S INCREMENT WILL BE

$$Iteration\ 1 : \frac{n}{2} + 0 = \frac{n}{2}\ , increment\ i = i + 1$$

$$Iteration\ 2 : \frac{n}{2} + 1\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 3 : \frac{n}{2} + 2\ , \quad increment\ \ i = i + 1$$

$$Iteration\ 4 : \frac{n}{2} + 3\ , \quad increment\ \ i = i + 1$$

....

As we do not know how many iterations have taken place, lets consider the last iteration is $\boldsymbol{k}$.

Rewriting the iterations:

$$\boldsymbol{Iteration\ 1 : \frac{n}{2} + (1-1)\ , increment\ i = i + 1}$$

$$\boxed{\boldsymbol{Iteration\ 1}}$$

$$\boldsymbol{Iteration\ 2 : \frac{n}{2} + (2-1)\ , \qquad increment\ \ i = i + 1}$$

$$\boxed{\boldsymbol{Iteration\ 2}}$$

$$\boldsymbol{Iteration\ 3 : \frac{n}{2} + (3-1)\ , \qquad increment\ \ i = i + 1}$$

$$\boxed{\boldsymbol{Iteration\ 3}}$$

$$\boldsymbol{Iteration\ 4 : \frac{n}{2} + (4-1)\ , \qquad increment\ \ i = i + 1}$$

$$\boxed{\boldsymbol{Iteration\ 4}}$$

....

$$\boldsymbol{Iteration\ k : \frac{n}{2} + (k-1)\ , \qquad increment\ \ i = i + 1}$$

$$\boxed{\boldsymbol{Iteration\ k}}$$

$$\boldsymbol{And, \frac{n}{2} + (k-1) = n}$$

$\boldsymbol{, as\ n\ is\ the\ upper\ bound\ upto\ which\ loop\ will\ run}$

$$\boldsymbol{\frac{n}{2} + (k-1) = n}$$

$$\boldsymbol{\Longrightarrow \frac{n + 2k - 2}{2} = n}$$

$$\Longrightarrow n + 2k - 2 = 2n$$

$$\Longrightarrow 2k - 2 = n$$

$$\Longrightarrow 2k = n + 2$$

$$\Longrightarrow k = \frac{(n+2)}{2}$$

$$\Longrightarrow k = \frac{n}{2} + \frac{2}{2}$$

$$\Longrightarrow k = \frac{n}{2} + 1$$

$$Outer\ loop\ i\ runs = \frac{n}{2} + 1\ times.$$

$The\ upper\ bound\ of\ j\ \ become = j + \frac{n}{2} \leq n = j \leq n - \frac{n}{2}$

$$= j \leq \frac{2n - n}{2} = j \leq \frac{n}{2}$$

$Hence\ number\ of\ k = k + 1\ prints = \frac{n}{2}\left(\frac{n}{2} + 1\right)$

$$\frac{n^2}{4} + \frac{n}{2} = \frac{n^2 + 2n}{4} = O\left(\frac{n^2 + 2n}{4}\right) = O(n^2)$$

$Hence\ the\ time\ complexity = O\left(n^2\right)$