20.5. TIME COMPLEXITY CALCULATION FOR LOOP (EG-4).

$//outer\ loop\ executed\ \ n\ times$

$for(i = 1; i \leq n; i + 3)\{$

$\qquad k = k + 1\ ;\ //\ constant\ time.$

$\}$

SOLUTION

Here if we notice $i + 3\ is\ increment\ factor\ , where\ 1\ is\ lower\ bound\ and\ n\ is\ lower\ bound.$

$We\ can\ write\ it\ as : c_1\ \times 1 \leq f(n) \leq c \times n\ ,$

$Focusing\ on\ upper\ bound\ or\ Big\ O\ time\ complexity:$

$f(n) \leq c \times n\ , where\ n\ is\ g(n).$

$Note : At\ every\ incement\ of\ i + 3\ , k = k + 1\ prints.$

Hence when:

$f(1) \leq c \times n \Longrightarrow when\ i = 1$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 1 + 3 = 4$

$f(4) \leq c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 4 + 3 = 7$

$f(7) \leq c \times n$

$k = k + 1\ executes\ in\ 1\ unit\ of\ time$

$i\ increments\ 7 + 3 = 10$

....

$Here\ the\ loop\ runs\ either : 1 + (1 + 3) + (4 + 3) + \cdots + n$

$or : 1 + (1 + 3) + (4 + 3) + \cdots + n - 1$

$Here\ we\ will\ use\ \lfloor\ \rfloor\ which\ represents\ floor\ value$:

$.i.e.\ \lfloor 1.23 \rfloor = 1, \lfloor 1 \rfloor = 1, \lfloor 2.26 \rfloor = 2$

WHEN IT RUN UP TO $n\ time.$

$$for\ n = 1\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{1}{3} \right\rfloor + 1\right) = 0 + 1 = 1$$

$$for\ n = 2\ , k = k + 1\ will\ print\ at\ i = \ 1, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{2}{3} \right\rfloor + 1\right) = 0 + 1 = 1$$

$$for\ n = 3\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{3}{3} \right\rfloor + 1\right) = 1 + 1 = 2$$

$$for\ n = 4\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{4}{3} \right\rfloor + 1\right) = 1 + 1 = 2$$

$$for\ n = 5\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{5}{3} \right\rfloor + 1\right) = 1 + 1 = 2$$

$$for\ n = 6\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, 6, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{6}{3} \right\rfloor + 1\right) = 2 + 1 = 3$$

$$for\ n = 7\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, 6, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{7}{3} \right\rfloor + 1\right) = 2 + 1 = 3$$

$$for\ n = 8\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, 6, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{8}{3} \right\rfloor + 1\right) = 2 + 1 = 3$$

$$for\ n = 9\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, 6, 9, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{9}{3} \right\rfloor + 1\right) = 3 + 1 = 4$$

$$for\ n = 10\ , k = k + 1\ will\ print\ at\ i = \ 1, 3, 6, 9, \left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1$$

$$\Rightarrow \left(\left\lfloor \frac{10}{3} \right\rfloor + 1\right) = 3 + 1 = 4$$

$$The\ innermost\ statement\ will\ be\ printed : O\left(\left(\left\lfloor \frac{n}{3} \right\rfloor\right) + 1\right) = O(n)$$

WHEN IT RUN UP TO $n-1$ $time.$

$\boldsymbol{If\ we\ notice\ for\ (n-1)time,}$

$$\boldsymbol{for\ n=1\ ,k=k+1\ will\ print\ at\ i=\ 1,\left(\left\lfloor\frac{n-1}{3}\right\rfloor\right)+1}$$

$$\Rightarrow\left(\left\lfloor\frac{1-1}{3}\right\rfloor+1\right)=\mathbf{0+1=1}$$

$$\boldsymbol{for\ n=2\ ,k=k+1\ will\ print\ at\ i=\ 1,\left(\left\lfloor\frac{n-1}{3}\right\rfloor\right)+1}$$

$$\Rightarrow\left(\left\lfloor\frac{2-1}{3}\right\rfloor+1\right)=\mathbf{0+1=1}$$

$$\boldsymbol{for\ n=3\ ,k=k+1\ will\ print\ at\ i=\ 1,\left(\left\lfloor\frac{n-1}{3}\right\rfloor\right)+1}$$

$$\Rightarrow\left(\left\lfloor\frac{3-1}{3}\right\rfloor+1\right)=\mathbf{0+1=1}$$

$$\boldsymbol{for\ n=4\ ,k=k+1\ will\ print\ at\ i=\ 1,4\left(\left\lfloor\frac{n-1}{3}\right\rfloor\right)+1}$$

$$\Rightarrow\left(\left\lfloor\frac{4-1}{3}\right\rfloor+1\right)=\mathbf{1+1=2}$$

$$\boldsymbol{for\ n=5\ ,k=k+1\ will\ print\ at\ i=\ 1,4\left(\left\lfloor\frac{n-1}{3}\right\rfloor\right)+1}$$

$$\Rightarrow\left(\left\lfloor\frac{5-1}{3}\right\rfloor+1\right)=\mathbf{1+1=2}$$

$$for\ n = 6\ , k = k + 1\ will\ print\ at\ i = \ 1, 4 \left(\left\lfloor \frac{n-1}{3} \right\rfloor\right) + 1$$

$$\Longrightarrow \left(\left\lfloor \frac{6-1}{3} \right\rfloor + 1\right) = 1 + 1 = 2$$

$$for\ n = 7\ , k = k + 1\ will\ print\ at\ i = \ 1, 4, 7 \left(\left\lfloor \frac{n-1}{3} \right\rfloor\right) + 1$$

$$\Longrightarrow \left(\left\lfloor \frac{7-1}{3} \right\rfloor + 1\right) = 2 + 1 = 3$$

$$The\ innermost\ statement\ will\ be\ printed : O\left(\left(\left\lfloor \frac{n-1}{3} \right\rfloor\right) + 1\right) = O(n)$$
