# 20.1. ASYMPTOTIC ANALYSIS

# NESTED FOR LOOP.

Approach:

Finding $\boldsymbol{Big}\ (\boldsymbol{O})\ \boldsymbol{i.e.upto\ n\ times\ run\ of\ the\ particular\ code\ or\ we\ can\ tell\ traverse\ to\ the\ last}$ .

$$\boldsymbol{for(i = 1; i \leq n; i++)\{}$$

$$\boldsymbol{for(j = 1; i \leq n; i++)\{}$$

$$\boldsymbol{c = c + 1;}$$

$$\}$$

$$\}$$

**SOLUTION:**

$\boldsymbol{1. Inner\ most\ loop's\ statement \Longrightarrow c = c + 1\ which\ runs\ at\ O(1)\ time.}$

$\boldsymbol{2. No. of\ inputs\ in\ outer\ for\ loop\ takes\ 1\ to\ n\ times.}$

$\boldsymbol{lets\ see\ the\ inner\ loop\ and\ runtime\ of}$

$inner\ loop's\ statement.$

$$f(1) \leq c \times n \Rightarrow when\ i = 1$$

$$f(1) \leq c \times n \Rightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Rightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Rightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$\dots \dots \dots .$

$$f(n) \leq c \times n \Rightarrow when\ j = n$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e.\ when\ i = 1\ ,the\ inner\ most\ loop\ statement$$

$$run\ (1 + 1 + 1 + 1 + \cdots + n) = n\ times$$

$$f(2) \leq c \times n \Rightarrow when\ i = 2$$

$$f(1) \leq c \times n \Rightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Rightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Rightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$\dots \dots \dots .$

$$f(n) \leq c \times n \Rightarrow when\ j = n$$

$$c = c + 1 \; runs \; 1 \; unit \; of \; time.$$

$$i.e. \; when \; i = 2 \;, the \; inner \; most \; loop \; statement$$

$$run \; (1 + 1 + 1 + 1 + \cdots + n) = n \; times$$

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$$f(n) \leq c \times n \Rightarrow when \; i = n$$

$$f(1) \leq c \times n \Rightarrow when \; j = 1$$

$$c = c + 1 \; runs \; 1 \; unit \; of \; time.$$

$$f(2) \leq c \times n \Rightarrow when \; j = 2$$

$$c = c + 1 \; runs \; 1 \; unit \; of \; time.$$

$$f(3) \leq c \times n \Rightarrow when \; j = 3$$

$$c = c + 1 \; runs \; 1 \; unit \; of \; time.$$

... ... ....

$$f(n) \leq c \times n \Rightarrow when \; j = n$$

$$c = c + 1 \; runs \; 1 \; unit \; of \; time.$$

$$i.e. \; when \; i = n \;, the \; inner \; most \; loop \; statement$$

$$run \; (1 + 1 + 1 + 1 + \cdots + n) = n \; times$$

We can add n to n times $[n + n + n + \cdots + n]$gives $n^2$.

$Eg: if \; n = 3 \; \Rightarrow if \; we \; add \; 3 \; times \; 3 = 3 + 3 + 3 = 9 = 3^2$

$Hence\ , we\ can\ tell\ that:$

$upper\ bound\ of\ 1st\ loop = g(n) = n$

$\times$

$upper\ bound\ of\ 2nd\ \ loop = g(n) = n$

$$= O\left(n^2\right)$$

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