PROPERTIES OF BIG-THETA

The following are the properties of the big – theta notation:

1. $\boldsymbol{If\ f(n) = \text{O}\big(g(n)\big)\ and\ g(n) = \text{O}\big(\mathrm{f}(\mathrm{n})\big)}$ $\mathbf{, then\ f(n) = \Theta(n)}$.

2. If we deduce the equation of Big Theta $\mathbf{\Theta(n)}$:

$$\boldsymbol{c_1 g(n) \le f(n) \le c_2 g(n)\ \ for\ all\ n \ge n_0}$$

We get:

- $\boldsymbol{c_1 g(n) \le f(n)\ or\ f(n) \ge c_1 g(n), for\ all\ n \ge n_0\ is\ \Omega(n)}$.
- $\boldsymbol{f(n) \le c_2 g(n), for\ all\ n \ge n_0\ is\ \text{O}(n)}$.

Hence both upper bound and lower bound must exists to have Theta that is tight upper bound and tight lower bound must exist to produce tight bound.

3. For any polynomial of the order of $\boldsymbol{k}$, one can show that

$$\boldsymbol{f(n)\ is\ in\ \Theta\big(n^k\big)}.$$

We can see the third point through an example:

Example: Show $log(n!) = \Theta(nlogn)$

Solution:

$$n! = (n-0) \times (n-1) \times (n-2) \ldots\ldots n$$

$$= (n-0) \times (n-1) \times (n-2) \ldots\ldots (n-(n-1)) \; [$$

$Note: (n-(n-1)\,) = n]$

$$= n\left(1-\frac{0}{n}\right) \times \left(1-\frac{1}{n}\right) \times \left(1-\frac{2}{n}\right) \ldots\ldots \left(1-\left(1-\frac{1}{n}\right)\right)$$

$$= n^n \prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)$$

Therefore,

$$log(n!) = log\left(n^n \prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)\right)$$

$$= log(n^n) + log\left(\prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)\right)$$

$$= nlogn + log\left(\prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)\right)$$

Now, $f(n) = nlogn + log\left(\prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)\right)$

We can simply tell that, the $f(n)$ *is in* $\Theta(nlogn\ )$

Thus , asymptotic notations are helpful is representing the order of growth of an algorithm .

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