20.23. WHEN THREE NESTED FOR LOOP $\neq O(n^3)$

LET'S TAKE AN EXAMPLE:

$for(i = 1; i \leq n ; i++)\{$

$for(j = 1; j \leq i^2 ; j++)\{$

$for\left(k = 1; k \leq \frac{n}{2} ; k++\right)\{$

$c = c + 1;$

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SOLUTION

OUTER LOOP RUNS 1 TO N, J LOOP RUNS:

$i = 1 , j = 1$

$i = 2 , j = 2^2 = 4$

$i = 3 , j = 3^2 = 9$

$\ldots n^2\ times,$

$$And\ for\ every\ loop\ of\ j\ k\ runs \frac{n}{2}\ i.e.$$

$$j = 1\ , k = 1\ to\ \frac{n}{2}\ \ , c = c + 1 = \sum_{i=1}^{\frac{n}{2}} 1 = \frac{n}{2}$$

$$j = 4\ , k = 1\ to\ \frac{n}{2}\ , c = c + 1 = \sum_{i=1}^{\frac{n}{2}} 1 = \frac{n}{2}$$

....

$$j = n^2\ , k = 1\ to\ \frac{n}{2}\ , c = c + 1 = \sum_{i=1}^{\frac{n}{2}} 1 = \frac{n}{2}$$

$$Hence\ we\ can\ formulate:$$

$$1 \times \frac{n}{2} + 4 \times \frac{n}{2} + 9 \times \frac{n}{2} + \cdots + n^2 \times \frac{n}{2}$$

$$\Rightarrow \frac{n}{2}(1 + 4 + 9 + \cdots + n^2)$$

$$\Rightarrow \frac{n}{2} \times \sum_{n=1}^{n} n^2$$

$$\Rightarrow \frac{n}{2} \times \frac{2n^3 + 3n^2 + n}{6}$$

$$\Rightarrow \frac{2n^4 + 3n^3 + n^2}{12}$$

$$O\left(\frac{2n^4 + 3n^3 + n^2}{12}\right) = O(n^4)$$

HENCE IT IS PROVED THAT 3 FOR LOOP DOES NOT REPRESENTS $O(n^3)\ always.$