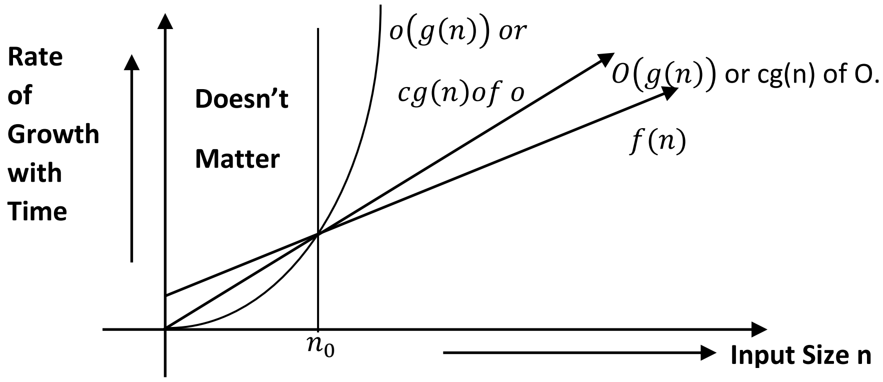
10. LITTLE – OH NOTATION



DEFINITION: Let f and g be two functions that map a set of natural numbers to a set of positive real numbers, $f: \mathbb{N} \longrightarrow \mathbb{R}$.

Let $o(g)$be the set of all functions with a similar rate of growth.

The relation $f(n) = o(g(n))$ holds good, if there exist two positive constants c and n_0 such that $f(n) < c \times g(n)$.

Some points over little-oh notion:

- The little-oh notion is used very rarely.
- Here the value of c is very small.
- The $little - oh$ notation can be used instead of the big-Oh notation as the little-oh notation represents a ***loose upper bound***.

LITTLE OH DEFINITION IN LIMITS - LITTLE OH RATIO THEOREM

DEFINITION: The function $f(n) = o\big(g(n)\big)\ if\ \lim_{n\to\infty}\frac{f(g)}{g(n)} = 0,$

$$which\ implies\ that\ f(n) = o\big(g(n)\big).$$

EXAMPLES OF LITTLE OH

Example

$$Let\ f(n) = 7n + 6. Show\ that\ f(n)\ is\ in\ o(n^2).$$

Solution

As we know, $f(n) = o(n^2)\ as\ \lim_{n\to\infty}\frac{7n+2}{n^2}$

$$\lim_{n\to\infty}\frac{7n+2}{n^2}$$

$$\lim_{n\to\infty}\frac{7n}{n^2}+\frac{2}{n^2}$$

$$\lim_{n\to\infty} \frac{7}{n} + \frac{2}{n^2}$$

$$As\ per\ infinity\ theory\ of\ limit\ \lim_{n\to\infty} \frac{c}{x^a} = 0$$

$$\lim_{n\to\infty} \frac{7}{n} + \lim_{n\to\infty} \frac{2}{n^2}$$

$$0 + 0$$

0

Therefore, we can say that $7n + 6\ \in o(n^2)$.

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