20.23. LOG N COMPLEXITY

EXAMPLE 1

$for(i = 1; i \leq n\,; i = i * 1)\{$

$c = c + 1;$

$\}$

ANSWER

This is a perception building as we know the increment will go from 1 *to* n *as* $1^n = 1$.

$1^n \Longrightarrow\ log_1(1) = n = 1\ i.e\ \log_a a = 1$

Similarly,

$for(i = 1; i \leq n\,; i = i * 2)\{$

$c = c + 1;$

$\}$

ANSWER

$Here\ c = c + 1\ prints\ 1\ time\ , then\ i\ will\ be\ 1 \times 2 = 2\ ,$

$then\ \ 2 \times 2 = 4\ , 4 \times 2 = 8 \ldots till\ 2^k \geq n$

$i.e. number\ of\ iteration\ will\ be\ 2^k\ but\ the\ iteration$

$will\ not\ be\ greater\ than\ that\ of\ n.$

i	$i \times 2$	n
1	$1 \times 2 = 1$	n
2	$2 \times 2 = 4$	n
$\ldots..$	$\ldots\ldots$	$\ldots..$
2^{k-1}	$2^{k-1} \times 2^k = 2^k \geq$	n

THEREFORE,

$2^k \geq n$

$k = log_2 n\ or\ \log n$

$Where\ k\ is\ number\ of\ iterations\ that\ will\ run$

$upto\ n.$

$Now\ say\ n\ =\ 5:$

$k = log_2 5 = 2.3$

BUT THE ACTUAL ITERATIONS THAT WE HAVE IS:

$1\ , 2, 4\ i.e. 3\ times.$

HENCE EITHER WE DO: $\lfloor \log n \rfloor + 1\ or\ \lceil \log n \rceil$

$floor(logn) + 1\ or\ ceil(logn)$

HENCE NUMBER OF ITERATIONS:

$\lfloor \log n \rfloor + 1\ or\ \lceil \log n \rceil$

AND COMPLEXITY STANDS AS:

$$O(\lfloor \log n \rfloor + 1)\ or\ O(\lceil \log n \rceil) = O(log\ n)$$

EXAMPLE 2

$for(i = 1; i \leq n\ ; i = i * 3)\{$

$c = c + 1;$

$\}$

$Number\ of\ iterations$: $\lfloor \log_3 n \rfloor + 1\ or\ \lceil \log_3 n \rceil$

COMPLEXITY:

$$O(\lfloor \log_3 n \rfloor + 1)\ or\ O(\lceil \log_3 n \rceil) = O(log\ n)$$