20.8. TIME COMPLEXITY CALCULATION NESTED FOR LOOP (EG-7).

//*outer loop executed n times*

$for(i = 1; i \leq n; i + +)\{$

  //*inner loop executes n times*

 $for(j = 1; j \leq n/2; j + +)\{$

    $k = k + 1 ; //$ *constant time*.

}

SOLUTION:

1. *Inner most loop's statement* $\Rightarrow k = k + 1$ *which runs at* $O(1)$ *time*.

2. *No. of inputs in outer for loop takes* 1 *to n times*.
lets see the inner loop and runtime of
inner loop's statement.

$f(1) \leq c \times n \Rightarrow when\ i = 1$

$\qquad f(1) \leq c \times n \Rightarrow when\ j = 1$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad f(2) \leq c \times n \Rightarrow when\ j = 2$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad f(3) \leq c \times n \Rightarrow when\ j = 3$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad ... ...$

$\qquad f\left(\dfrac{n}{2}\right) \leq c \times n \Rightarrow when\ j = \dfrac{n}{2}$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$i.e.when\ i = 1\ ,the\ inner\ most\ loop\ statement$

$$run\ \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

$f(2) \leq c \times n \Rightarrow when\ i = 2$

$\qquad f(1) \leq c \times n \Rightarrow when\ j = 1$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad f(2) \leq c \times n \Rightarrow when\ j = 2$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad f(3) \leq c \times n \Rightarrow when\ j = 3$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$\qquad ... ...$

$\qquad f\left(\dfrac{n}{2}\right) \leq c \times n \Rightarrow when\ j = \dfrac{n}{2}$

$\qquad c = c + 1\ runs\ 1\ unit\ of\ time.$

$i.e.when\ i = 2\ ,the\ inner\ most\ loop\ statement$

$$run\ \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

$.....$

$f(n) \leq c \times n \Rightarrow when\ i = n$

$$f(1) \leq c \times n \Rightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Rightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Rightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$\dots\ \dots\ \dots$$

$$f(n-1) \leq c \times n \Rightarrow when\ j = n - 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e.\ when\ i = n\ ,the\ inner\ most\ loop\ statement$$

$$run\ \left(1 + 1 + 1 + 1 + \cdots + \frac{n}{2}\right) = \frac{n}{2}\ times$$

Therefore printing the inner most statement $(k = k + 1)$

$$n\ times\ \frac{n}{2} = \frac{n}{2} + \frac{n}{2} + \cdots + \frac{n}{2} = n\left(\frac{n}{2}\right) = \frac{n^2}{2}\ times\ ,$$

$$hence\ O\left(\frac{n^2}{2}\right) = O(n^2)$$

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