20.1. ASYMPTOTIC ANALYSIS

NESTED FOR LOOP.

Approach:

Finding $Big\ (O)\ i.e. upto\ n\ times\ run\ of\ the\ particular$ $code\ or\ we\ can\ tell\ traverse\ to\ the\ last\ .$

$$for(i = 1; i \leq n; i++)\{$$
$$\quad for(j = 1; i \leq n; i++)\{$$
$$\quad\quad c = c + 1;$$
$$\quad \}$$
$$\}$$

SOLUTION:

$1. Inner\ most\ loop's\ statement \Rightarrow c = c + 1\ which\ runs\ at$ $O(1)\ time.$

$2. No. of\ inputs\ in\ outer\ for\ loop\ takes\ 1\ to\ n\ times.$
$lets\ see\ the\ inner\ loop\ and\ runtime\ of$
$inner\ loop's\ statement.$

$$f(1) \le c \times n \Rightarrow when\ i = 1$$

$$f(1) \le c \times n \Rightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \le c \times n \Rightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \le c \times n \Rightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

… … ….

$$f(n) \le c \times n \Rightarrow when\ j = n$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = 1\ , the\ inner\ most\ loop\ statement$$

$$run\ (1 + 1 + 1 + 1 + \cdots + n) = n\ times$$

$$f(2) \le c \times n \Rightarrow when\ i = 2$$

$$f(1) \le c \times n \Rightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \le c \times n \Rightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \le c \times n \Rightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

… … ….

$$f(n) \le c \times n \Rightarrow when\ j = n$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = 2\ , the\ inner\ most\ loop\ statement$$

$$run\ (1 + 1 + 1 + 1 + \cdots + n) = n\ times$$

.....

$$f(n) \leq c \times n \Longrightarrow when\ i = n$$

$$f(1) \leq c \times n \Longrightarrow when\ j = 1$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(2) \leq c \times n \Longrightarrow when\ j = 2$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$f(3) \leq c \times n \Longrightarrow when\ j = 3$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

… … ….

$$f(n) \leq c \times n \Longrightarrow when\ j = n$$

$$c = c + 1\ runs\ 1\ unit\ of\ time.$$

$$i.e. when\ i = n\ , the\ inner\ most\ loop\ statement$$

$$run\ (1 + 1 + 1 + 1 + \cdots + n) = n\ times$$

We can add n to n times $[n + n + n + \cdots + n]$gives n^2.

$$Eg: if\ n = 3\ \Longrightarrow if\ we\ add\ 3\ times\ 3 = 3 + 3 + 3 = 9 = 3^2$$

$$T(n) = \sum_{i=1}^{n} 1 \times \sum_{j=1}^{n} 1 = \sum_{i=1}^{n} n = [n + n + n + \cdots + n] = n^2$$

$$Hence\ , we\ can\ tell\ that:$$

$$upper\ bound\ of\ 1st\ loop = g(n) = n$$

$$\times$$

$$upper\ bound\ of\ 2nd\ \ loop = g(n) = n$$

$$= O(n^2)$$

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