21.A .SOME MORE EXAMPLES OF FOR LOOP TIME COMPLEXITIES

$\boldsymbol{Suppose\ we\ have:}$

$\boldsymbol{p = 0;}$

$\boldsymbol{for\ (i = 1; p < n; i++)\{}$

$\boldsymbol{k = k + 1; --statement}$

$\boldsymbol{p = p + i;}$

$\}$

SOLUTION:

THEN WHAT IS THE UPPER BOUND HERE?

$\boldsymbol{p}$	$\boldsymbol{i}$	$\boldsymbol{n}$	$\boldsymbol{i++}$	$\boldsymbol{p+i}$
0	1	n	2	$0+1$
1	2	n	3	0+1+2
3	3	n	4	0+1+2+3
….	…	…	….	…
$0+1+2+3+\cdots+k-2$	k-1	n	k	0+1+2+3+…+ k-2+k-1

$$\boldsymbol{p = 0 + 1 + 2 + 3 + \cdots + k - 2 + k - 1}$$

$$\boldsymbol{\Rightarrow 1 + 2 + 3 + \cdots + k - 1 + k}$$

$$\boldsymbol{p} = 1 + 2 + 3 + \cdots + k - 1 + k = \frac{k(k+1)}{2} \leq n$$

$$\Rightarrow \frac{k(k+1)}{2} \leq n$$

$$\Rightarrow \frac{k^2 + k}{2} \leq n$$

$$\Rightarrow k^2 + k \leq 2n$$

$$\Rightarrow k^2 + k - 2n \leq 0$$

By quadratic equation:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -2n}}{2}$$

$$\Rightarrow \frac{-1 \pm \sqrt{1 + 8n}}{2}$$

WE GET:

$$\frac{-1 + \sqrt{1 + 8n}}{2} \quad and \quad \frac{-1 - \sqrt{1 + 8n}}{2}$$

AND THE GENERAL TERM UP TO WHICH $(\boldsymbol{k} = \boldsymbol{k} + \mathbf{1})$***will get printed***:

$$\textit{ceil of } \left\lceil \frac{-1+\sqrt{1+8n}}{2} \right\rceil$$

$$\boldsymbol{Hence\ time\ complexity\ is}\ \sqrt{\boldsymbol{n}} = \boldsymbol{O}\left(\sqrt{\boldsymbol{n}}\right)$$