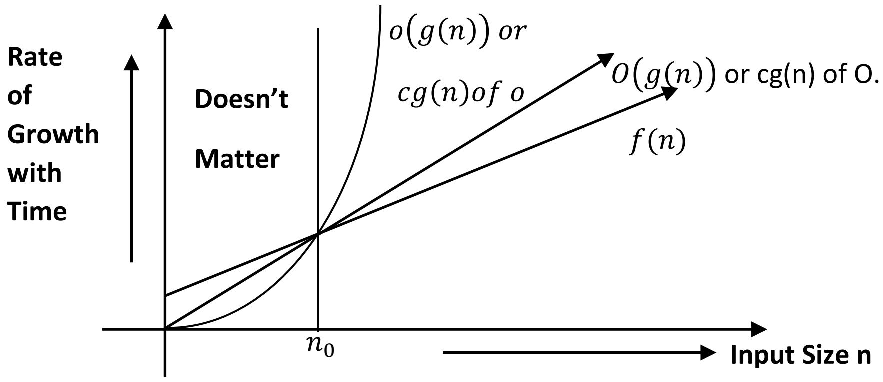
# 10. LITTLE – OH NOTATION




**DEFINITION:** Let $f$ *and* $g$ be two functions that map a set of natural numbers to a set of positive real numbers, $f: \mathbb{N} \longrightarrow \mathbb{R}$.

Let $o(g)$ be the set of all functions with a similar rate of growth.

The relation $f(n) = o(g(n))$ holds good, if there exist two positive constants $c$ *and* $n_0$ such that $f(n) < c \times g(n)$, for all n $> n_0$.

## Some points over little-oh notion:

- The little-oh notion is used very rarely.
- Here the value of $c$ is very small.
- The $little - oh$ notation can be used instead of the big-Oh notation as the little-oh notation represents a ***loose upper bound***.

# LITTLE OH DEFINITION IN LIMITS - LITTLE OH RATIO THEOREM

**DEFINITION:** The function $f(n) = o\big(g(n)\big)\ if\ \lim_{n\to\infty} \frac{f(g)}{g(n)} = 0,$

$$which\ implies\ that\ f(n) = o\big(g(n)\big).$$

# EXAMPLES OF LITTLE OH

## Example 1

$$Let\ f(n) = 7n + 6. Show\ that\ f(n)\ is\ in\ o(n^2).$$

### Solution

As we know, $f(n) = o(n^2)\ as\ \lim_{n\to\infty} \frac{7n+2}{n^2}$

$$= \lim_{n\to\infty} \frac{7n+2}{n^2}$$

$$= \lim_{n\to\infty} \frac{7n}{n^2} + \frac{2}{n^2}$$

$= \lim_{n \to \infty} \frac{7}{n} + \frac{2}{n^2}$

$As\ per\ infinity\ theory\ of\ limit\ \lim_{n \to \infty} \frac{c}{x^a} = 0$

$= \lim_{n \to \infty} \frac{7}{n} + \lim_{n \to \infty} \frac{2}{n^2}$

$= 0 + 0$

$= 0$

Therefore, we can say that $7n + 6 \in o(n^2)$.

## Example 2

$$Let\ f(n) = 7n + 6. Show\ that\ f(n)\ is\ in\ o(n^2).$$

[ Without Limit]

## Solution

$7n + 6 < c \times n^2$

One can write it as:

$f(n) < 7n^2 + 6n^2$
$f(n) < 13n^2$

Now c = 13

$\Longrightarrow 13n^2 - 7n - 6 < 0$

$\Longrightarrow 13n^2 + 6n - 13n - 6 < 0$

$\Longrightarrow 13n^2 + 6n - 13n - 6 < 0$

$\Longrightarrow n(13n + 6) - 1(13n + 6) < 0$

$\Longrightarrow (13n + 6)(n - 1) < 0$

$$= 13n + 6 < 0$$
$$= 13n < -6$$
$$= n < -\frac{6}{13}$$

$$= n - 1 < 0$$
$$= -1 < -n$$
$$= n > 1$$

**Hence, we got $n > 1$ *and* $c = 13$**

Now if we see it through a table

| N | $7n + 6$ | $13n^2$ |
|---|---|---|
| 2 | 20 | 52 |
| 3 | 27 | 117 |
| 4 | 34 | 208 |
| 5 | 41 | 325 |

Hence, we can tell $c \times g(n)$ *is loosely bound and* $f(n) = o(n^2)$.

## NOW COMPARE IF IT IS TIGHTLY UPPER BOUND $O(n)$ THEN:

$\Longrightarrow f(n) \leq 7n + 6n = 13n$
Therefore
$\Longrightarrow 7n + 6 \leq 13n$
$\Longrightarrow 6 \leq 6n$
$\Longrightarrow 1 \leq n$
$\Longrightarrow n \geq 1$

| N | $f(n) = 7n + 6$ | $g(n) = 13n$ |
|---|---|---|
| 1 | **13** | **13** |
| 2 | 20 | 26 |
| 3 | 27 | 39 |
| 4 | 34 | 52 |
| 5 | 41 | 65 |

Hence, we see $O(n)$ *is tightly upper bound other than* $o(n)$*which is loosely upper bound and graph to see the difference* .

****************