20.19. TIME COMPLEXITY CALCULATION NESTED FOR LOOP (MORE THAN TWO LOOP).

EXAMPLE 5

$$
\begin{aligned}
&for(i = 1; i \leq n; i++)\{\\
&\qquad for(j = 1; j \leq i; j++)\{\\
&\qquad\quad for(k = 1; k \leq i; k++)\{\\
&\qquad\qquad\quad for(l = 1; l \leq i; l++)\{\\
&\qquad\qquad k = k + 1;\\
&\qquad\quad \}\\
&\qquad \}\\
&\}
\end{aligned}
$$

ANSWER

$\boldsymbol{f(1) \leq n \text{ , } where\ i = 1}$

$\boldsymbol{f(1) \leq i \text{ , } where\ i = 1, j = 1}$

$\boldsymbol{f(1) \leq i, where\ i = 1, k = 1}$

$\boldsymbol{f(1) \leq i, where\ i = 1 \text{ , } l = 1}$

$\mathrm{c} = \mathrm{c} + \mathbf{1} \text{ , } ------\rightarrow (\mathbf{1})$

$$\boldsymbol{T(n) = \sum_{i=1}^{1} 1 = 1}$$

$\boldsymbol{f(2) \leq n \text{ , } where\ i = 2}$

$\boldsymbol{f(1) \leq i \text{ , } where\ i = 2, j = 1}$

$\boldsymbol{f(1) \leq i, where\ i = 2, k = 1}$

$\boldsymbol{f(1) \leq i, where\ i = 2, l = 1}$

$\mathrm{c} = \mathrm{c} + \mathbf{1} \text{ , } ------\rightarrow (\mathbf{1})$

$\boldsymbol{f(2) \leq i, where\ i = 2, l = 2}$

$\mathrm{c} = \mathrm{c} + \mathbf{1} \text{ , } ------\rightarrow (\mathbf{1})$

$$\boldsymbol{T(n) = \sum_{i=1}^{2} 1 = 2}$$

$\boldsymbol{f(2) \leq i, where\ i = 2, k = 2}$

$\boldsymbol{f(1) \leq i, where\ i = 2, l = 1}$

$\mathrm{c} = \mathrm{c} + \mathbf{1} \text{ , } ------\rightarrow (\mathbf{1})$

$\boldsymbol{f(2) \leq i, where\ i = 2, l = 2}$

$\mathrm{c} = \mathrm{c} + \mathbf{1} \text{ , } ------\rightarrow (\mathbf{1})$

$$\boldsymbol{T(n) = \sum_{i=1}^{2} 1 = 2}$$

$$f(2) \leq i \text{ , where } i = 2, j = 2$$

$$f(1) \leq i, \text{where } i = 2, k = 1$$

$$f(1) \leq i, \text{where } i = 2, l = 1$$

$$c = c + 1 \text{ , } ------\rightarrow (1)$$

$$f(2) \leq i, \text{where } i = 2, l = 2$$

$$c = c + 1 \text{ , } ------\rightarrow (1)$$

$$T(n) = \sum_{i=1}^{2} 1 = 2$$

$$f(2) \leq i, \text{where } i = 2, k = 2$$

$$f(1) \leq i, \text{where } i = 2, l = 1$$

$$c = c + 1 \text{ , } ------\rightarrow (1)$$

$$f(2) \leq i, \text{where } i = 2, l = 2$$

$$c = c + 1 \text{ , } ------\rightarrow (1)$$

$$T(n) = \sum_{i=1}^{2} 1 = 2$$

$$T(n) = \sum_{i=1}^{2} 1 + \sum_{i=1}^{2} 1 + \sum_{i=1}^{2} 1 + \sum_{1=1}^{2} 1 = 2 + 2 + 2 + 2 = 8$$

$$\text{Hence it goes like } 1 + 8 + 27 + \cdots + n^3 = \sum_{n=1}^{n} n^3$$

$$\textit{And we know that} \sum_{n=1}^{n} n^3 = 1 + 8 + 27 + \cdots + n^3 =$$

$$= \frac{n^4 + 2n^3 + n^2}{4} = O(n^4)$$

$$\textit{Deduction of} \sum_{n=1}^{n} n^3 = (1^3 + 2^3 + 3^3 + \cdots + n^3) = \left(\frac{n(n+1)}{2}\right)^2$$

Solution:

$$\sum_{n=1}^{n} n^2 = \left(1^3 + 2^3 + 3^3 + \cdots + n^3\right)$$

Lets take : Binomial Series of Expansion

$$(a+b)^n = n_{C_0} a^n b^0 + n_{C_1} a^{n-1} b^1 + n_{C_2} a^{n-2} b^2 + \cdots + n_{C_n} a^0 b^n$$

Now if we take: $(n+1)^4$

$$= 4_{C_0} \times n^4 \times 1^0 + 4_{C_1} \times n^3 \times 1^1 + 4_{C_2} \times n^2 \times 1^2 + 4_{C_3} \times n^1 \times 1^3 + 4_{C_4} \times n^0 \times 1^4$$

$$= 1 \times n^4 \times 1 + 4 \times n^3 \times 1 + 6 \times n^2 \times 1 + 4 \times n^1 \times 1 + 1 \times 1 \times 1$$

$$= n^4 + 4n^3 + 6n^2 + 4n + 1$$

$$= (n+1)^4 - n^4 = n^4 + 4n^3 + 6n^2 + 4n + 1 - n^4 = 4n^3 + 6n^2 + 4n + 1$$

Now, putting $n = 1, 2, 3, 4, 5, \ldots, n-1, n$ *and Adding it we get*:

$\Rightarrow (1+1=2)^4 - 1^4 = 4\times 1^3 + 6\times 1^2 + 4\times 1 + 1$

Hence,

$2^4 - 1^3 = 4\times 1^3 + 6\times 1^2 + 4\times 1 + 1$

$3^4 - 2^4 = 4\times 2^3 + 6\times 2^2 + 4\times 2 + 1$**[As we add** 2^4 *and* 2^4 *gets cancelled***]**

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$n^4 - (n-1)^4 = 4\times (n-1)^3 + 6\times (n-1)^2 + 4\times (n-1) + 1$

$(n+1)^4 - n^4 = 4\times n^3 + 6\times (n)^2 + 4\times (n) + 1$**[As we add** n^4 *and* n^4 *gets cancelled***]**

$$(n+1)^4 - 1^4 = 4(1^3 + 2^3 + \cdots + (n-1)^3 + n^3) + 6(1^2 + 2^2 + \cdots + (n-1)^2 + n^2) + 4(1+2+3+\cdots+n) + (1\times n = n)$$

$$(n+1)^4 - 1^4 = 4\times \sum_{n=1}^{n} n^3 + 6\times \sum_{n=1}^{n} n^2 + 4\times \sum_{n=1}^{n} n + n$$

$$\Rightarrow 4\times \sum_{n=1}^{n} n^3 = (n+1)^4 - 1^4 - 6\times \sum_{n=1}^{n} n^2 - 4\times \sum_{n=1}^{n} n - n$$

$$\Rightarrow 4\times \sum_{n=1}^{n} n^3 = (n+1)^4 - 1^4 - \left(6\times \frac{2n^3+3n^2+n}{6}\right) - \left(4\times \left(\left(\frac{n^2+n}{2}\right)\right)\right) - n$$

$$\Rightarrow 4\times \sum_{n=1}^{n} n^3 = (n+1)^4 - 1^4 - (2n^3+3n^2+n) - \left(2(n^2+n)\right) - n$$

$$\Rightarrow 4\times \sum_{n=1}^{n} n^3 = n^4 + 4n^3 + 6n^2 + 4n + 1 - 1 - (2n^3+3n^2+n) - (2n^2+2n) - n$$

$$\Rightarrow 4 \times \sum_{n=1}^{n} n^3 = n^4 + 4n^3 + 6n^2 + 4n - \left(2n^3 + 3n^2 + n\right) - \left(2n^2 + 2n\right) - n$$

$$\Rightarrow 4 \times \sum_{n=1}^{n} n^3 = n^4 + 4n^3 + 6n^2 + 4n - 2n^3 - 3n^2 - n - 2n^2 - 2n - n$$

$$\Rightarrow 4 \times \sum_{n=1}^{n} n^3 = n^4 + 2n^3 + n^2$$

$$\Rightarrow \sum_{n=1}^{n} n^3 = \frac{n^4 + 2n^3 + n^2}{4}$$

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