

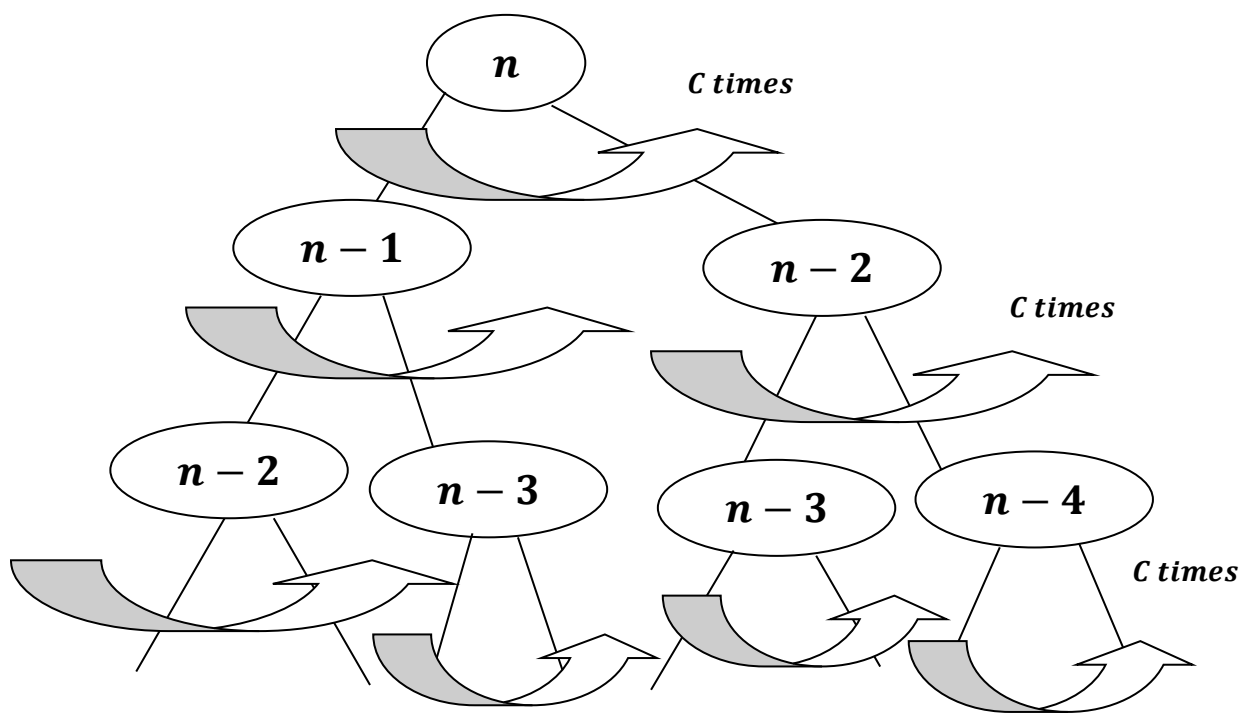
Fibonacci Series Time Complexity with Recursion Tree

*In fibonacci series : $T(n) = T(n - 1) + T(n - 2)$, where `n` is greater than 1 ,
when `n` = 0 , $T(n) = 0$ and $T(n) = 1$, when `n` = 1, these are base cases for
fibonacci series.*

$$T(n) = \begin{cases} 0 & \text{if}(n = 0) \\ 1 & \text{if}(n = 1) \\ T(n - 1) + T(n - 2) & \text{if}(n > 1) \end{cases}$$

To start with: –

If the recursion goes from `n` to `1` times, considering every nodes, we get:



| <i>Level</i> | <i>No. of problems</i> | <i>Problem size</i> | <i>Work Done</i> |
|---------------------|-------------------------------|-------------------------------|-----------------------------------|
| 0 | <i>c</i> | 1 | <i>c</i> |
| 1 | <i>c</i> | 2 | <i>2c</i> |
| 2 | <i>c</i> | 4 | <i>4c</i> |
| . | . | . | . |
| . | . | . | . |
| <i>n - 1</i> | <i>c</i> | <i>2ⁿ⁻¹</i> | <i>c × 2ⁿ⁻¹</i> |

Here `c` is constant.

Hence we have assumed : $T(n - 1) \approx T(n - 2)$ for previous problem , because of the same growth as shown in recurrence tree,

Now we can see the growth = $c + 2c + 4c + \dots c \times 2^{n-1}$

We can rewrite it as

$$: 2^0 \times c + 2^1 \times c + 2^2 \times c + \dots + c \times 2^{n-1}$$

$$\Rightarrow c\{1 + 2 + 2^2 + \dots + 2^{n-1}\}$$

if we see it, $1 + 2 + 2^2 + \dots$ is in geometric finite series:

$$\Rightarrow \frac{x^{n+1} - 1}{x - 1} = \frac{2^{n-1+1} - 1}{2 - 1} = 2^n - 1 = O(2^n - 1) = O(2^n) \text{ is time complexity.}$$
