$\textbf{The Handshake Problem}$
$\textbf{–Time Complexity}$

$\boldsymbol{As\ we\ see\ the\ the\ program\ ,the\ return\ value\ is}$:

$\boldsymbol{return\ handshake(n-1)+(n-1)};$

$\boldsymbol{Hence\ ,handshake(n-1)\ \Rightarrow T(n-1)}$.

$\boldsymbol{And\ now\ come\ to\ the\ base\ case}$:

$\boldsymbol{i.e. when\ 0\ person\ or\ 1\ person\ in\ the\ room\ returns\ 0\ handshakes}$.

$\boldsymbol{Hence\ base\ case\ always\ \ runs\ once\ after}$ \`$\boldsymbol{n}$-1\` $\boldsymbol{times\ recursion\ is\ executed\ hence}$,

$$\boldsymbol{T(n)\ \ =T(n-1)+1}$$

$\boldsymbol{Here\ 1\ is\ added\ for\ base\ case}$.

$$Solution\ for\ the\ above\ recurrence\ relation$$

$If\ we\ apply\ substitution\ method$:

$$T(n) = T(n-1) + 1$$

$$\therefore T(n-1) = T(n-1-1) + 1 = T(n-2) + 1$$

$$\therefore T(n-2) = T(n-2-1) + 1 = T(n-3) + 1$$

$Hence\ if\ it\ runs\ upto\ `k`\ times\ , we\ get$:

$$\therefore T(n-k) = T(n-k-1) + 1$$

$We\ can\ substitute\ them\ as$:

$$T(n) = T(n-1) + 1$$
$$= T\big((n-1)(n-2)\big) + 1$$
$$= T\big((n-1)(n-2)(n-3)\big) + 1$$

$\boldsymbol{upto\ `i`\ we\ get}$:

$$= T\big((n-1)(n-2)(n-3)\ + \cdots + (n-i)\ \big) + 1$$

$\boldsymbol{Let\ i = n-1\ and\ then\ , we\ will\ have}$:

$$= T\left((n-1)(n-2)(n-3) + \cdots + \big(n-(n-1)\big)\right) + 1$$

$$= T\big((n-1)(n-2)(n-3) + \cdots + (1)\big) + 1$$

$$or, T\big((1)(2)(3) + \cdots + (n-1)\big) + 1$$

$$= \sum_{n=1}^{n-1} n + 1$$

$\boldsymbol{By\ arithmetic\ progression\ we\ get}$:

$$\Rightarrow \sum_{n=1}^{n} n = \frac{1}{2}\ n(n+1)\ , hence:$$

$$\Rightarrow \sum_{n=1}^{n-1} n = \frac{1}{2}(n-1)\big((n-1)+1\big) = \frac{1}{2} n \times (n-1)$$

$$= \frac{n(n-1)}{2} + 1$$

$$= \frac{n^2 - n}{2} + 1$$

$$= O\left(\frac{n^2 - n}{2}\right) + O(1)$$

$$= O\left(\frac{n^2 - n}{2}\right)$$

$$= \frac{1}{2} \times O(n^2 - n)$$

$$= O(n^2)$$

By Recurrence Tree

$T(n) = T(n-1) + a$ *, where a is constant.*

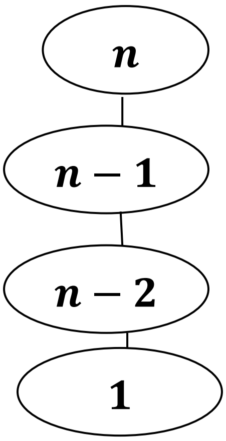
As it runs n times , we can say it will be :

$T(n) = T(n-1) + n$

And $n = 1$ *then* $T(n)$ *or* $T(1) = 1$, *i.e. runs at constant time.*

Hence,

$$T(n) = \begin{cases} 1 & , for\ n = 1 \\ T(n-1) + n & , for\ n > 1 \end{cases}$$



$Level$	$No. of$ $problems$	$Problem$ $size$	$Work$ $Done$
0	1	n	n
1	1	$n-1$	$n-1$
2	1	$n-2$	$n-2$
.	.	.	.
.	.	.	.
$n-1$	1	1	1

$$\Longrightarrow n + (n-1) + (n+2) + \cdots + 1 = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$= O\left(\frac{n^2 - n}{2}\right)$$

$$= O\left(\frac{n^2 - n}{2}\right)$$

$$= \frac{1}{2} \times O(n^2 - n)$$

$$= O(n^2)$$

… ….