Infix to Postfix Conversion

To start with we all have functionaries:

```
typedef struct Stack
{
    int top;
    int size;
    char *s;

} Stack;
```

Creation of Stack

```
void create(Stack *st, int cap)
{
    st->size = cap;
    st->top = -1;
    st->s = (char *)malloc(st->size * sizeof(char));
}
```

Check IsEmpty of Stack

```
int isEmpty(Stack st)
{
    if (st.top == -1)
    {
        return 1;
    }
    return 0;
}
```

Checking Stack is full

```
int isFull(Stack st)
{
    if (st.top == st.size - 1)
    {
        return 1;
    }
    return 0;
}
```

Pushing Element in Stack

```
void push(Stack *st, char x)
{
    if (isFull(*st))
    {
        cout << "Stack Overflow" << endl;
    }
    else
    {
        st->top++;
        st->s[st->top] = x;
    }
}
```

Poping Element in Stack

```
char pop(Stack *st)
{
    if (isEmpty(*st))
    {
        cout << "Stack Underflow" << endl;
        return '\0';
    }
    return st->s[st->top--];
}
```

As we are dealing with characters , hence return will be a character.

Suppose we have arithmetic expression:

$$(a + b) * (c\hat{}d)/e$$

1) ***At first it creates the stack.***

```
char *InToPost(char *infix)
{
    Stack st;
    int len = strlen(infix);
       // Calculate the length of the string
    char *postfix = (char *)malloc((len + 1) *
sizeof(char)); // Allocate the memory for postfix
string
    create(&st, len);
}
```

2) ***Now from priority checker it will return*** **0** ***for*** (.

```
int pre(char x)
{
  …….
    else if (x == '(' || x == ')')
    {
        return 0;
    }
  ….
}
```

3) *Now if its an operator and it must not come at first as we doing infix to postfix conversion.*

That is the expression is:

$(a + b) * (c\hat{}d)/e$

And isOperand() function takes a character in its parameter:

```
int isOperand(char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/'
|| x == '(' || x == ')' || x == '^' || x == '%')
    {
        return 0;
    }
    return 1;
}
```

if not a character but an operator then return 0 or false, otherwise 1 or true.

And as it returns 0 , it will skip the if part . If will run when the function returns 1 (true):

```
char *InToPost(char *infix)
{
      …..
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)//Skipped
        {
         …..
         }
     }
}
```

Hence it will now enter the else part:

```
char *InToPost(char *infix)
{
     int i = 0, j = 0;
      …..
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)//Skipped
        {
         …..
         }
        else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {
                push(&st, infix[i]);
                i++;
            }
```

```
                ……. .


    }
}
```

$$pre(infix[infix[0]]) = pre(\text{`(`}) = 0 > pre(st.s[st.top])$$

$$= -1 \text{ is true or } isEmpty(st) = true \text{ also } infix[0] = \text{`(`} \text{ is true.}$$

$$push \rightarrow \text{`(`} . And\ i++ = i + 1 = 0 + 1 = 1.$$

$$And\ also\ now\ top = top + 1 = -1 + 1 = 0.$$

Stack

(	→ $Top = 0$

4) *Now we have character `a`*

$$i = 1 \text{ and } j = 0$$

```
char *InToPost(char *infix)
{
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
            postfix[j] = infix[i];
            j++;
            i++;
```

```
        }
  ….
}
```

Now character `a` will return 1 (true) i.e. we have an operand and its type is character:

```
int isOperand(char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/'
|| x == '(' || x == ')' || x == '^' || x == '%')
    {
        return 0;
    }
    return 1;
}
```

As we return 1 if statements runs:

```
if (isOperand(infix[i])==1)
        {
            postfix[j] = infix[i];
            j++;
            i++;
        }
```

$$postfix\,[0] = infix[1] = a$$

$$j++ = j + 1 = 0 + 1 = 1$$

$$i = i + 1 = 1 + 1 = 2$$

Hence postfix formation: a

5) ***we have operator*** `+`

Hence now again the operand function will return **0**.

```
int isOperand(char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/'
|| x == '(' || x == ')' || x == '^' || x == '%')
    {
        return 0;
    }
    return 1;
}
```

And then it runs the else part:

```
char *InToPost(char *infix)
{
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
         ….
         }
         else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {
                push(&st, infix[i]);
                i++;
            }
         ….
}
```

$$pre(infix[infix[2]]) = pre(`+`) = 1 > pre(st.s[st.top])$$

$$= pre(st.s[0]) = pre(`(`) = 0 \; is \; true \; .$$

$$push \rightarrow `+` \; is \; true. And \; i++ = i + 1 = 2 + 1 = 3.$$

$$And \; also \; now \; top = top + 1 = \; 0 + 1 = 1.$$

Stack

(
+

→ $Top = 1$

6) ***we have operand `b`***

$$i = 3, j = 1$$

```
char *InToPost(char *infix)
{
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
            postfix[j] = infix[i];
            j++;
            i++;
        }
  ……
}
```

Now character `b` will return 1 (true) i.e. we have an operand

***and its type is character*:**

```
int isOperand(char x)
{
    if (x == '+' || x == '-' || x == '*' || x == '/'
|| x == '(' || x == ')' || x == '^' || x == '%')
    {
        return 0;
    }
    return 1;
}
```

$$postfix\,[1] = infix[3] = b$$

$$j++ = j + 1 = 1 + 1 = 2$$

$$i = i + 1 = 3 + 1 = 4$$

***Hence postfix formation*: ab**

7) *Now we have operator i.e.* `)`

***Now logic is we will not push*) *rather we will pop out the element* ` + ` *in stack and again pop out the opening brace* `)` .

Now 1st pop will continue less than* `(` *opening brace.

```
char *InToPost(char *infix)
{
    ….

    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
             ….;
        }
        else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {
                 ….;
            }
                 ---------------------
                  //It will run here
                   -------------------
            else if (pre(infix[i]) <=
pre(st.s[st.top]) && infix[i] == ')')
            {
                while (st.s[st.top] != '(')
                {
                    postfix[j] = pop(&st);
                    j++;
                }
                pop(&st);
                i++;
            }
….
}
```

$\boldsymbol{isOperand(infix[4] = `)`) = 0[False], hence\ else\ part\ runs.}$

$\boldsymbol{pre(infix[4] = `)`) = 0 \leq pre(s[top] = s[1] = ` + `)}$
$\boldsymbol{= 1\ is\ true\ and\ infix[4] = `)`}$

$\boldsymbol{is\ true.}$

$\boldsymbol{Then:}$

$\boldsymbol{While(top\ !=[not\ equal\ to]\ '('){}}$

$\boldsymbol{postfix[2] = pop()\ = ` + `;}$

$\boldsymbol{j++;}$

}

$\boldsymbol{j = j + 1 = 2 + 1 = 3\ and\ top = top - 1 = 1 - 1 = 0}$

Therefore, after pop function stack we have:

Stack

(	$\boldsymbol{\rightarrow Top = 0}$

$\boldsymbol{Now, again\ pop() \rightarrow `(`}$

$\boldsymbol{This\ pop\ function\ will\ make\ stake\ empty:}$

$\boldsymbol{top = top - 1 = 0 - 1 = -1}$

Stack

	$\boldsymbol{\rightarrow Top = -1}$

$$\textbf{And then } i++ = i + 1 = 4 + 1 = 5$$

$$\textbf{Hence postfix formation: } ab +$$

8) we have operator `*`

```
 while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
         …..;
        }
        else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {
                push(&st, infix[i]);
                i++;
            }
          …..
}
```

$$\textbf{isOperand}(\textbf{infix}[5] = `*`) = 0[\textbf{False}], \textbf{hence else part runs.}$$

$$\textbf{prefix}(\textbf{infix}[5] = `*`) = 2 > \textbf{pre}(\textbf{st.s}[\textbf{st.top}]) = -1$$

$$\textbf{push} \rightarrow *$$

Stack

*	→ $Top = 0$

$$top = top + 1 = -1 + 1 = 0 \; and \; i = i + + = 5 + 1 = 6$$

9) Now we can relate how (c^d)will be processed.

1st it will push (

Stack

*
(

$\rightarrow Top = 1$

$$top = top + 1 = 0 + 1 = 1 \; and \; i = i + 1 = 6 + 1 = 7.$$

2nd for c it will be put inside posfix array:

$$postfix[3] = infix[7] = c$$

$$i = i + 1 = 7 + 1 = 8$$

$$j = j + 1 = 3 + 1 = 4$$

Hence postfix formation: $ab + c$

3rd for ^ , it will again will be pushed inside the stack

Stack

*
(
^

$\rightarrow Top = 2$

$$top = top + 1 = 1 + 1 = 2 \; and \; i = i + 1 = 8 + 1 = 9.$$

$\boldsymbol{4rth\ for\ d\ it\ will\ be\ put\ inside\ posfix\ array}$:

$\boldsymbol{postfix[4] = infix[9] = d}$

$\boldsymbol{i = i + 1 = 9 + 1 = 10}$

$\boldsymbol{j = j + 1 = 4 + 1 = 5}$

$\boldsymbol{Hence\ postfix\ formation: ab + cd}$

$\boldsymbol{5th\ for}$ `(` :

Everything will get popped out from stack less than

`(` :

$\boldsymbol{postfix[j = 5] = pop()}$ = `^` ;

$\boldsymbol{top = top - 1 = 2 - 1 = 1}$

$\boldsymbol{j = j + 1 = 5 + 1 = 6}$

Stack

*
(

$\rightarrow \boldsymbol{Top = 1}$

And then `(` ***will be popped out***

$\boldsymbol{top = top - 1 = 1 - 1 = 0}$

Stack

*

$\rightarrow \boldsymbol{Top = 0}$

$i = i + 1 = 10 + 1 = 11$

Hence postfix formation: $ab + cd\hat{}$

10)*Now if we see the Stack` * ` is left to be popped out. Now there is no parenthesis left in the arithmetic expression. Hence, there must be a condition which will pop out the left over of stack and help in pushing the operator in stack when there is no parenthesis in the expression* :

```
char *InToPost(char *infix)
{
   ……
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {…..;}
        else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {….;}
            else if (pre(infix[i]) <=
pre(st.s[st.top]) && infix[i] == ')')
            {….;}
```

```
            else
            {

                while (pre(infix[i]) <=
pre(st.s[st.top]) && !isEmpty(st) && st.s[st.top] !=
'(' && st.s[st.top] != ')')
                {

                    postfix[j] = pop(&st);
                    j++;
                }
                push(&st, infix[i]);
                i++;
            }
           …..;

}
```

If you see the whole arithmetic expression:

$$(a + b) * (c\hat{}d)/e$$

Hence not only we have to pop $*$ but also must push $/$.

$$isOperand(infix[i] = infix[11] = /)$$
$$= 0 \text{ hence move to else part.}$$

$$pre(infix[i] = infix[11] = /) = 2 \leq pre(st.s[st.top]$$

$$= st.s[0] = * = 2) = true$$

$$!\, isEmpty(st) = true$$

And top of the stack doesnot contain (and) parenthesis.

$postfix[j = 6] = pop()\ = *$

$j++ = j + 1 = 6 + 1 = 7$

$Hence\ postfix\ formation: ab + cd\hat{}\ *$

$top = top - 1 = 0 - 1 = -1\ i.e.\ empty$

$Now\ coming\ out\ from\ the\ loop\ ,we\ get:$

$Push \rightarrow /\ \ into\ the\ stack.$

$top = \ -1 + 1 = 0$

Stack

/	→ $Top = 0$

$i = i++ = i + 1 = 11 + 1 = 12$

11) ***Now we have `e` left for assigning it to array.***

```
char *InToPost(char *infix)
{
    while (infix[i] != '\0')
    {
        if (isOperand(infix[i])==1)
        {
            postfix[j] = infix[i];
            j++;
            i++;
        }
  ….
```

```
}
```

$Now\ isOperand(infix[i] = infix[12]) = 1(true) will\ enter\ if's\ body.$

$postfix[j = 7] = infix\ [i = 12] = e$

$Hence\ postfix\ formation: ab + cd\hat{} * e$

$j = j + 1 = 7 + 1 = 8$

$i = i + 1 = 12 + 1 = 13$

12) *Now one element left in the stack to be popped out i.e./*

```
char *InToPost(char *infix)
{
   …..
    while (infix[i] != '\0')
    {
    if (isOperand(infix[i]) == 1)
        {…}

   else
        {
            if (pre(infix[i]) > pre(st.s[st.top]) ||
isEmpty(st) || infix[i] == '(')
            {…}
```

```
 else if (pre(infix[i]) <= pre(st.s[st.top]) &&
infix[i] == ')')
            {…}

     else
            {
                while (pre(infix[i]) <=
pre(st.s[st.top]) && !isEmpty(st) && st.s[st.top] !=
'(' && st.s[st.top] != ')')
                {….}
            }
        }
    }

//What ever left in the stack will be popped out.

while (!isEmpty(st))
    {
        postfix[j] = pop(&st);

        j++;
    }

    postfix[j] = '\0';

    free(st.s);

    return postfix;
}
```

we have already finished elements in i to be compared:

i.e. what ever left in the stack now, will be popped out till its empty.

$Postfix[j = 8] = pop() =/$

$top = top - 1 = 0 - 1 = -1.$

i.e. stack is empty.

$j = j + 1 = 9$

And 9th element will have a null terminator '\0'.

It implies we have finished our post fix conversion or end of string. Null teminator marks the end of the string .

$Hence, postfix[j = 9] = '/0'$.

Hence postfix formation: $ab + cd\hat{} * e/$

Now, free the stack memory $= free(st.s)$ *and return the post fix expression.*

Now, question may arise on:

```
                while (pre(infix[i]) <=
pre(st.s[st.top]) && !isEmpty(st) && st.s[st.top] !=
'(' && st.s[st.top] != ')')
                {

                    postfix[j] = pop(&st);
                    j++;
                }
                push(&st, infix[i]);
                i++;
            }
```

***Now consider three arithmetic expression*:**

1. $(a+b)/(c-b)+e$

2. $(a+b)+(c-b)+e$

3. $(a+b)+(c-b)/e$

Consider 1st expression :

we know how $(a+b)$ ***and*** $(c-b)$ ***will be executed***

***The stack remains*:**

/	$\rightarrow Top = 0$

And we have $ab+cb-$ ***as half converted postfix expression*.**

Now we will have a comparison between $+$ ***and*** $/$.

$pre(infix[i=11]='+') = 1 \leq pre(st[s.top]='/')$
$= 2$ ***is true*.**

***The above loop runs based on this condition*(*as this satisfy the requirement*).**

Consider 2nd expression :

we know how $(a+b)$ ***and*** $(c-b)$ ***will be executed***

The stack remains:

$+$	$\rightarrow Top = 0$

And we have $ab+cb-$ ***as halfconverted postfix expression***.

Now we will have a comparison between $+$ ***and*** $/$.

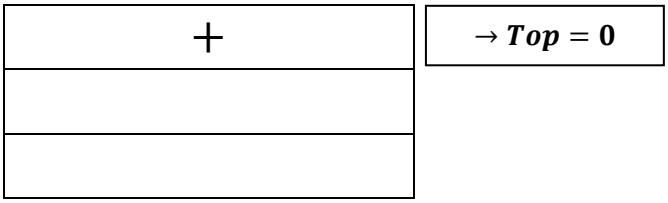
$pre(infix[i=11] = '+') = 1 \leq pre(st[s.top] = '+') = 1$ ***is true***.

The above loop runs based on this condition(***as this satisfy the requirement***)

Consider 3rd expression :

we know how $(a+b)$ ***and*** $(c-b)$ ***will be executed***

The stack remains:



And we have ab + cb − as halfconverted postfix expression.

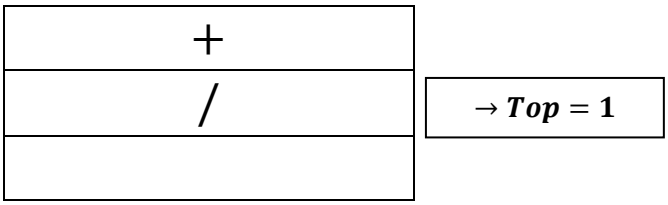
Now we will have a comparison between + and /.

$$pre(infix[i = 11] = '/') = 2 \leq pre(st[s.top] = '+') = 1 \text{ is false.}$$

What it will do is : it will push the next element in the stack :

```
if (pre(infix[i]) > pre(st.s[st.top]) || isEmpty(st)
|| infix[i] == '(')
            {
                push(&st, infix[i]);
                i++;
            }
```

Now stack will be :



After the postfix operation we will get:

```
if (isOperand(infix[i]) == 1)
        {
            postfix[j] = infix[i];
            j++;
            i++;
        }
```

$$ab + cb - e$$

And then we have finished elements in i to be compared:

Hence everything will be popped out one by one:

1. $pop \rightarrow /$

```
while (!isEmpty(st))
    {
        postfix[j] = pop(&st);

        j++;
    }
```

+

$\rightarrow Top = 0$

***Hence post fix expression now*: $ab + cb - e/$**

2. ***pop*** $\rightarrow$ $+$

```
while (!isEmpty(st))
    {
        postfix[j] = pop(&st);

        j++;
    }
```


$\rightarrow Top = -1$

***Hence post fix expression now*: $ab + cb - e/+$**

Analysis of Algorithm

After analysing the workings of the program we can generate the algorithmic structure :

Infix To Postfix

Algorithm

Step 1: Add) to the end of the infix expression

Step 2: Push (on to the stack

Step 3: Repeat until each character in the infix notation is scanned

If a (is encountered, push it on the stack.

If an operand (whether a digit or a character) is encountered, add it postfix expression.

If a) is encountered, then:

a. Repeatedly pop from stack and add it to the postfix expression until a (is encountered.

b. Discard the (. That is, remove the (from stack and do not add it to the postfix expression.

If an operator is encountered,

a. Repeatedly pop from stack and add each operator (popped from the stack) to the

postfix expression which has the same precedence or a higher precedence than

b. Push the operator to the stack

[***END OF IF***]

Step* 4: *Repeatedly pop from the stack and add it to the postfix expression until the stack is empty

Step* 5: *EXIT

Time Complexity Analysis

For push and pop we know it will be $O(1)$ complexity .

But for *postfix character addition in an array.*

***We see each character is added 1 time consuming 1 unit of time*:**

```
postfix[j] = infix[i];
postfix[j] = pop(&st);
```

if there is no parenthesis it takes n unit time to complete.

if 1 parenthesis then it takes $n-1$ unit of time to complete.

if 2 parenthesis then it takes $n-2$ unit of time to complete.

if k no. of parenthesis then it takes $n-k$ unit of time to complete, where k is constant .

Also if we compare it with memory = postfifix takes a single unit of memory and runs upto n times or $n-k$ times to generate a postfix expression.

Hence post fix take $O(n)$times complexity.

$\boldsymbol{Operation}$	$\boldsymbol{Time\ Complexity}$
$\boldsymbol{isEmpty}()$	$\boldsymbol{O(1)}$
$\boldsymbol{isFull}()$	$\boldsymbol{O(1)}$
$\boldsymbol{isOperand}()$	$\boldsymbol{O(1)}$
$\boldsymbol{ispre}()/\ \boldsymbol{ispriority}()$	$\boldsymbol{O(1)}$
$\boldsymbol{push}()$	$\boldsymbol{O(1)}$
$\boldsymbol{pop}()$	$\boldsymbol{O(1)}$
$\boldsymbol{infix-postfix}$	$\boldsymbol{O(n)}$

Space Complexity Analysis

if we have k no. of characters and in which we have n number of operators.

We know when we add anything in array it takes $\boldsymbol{O(1)}$,
i. e. same unit of space used to input data in the auxiliary space created for array.

Where as n times the stack is used for push operation consuming n space complexity.
Hence if we sum up then we get: $\boldsymbol{O(1) + O(n) = O(n)}$
space complexity.