Probability – I

Random Experiment

Random Experiment is an experiment in which: -

i) **All the outcomes of the experiment are known in advance.**

ii) **The exact outcome of any specific performance of the experiment is unpredictable i.e. not known in advance.**

Sample Space

A sample space, denoted by 'S' associated with a random experiment is a set of points such that:

i) **Each element of 'S' denotes an outcome of the experiment,**

ii) **Any performance of the experiment results in an outcome that corresponds to exactly one element of 'S'.**

In other words, sample space consists of all possible outcomes that associated with the random experiment.

Suppose a random experiment: -

1) **<u>Tossing a coin:</u>**
Outcome: 2 possible outcomes 'H', 'T'.

Where H stands for head and T stands for tail.

Therefore, let Sample space be 'S'

And S= {H, T}.

2) **Six-faced die thrown observing the number of possible outcomes: -**
i.e. either 1,2,3,4,5,6.

Then: S= {1,2,3,4,5,6}.

3) **In the random experiment of a coin and die tossed together: -**

Note: - A coin has two outcomes.

S= {H, T}

A die has 6 outcomes:

S= {1,2,3,4,5,6}

Now if a coin and a die are tossed together then it will have: $6 \times 2 = 12\ outcomes$.

$$S = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}$$

We can also write it as: -

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

4) Find the sample space associated with the experiment of rolling pair of dice once.
Also find the element of the sample space.

Ans:
A pair of dice is 2 dice at once.

1st die's possible outcome: -

$$S = \{1, 2, 3, 4, 5, 6\}$$

2nd die's possible outcome: -

$$S = \{1, 2, 3, 4, 5, 6\}$$

Therefore, simply no. of possible outcome when two dice rolls over:

$$6 \times 6 = 36\ outcomes.$$

Together we may denote the outcome by an ordered pair (x, y) where x appears on first die and y appears on the second die.

Then, the sample space is given by: -

$$S = \{(x, y); x, y\ \epsilon\ \{1, 2, 3, 4, 5, 6\}\}$$

Written in set builder form.

	1	2	3	4	5	6
1	$(1,1)$	$(1,2)$	$(1,3)$	$(1,4)$	$(1,5)$	$(1,6)$
2	$(2,1)$	$(2,2)$	$(2,3)$	$(2,4)$	$(2,5)$	$(2,6)$
3	$(3,1)$	$(3,2)$	$(3,3)$	$(3,4)$	$(3,5)$	$(3,6)$
4	$(4,1)$	$(4,2)$	$(4,3)$	$(4,4)$	$(4,5)$	$(4,6)$

5	$(5,1)$	$(5,2)$	$(5,3)$	$(5,4)$	$(5,5)$	$(5,6)$
6	$(6,1)$	$(6,2)$	$(6,3)$	$(6,4)$	$(6,5)$	$(6,6)$

5) An experiment consists of recording boy-girl composition of families with 2 children.

i) What is the sample space if we are interested in knowing whether the elder child is a boy or girl?

ii) What is the sample space if we are interested in knowing whether it is a boy or a girl in the order of their births?

iii) What is the sample space if we are interested in number of girls in the family?

Solution

i) The elder child may either be a boy or girl. Hence, there are only a two possible outcomes.

The sample space is:

$$S = \{Boy, Girl\}$$

Or,

$$S = \{B, G\}$$

ii) As there are 2 children, there are 4 possible outcomes : -

$$\boldsymbol{Boy - Boy, Boy - Girl, Girl - Boy, Girl - Girl.}$$

Hence Sample space is:

$$\boldsymbol{S = \{BB, BG, GB, GG\}}$$

iii) As we are interested in knowing only the number of girls, there are 3 possible outcomes:

$$\boldsymbol{-no\ girl, 1\ girl, 2\ girl.}$$

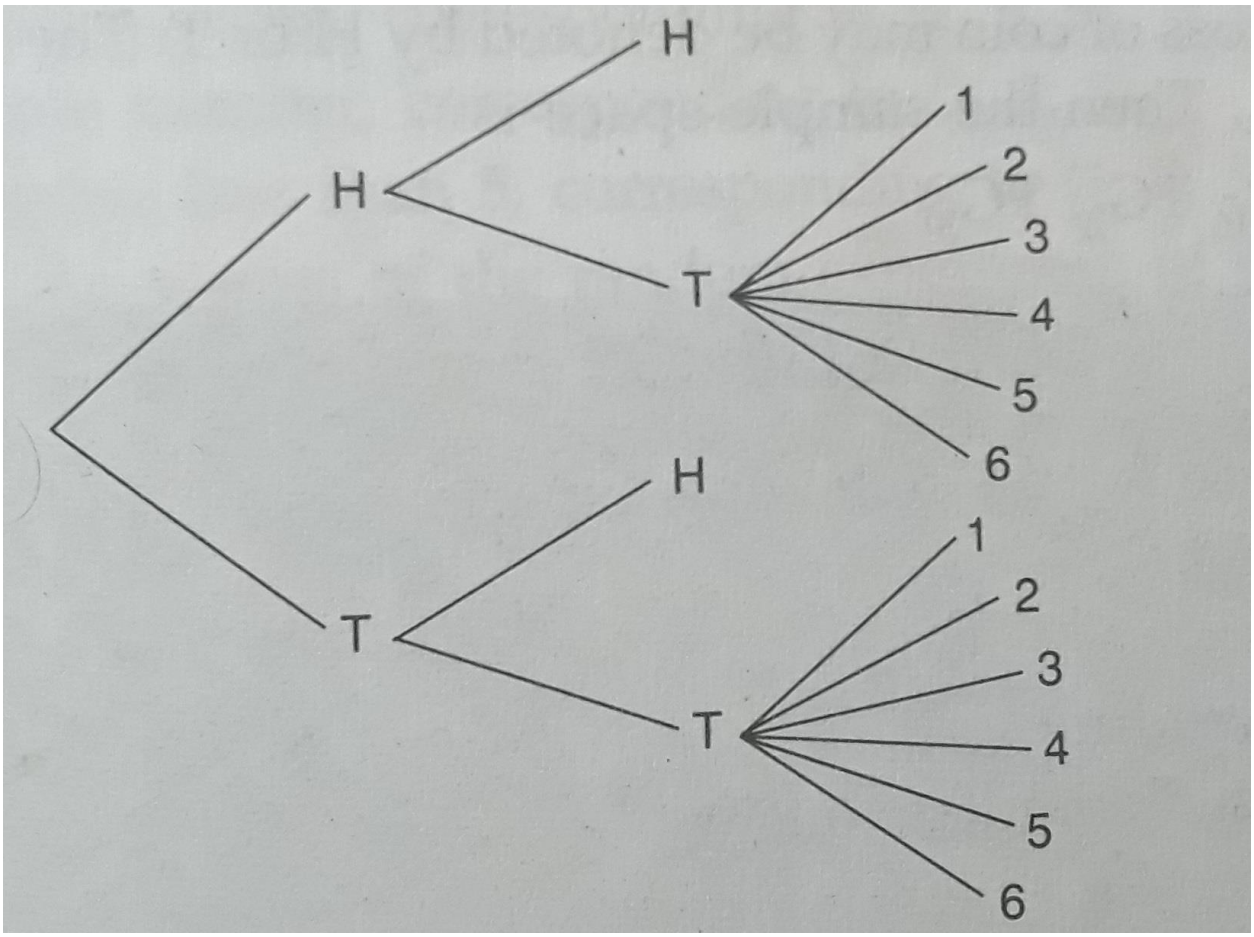
$$\boldsymbol{Hence\ Sample\ Space\ is: -}$$

$$\boldsymbol{S = \{0, 1, 2\}}$$

6. A coin is tossed twice. If the second throw results in a tail, we throw a die. Describe the sample space. How many outcomes are possible in this experiment?

Ans:

We can draw the following diagram:



Hence the sample space is:

$$\boldsymbol{S = \{HH, HT1, HT2, HT3, HT4, HT5, HT6, TH, TT1, TT2, TT3, TT4, TT5, TT6\}}$$

There are 14 possible outcomes.

7. A coin is tossed. If the result is a head, a die is thrown. If the die shows up the numbers 1 or 3, the die is thrown again and if it shows the number 5, then a coin is tossed.

Write the sample space of this experiment. How many outcomes possible in this experiment?

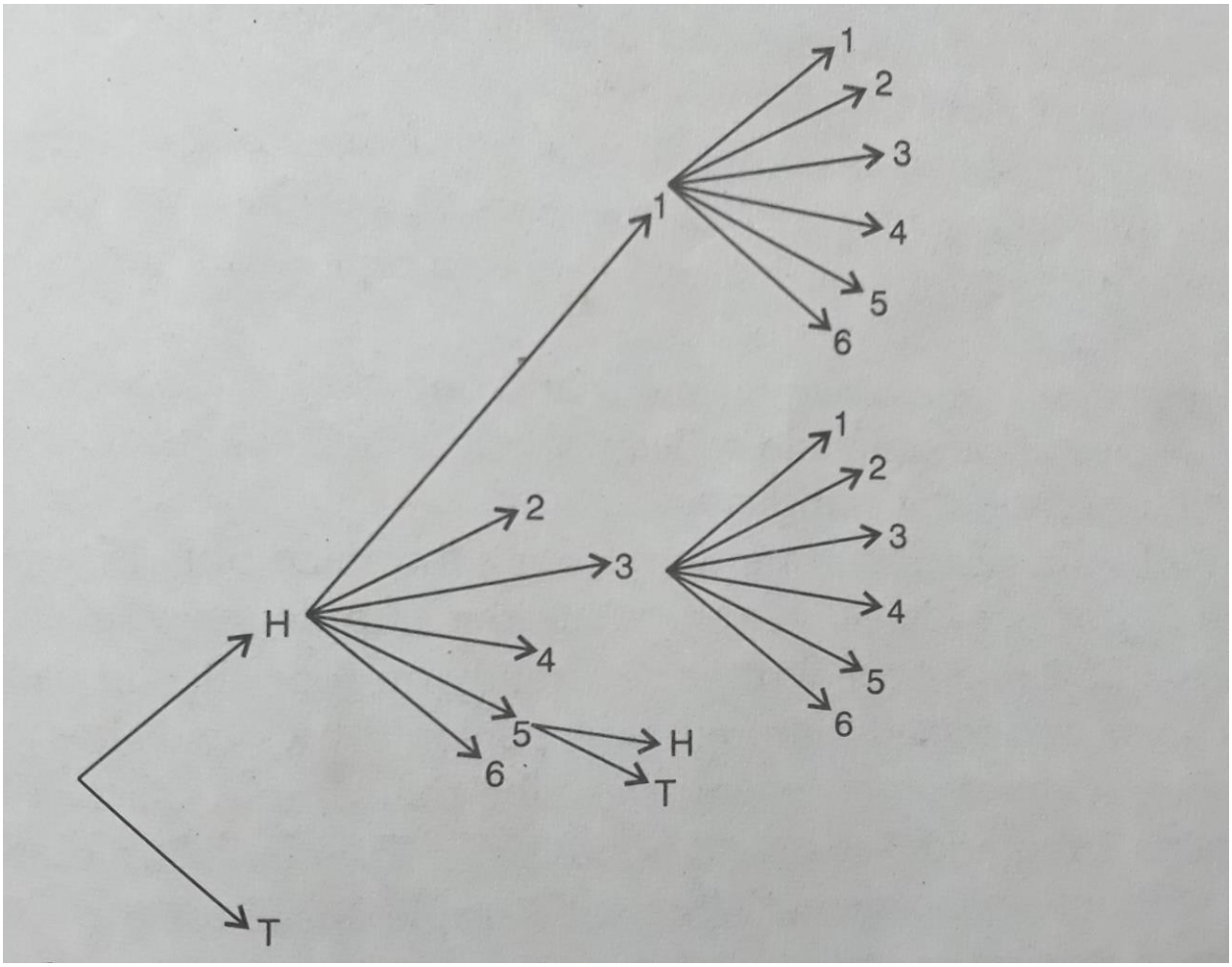
Solution:

1. **A coin is tossed, if the outcome is a tail (T) . The experiment is over.**

2. **If the outcome is a head (H), a die is thrown.**

a. **If the die shows up an even number i.e. 2,4,6 the experiment is over.**

b. **If the die shows up the numbers 1 or 3 the die is thrown again and if it shows up the number 5, a coin is tossed.**



Here is the sample space is:

$\boldsymbol{S = \{T, H2, H4, H6, H11, H12, H13, H14, H15, H16, H31,}$

$\boldsymbol{H32, H33, H34, H35, H36, H5H, H5T\}}$

Here are 18 possible outcomes.

8) We wish to choose one out of 2 boys and 3 girls. A coin is tossed. If it comes up heads, a boy is chosen otherwise a girl is chosen.

Describe the sample space.

Solution: The outcome of toss of coin may be denoted by H or T. The boys may be called B_1, B_2 and the girls G_1, G_2, G_3.

Then the sample space is: -

$$S = \{HB_1, HB_2, TG_1, TG_2, TG_3\}$$