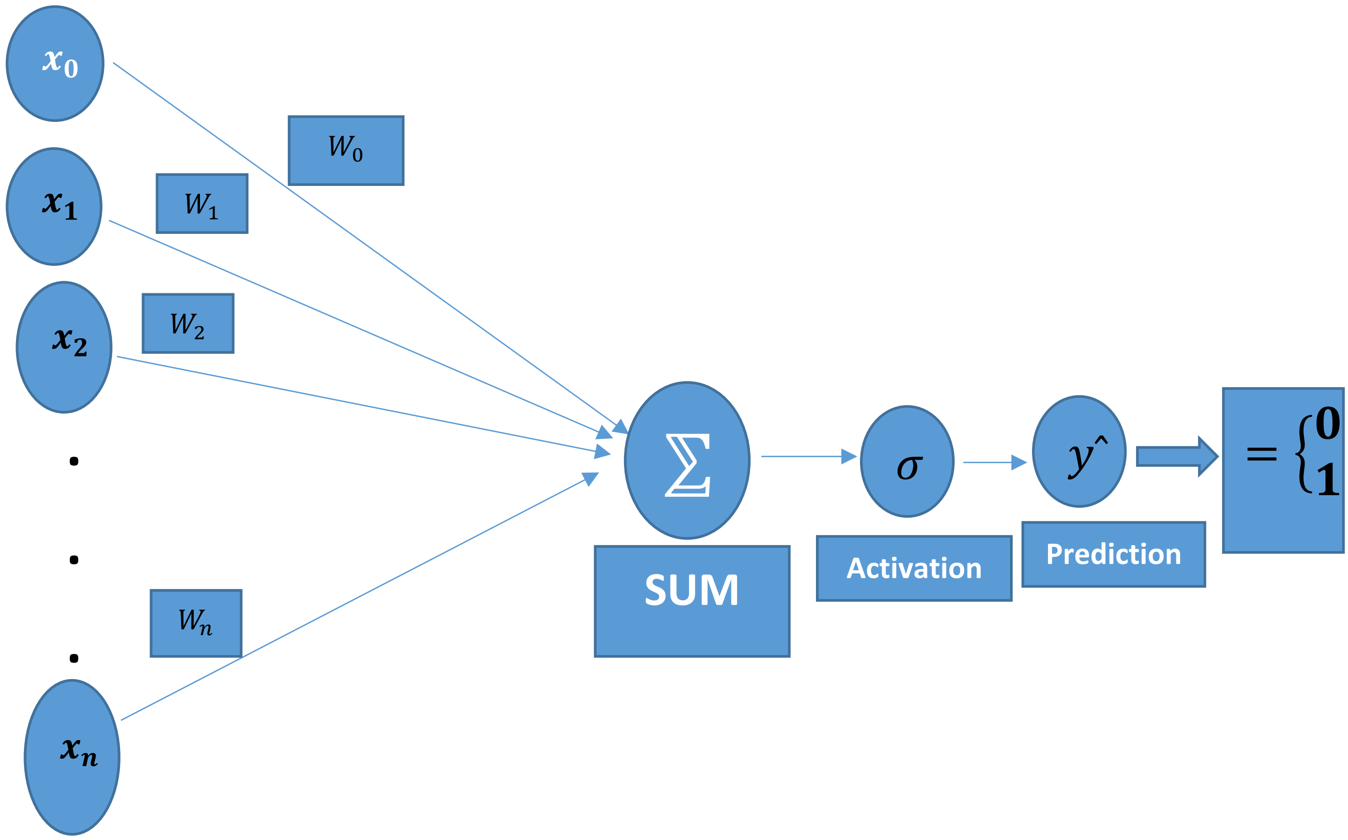
Sigmoid Function

Note in Linear regression model or in straight line we got formula as:

$\boldsymbol{y = mx + c,}$

Notably in Logistic regression model it is a binary classification model of Artificial Intelligence.

Where,



$w_1, w_2, .., w_n$ *are weights and* $x_1, x_2, .., x_n$ *are inputs that are multpilied* according to the weights given like:

$$y^{\wedge} = \sigma(x) = \sigma(w_0 x_0 + w_1 x_1 + \cdots + w_n x_n)$$

The sigmoid function

$$\sigma(x) = \frac{1}{1 + e^{-x}} - eqn(i)$$

or,

$$=> \sigma(x) = \frac{1}{1 + \frac{1}{e^x}}$$

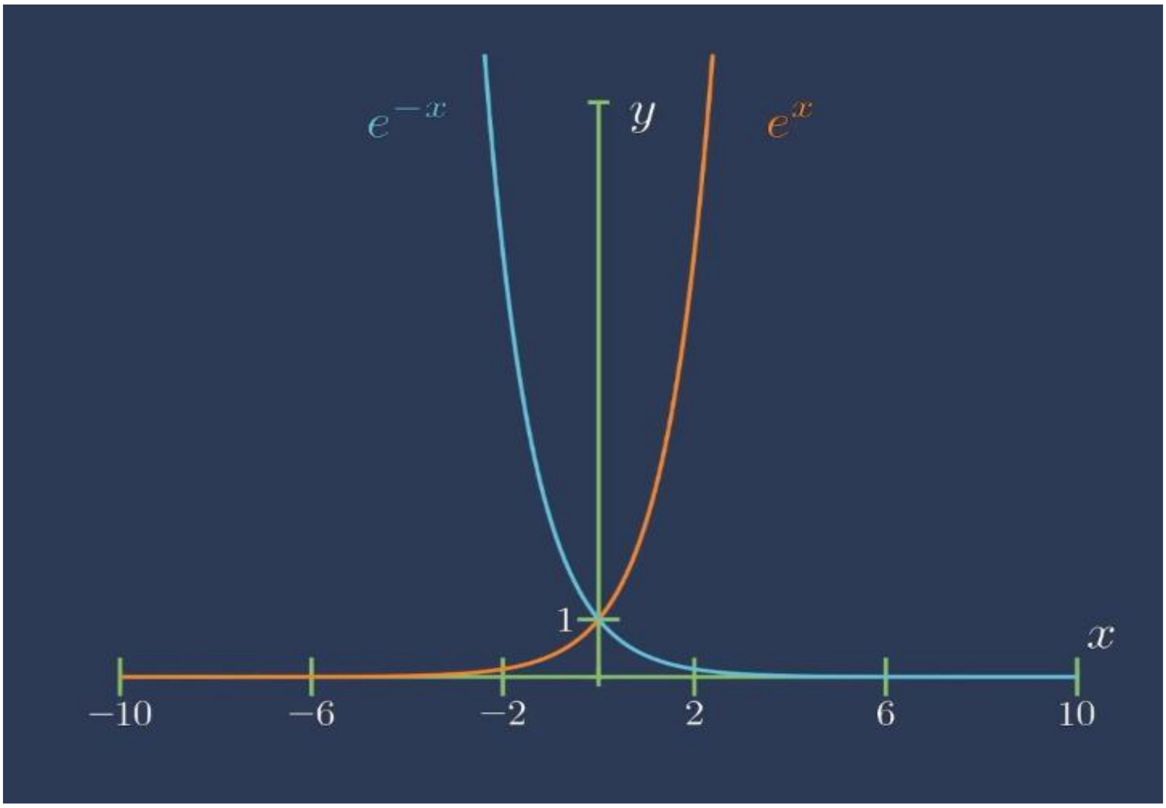
$$=> \sigma(x) = \frac{1}{\frac{e^x + 1}{e^x}}$$

$$=> \sigma(x) = \frac{e^x}{e^x + 1}$$

Hence,

$$\sigma(x) = \frac{e^x}{e^x + 1} - eqn(ii)$$

Q) What is the performance of e^{-x} $in\ a\ graph$?



Therefore, we get three types of limits of e^{-x} :

1. $\lim\limits_{x \to -\infty} e^{-x}$

2. $\lim\limits_{x \to 0} e^{-x}$

3. $\lim\limits_{x \to \infty} e^{-x}$

$$\boldsymbol{\lim_{x \to -\infty} e^{-x}}$$

Let,

$$f(u) = e^{u}, g(x) = -x$$

We know by chain rule of limit:

If $\lim\limits_{u \to b} f(u) = L\ , and, \lim\limits_{x \to a} g(x) = b$

$And\ f(x) is\ continuous\ at\ x = b, Then:$

$\lim_{x \to a} f\big(g(x)\big) = L$

According to chain rule:

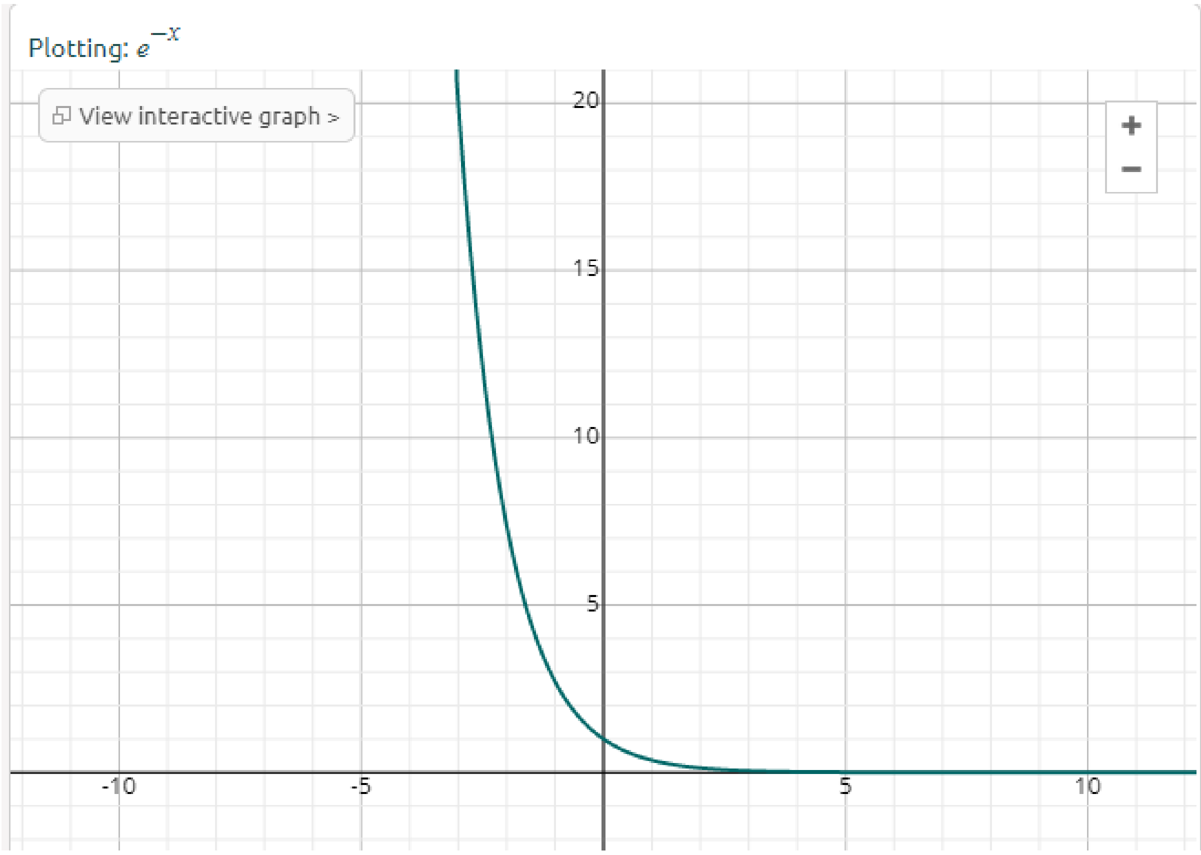
$\lim_{x \to -\infty} (-x) = -(-\infty)$

$=> \lim_{x \to -\infty} (-x) = +\infty$

Also,

$\lim_{u \to \infty} (e^u) = e^\infty = \infty$

$Therefore\ ,\ \boldsymbol{\lim_{x \to -\infty} e^{-x}} = \infty$



$$\lim_{x \to 0} e^{-x}$$

$$\lim_{x \to 0} e^{-x} = e^{0} = 1$$

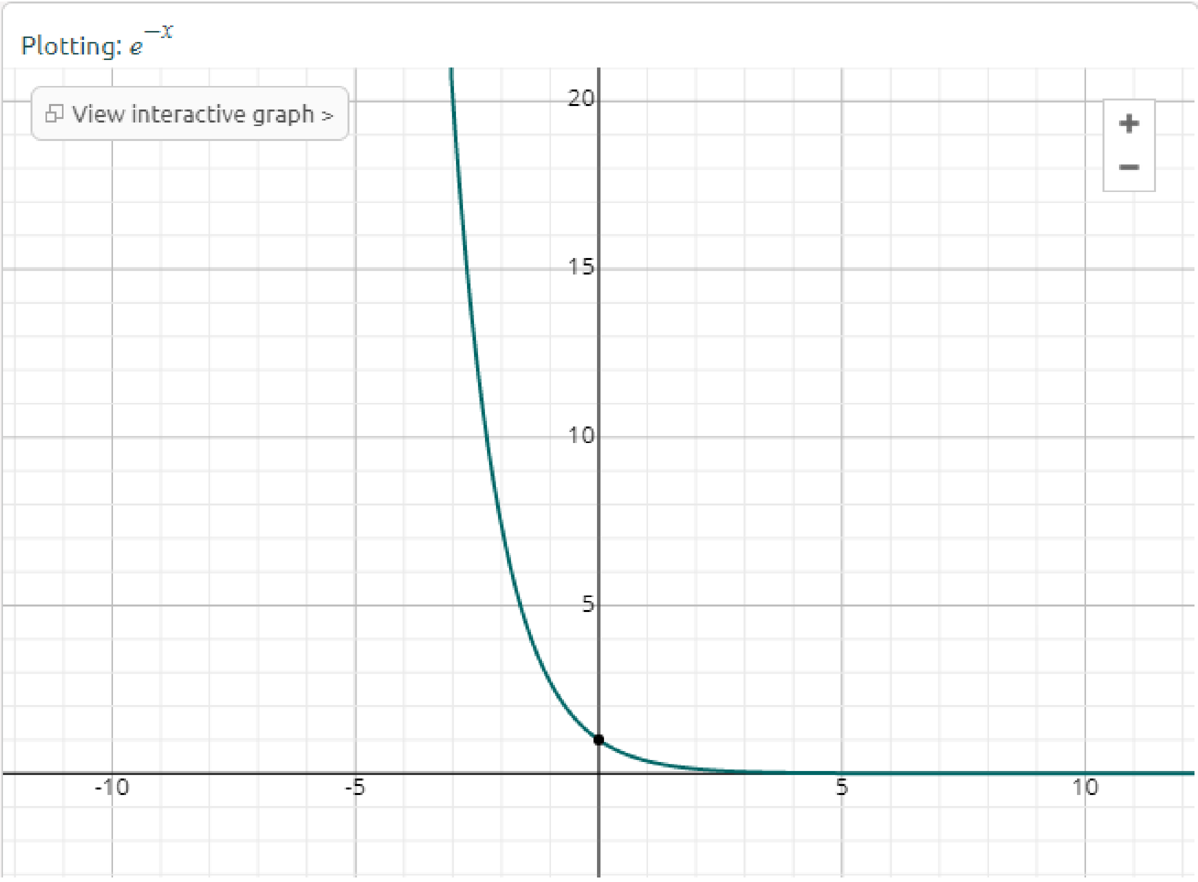


Fig: View the '•' in the above graph.

$$\lim_{x \to +\infty} e^{-x}$$

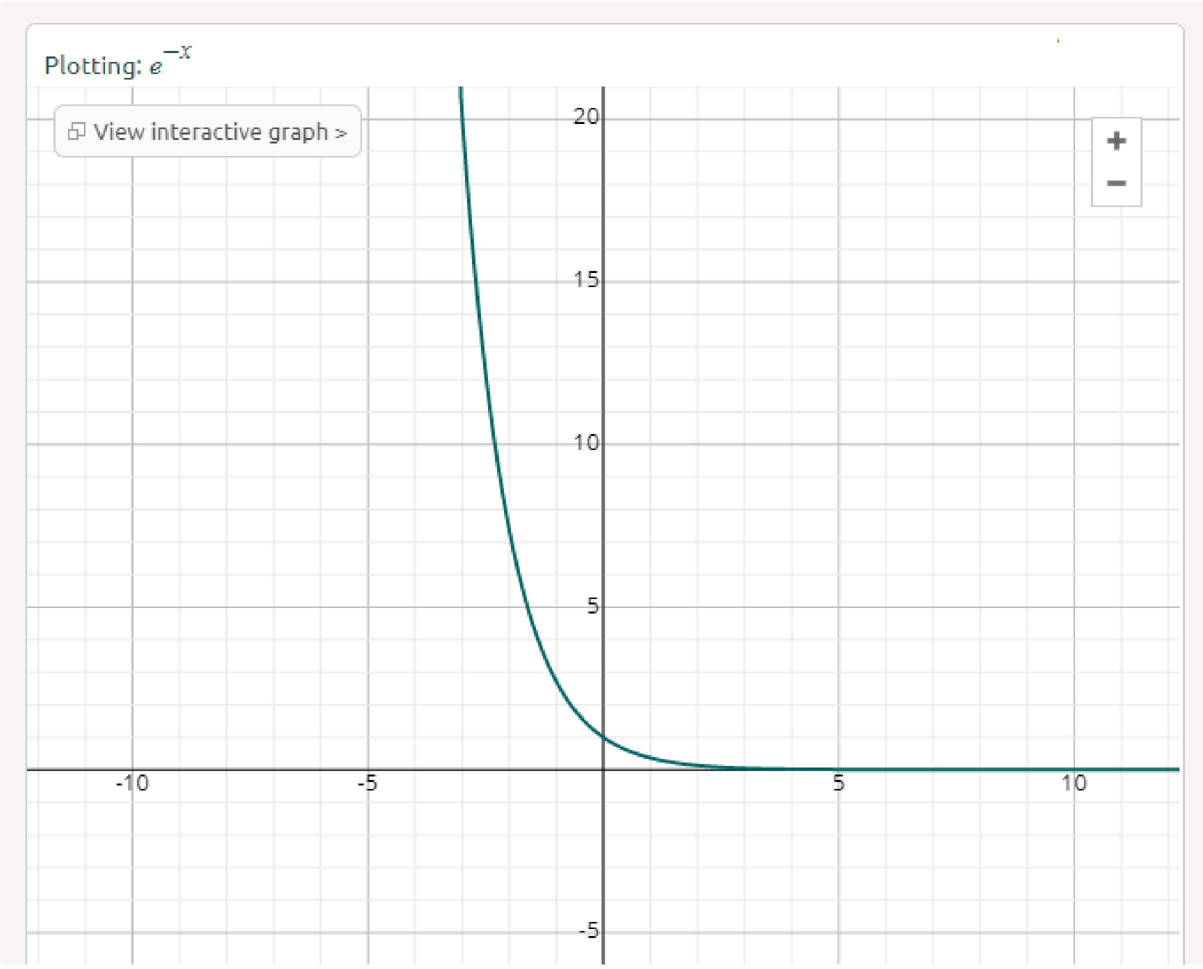
$$\lim_{x \to +\infty} e^{-x}$$

$$=> \lim_{x \to +\infty} \frac{1}{e^{x}}$$

$$=> \lim_{x\to+\infty} \frac{1}{e^{\infty}}$$

$we\ know\ e^{\infty} = \ \infty\ and\ from\ infinity\ property,$
$\frac{c}{\infty} = 0\ where\ c\ is\ any\ constant. Therefore,$

$$=> \lim_{x\to+\infty} \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$



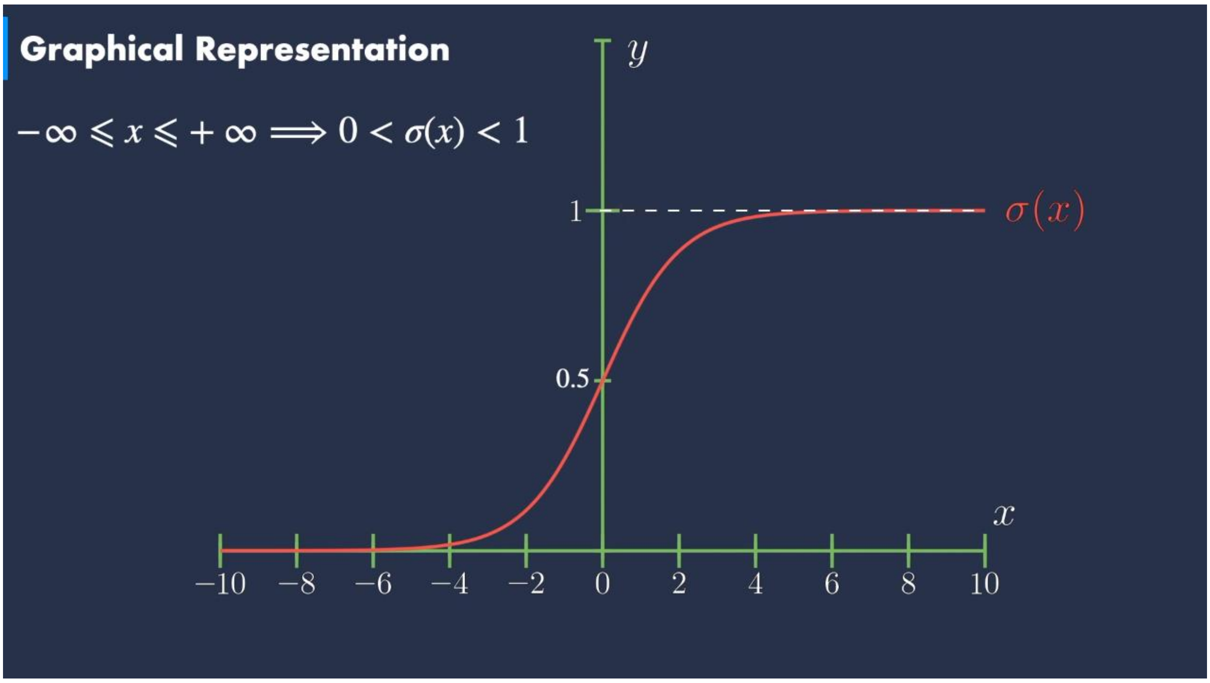
Now,

1. $\lim\limits_{x \to -\infty} \sigma(x) = \lim\limits_{x \to -\infty} \frac{1}{1+e^{-x}} = \frac{1}{1+\infty} = \frac{1}{\infty} = 0$

Note:- $1 + \infty = 1 + 2 + 3 + \cdots + \infty = \infty$

2. $\lim\limits_{x \to 0} \sigma(x) = \lim\limits_{x \to 0} \frac{1}{1+e^{-x}} = \frac{1}{1+e^{0}} = \frac{1}{1+1} = \frac{1}{2} = 0.5$

3. $\lim\limits_{x \to +\infty} \sigma(x) = \lim\limits_{x \to +\infty} \frac{1}{1+e^{-x}} = \frac{1}{1+0} = \frac{1}{1} = 1$



Therefore, we get, when:

$-\infty \leq x \leq +\infty$ *then* $0 < \sigma(x) < 1$