

Space Complexity of Array Operation

We already know that space we need for fixed component and variable part more precisely Auxiliary space or extra space we need and input space i. e. the input components.

1. Initialization:

```
int a[5] = { 1, 2, 3, 4, 5 };
```

for $a[1] = 1 \rightarrow O(1)$ space complexity.

for $a[2] = 2 \rightarrow O(1)$ space complexity.

for $a[3] = 3 \rightarrow O(1)$ space complexity.

for $a[4] = 4 \rightarrow O(1)$ space complexity.

for $a[5] = 5 \rightarrow O(1)$ space complexity.

We see that we not reusing the space rather we are expanding the auxiliary space to enter each element at $O(1)$ constant space complexity.

Hence we can sum up like :

$$O(1) + O(1) + O(1) + O(1) + O(1) = O(1)$$

or we can say , $O(5) = O(1)$ as 5 is the size of the array.

```
int n;  
cin >> n;  
a[n];
```

Now say we if the array size is n . Then the space complexity is $O(n)$.

Similarly ,

```
int n;  
cin >> n;  
int *a = (int *)malloc(n * sizeof(int));
```

And

```
int n;  
cin >> n;  
int *a = new int[n];
```

The space complexity of array having dynamic size n is $O(n)$.

2.Traversal of arrays

Array Traversal looks like:

```
// Array Traversal
    for (int i = 0; i < size; i++)
    {
        cout << a[i] << endl;
    }
}
```

There is no insertion or deletion of any elements in arrays hence it takes no space taken.

Therefore no space complexity generated for traversal of arrays.

The only addition space requires that is storing int i the input space = $O(1)$ space complexity.

3.Overriding ith element

```
int a[5] = { 1, 2, 3, 4, 5 };  
arr.arr[0] = 5;
```

$a[0] = 1 = 5$ gets overridden as only it modifies the 1st element of the array. Hence it doesnot change the fact that the space remaining constant i. e. $O(1)$.

4. Insert Element in Array

```
//Insert Elements in the array  
for (int i = 0; i < n; i++)  
{  
    cin >> a[i];  
}
```

When we are inserting each element in each auxiliary space as arrays are arranged in contiguous memory location.

n	$\rightarrow O(1)$
.	$\rightarrow O(1)$
.	$\rightarrow O(1)$
3	$\rightarrow O(1)$
2	$\rightarrow O(1)$
1	$\rightarrow O(1)$

space remaining constant i.e. auxiliary space already created during creation of array and we input the elements in that auxiliary space which has been already created.

Hence values gets inserted in that constant amount of space i.e.,

$$O(1) + O(1) + O(1) + \dots + O(1) = O(1).$$

The only additional space required is for integer 'i' in stack. space is already allocated when we declare $a[size]$.

And to that constant size we just input the elements which

doesnot allocate further or extra space rather it use the same space i. e. $O(1)$.

6. Insert Element at position In an Array

```
//Increment the array
    size=size+1;

    // Insert Elements in the array at a given
position
    for (int i = size-1 ; i > pos; i--)
    {

        a[i] = a[i - 1];

    }
    a[pos] = elem;
```

1st We increment the size of the array: $size = size + 1$.

2nd we shift the element from position to last index:

suppose $size = 5$ and we have indexes: $a[0]$, $a[1]$, $a[2]$, $a[3]$ and $a[4]$, as size increased , now we have array indexes: $a[0]$, $a[1]$, $a[2]$, $a[3]$, $a[4]$, $a[5]$ and $size = 6$.

Now we will put the at index 1. Hence we will swap elements:

- $a[5] = a[4]$ [values get swapped] $\rightarrow O(1)$ space complexity
- $a[4] = a[3]$ [values get swapped] $\rightarrow O(1)$ space complexity
- $a[3] = a[2]$ [values get swapped] $\rightarrow O(1)$ space complexity
- $a[2] = a[1]$ [values get swapped] $\rightarrow O(1)$ space complexity

Now we just will do is overriding the i th element:

- $a[1] = \text{elem}$ (User Input) $\rightarrow O(1)$ space complexity

Therefore , the space consumed is $O(1) + O(1) + O(1) + \dots + O(1) = O(1)$ complexity.

Analysis goes like:

$\text{size} = \text{size} + 1$ doesnot require any additional space.

The second line of code, $\text{for}(\text{int } i = \text{size} - 1; i > \text{pos}; i--)$, iterates through the last element to the element before the specified position. For each iteration , the value of the current is copied to the next element. This does not require any additional space, as the values of the elements are simply copied to the auxiliary space created for the array.

The third line of code $a[\text{pos}] = \text{elem}$, assigns the value of the new element to the specified position of the array.

This doesnot require any additional space , as the value of the new element is simply being stored in the auxiliary

array.

The only space required is integer i variable.

Therefore for array , space complexity is $O(1)$.

7.Delete Element at position In an Array

```
for (int i = pos-1 ; i <size-1; i++)  
{  
  
    a[i] = a[i + 1];  
  
}  
size=size-1;
```

Here also we just swap the elements elements which doesnot acquire any additional space and decrease the size .

- *Suppose we have size = 5 and And indices: a[0], a[1], a[2], a[3] and a[4].*
- *And we want to delete element at index: 2*

- *Then we go through a process of Swapping:*
 - $a[2 - 1 = 1] = a[2 + 1 = 3] \rightarrow \text{elem: } 3 \rightarrow \text{takes } O(1) \text{ constant space.}$
 - $a[3 - 1 = 2] = a[3 + 1 = 4] \rightarrow \text{elem: } 4 \rightarrow \text{takes } O(1) \text{ constant space.}$
 - $a[4 - 1 = 3] = a[4 + 1 = 5] \rightarrow \text{elem: } 5 \rightarrow \text{takes } O(1) \text{ constant space.}$

And $a[0]$ will be remain untouched.

- *Then we decrement the size:*
 - $\text{size} = \text{size} - 1$, now size is 4, traversal will take place from $a[0] = 1$, $a[1] = 3$, $a[2] = 4$, and $a[3] = 5$.

The only additional space it needs is for i in stack which is negligible.

Hence the space complexity it produces :

$$O(1) + O(1) + O(1) + \dots + O(1) = O(1)$$

Hence the space complexity for the above code is $O(1)$.

*Hence the space complexity for an array
can be summed up like:*

SPACE COMPLEXITY OF ARRAY OPERATION

<i>Operation</i>	<i>Space Complexity</i>
<i>Initialization[fixed constant size]</i>	<i>$O(1)$</i>
<i>Initialization[n size]</i>	<i>$O(n)$</i>
<i>Traversal</i>	<i>$O(1)$</i>
<i>Overriding ith element</i>	<i>$O(1)$</i>
<i>Insert element in the array</i>	<i>$O(1)$</i>
<i>Insert element at a position</i>	<i>$O(1)$</i>
<i>Delete element at a position</i>	<i>$O(1)$</i>

Space Complexity of Two Dimensional Array

The space complexity of a two-dimensional array depends on its size and the data type it holds.

Let's assume that the two-dimensional array has dimensions N rows and M columns, and each element in the array occupies a fixed amount of space, regardless of the data type.

In this case, the space complexity of the two-dimensional array would be $O(N * M)$ because you would need to allocate space for N rows and M columns. Each element takes up a fixed amount of space, so the total space required is proportional to the product of N and M .

However, if the size of each element in the array is not fixed, and it can vary depending on the data type or other factors, then you need to consider the space complexity on a per-element basis. For example, if each element in the array is an object that requires K units of space, then the space complexity would be $O(N * M * K)$.

It's important to note that the space complexity only accounts for the storage required by the array itself and does not include any auxiliary data structures or variables used in algorithms or functions that operate on the array.

When row is not equal to column then space created by

*array is row * column , hence it is $O(\text{row} * \text{column})$ here.*

*if row = m and column = n , then space complexity = $O(m * n)$.*

*When row is equal to column then space created by array is row * column = n^2 , hence it is $O(n^2)$.*

Space Complexity of Three Dimensional Array

The space complexity of a three-dimensional array depends on its dimensions and the data type it holds.

Let's assume the three-dimensional array has dimensions N rows, M columns, and K depth, and each element in the array occupies a fixed amount of space, regardless of the data type.

In this case, the space complexity of the three-dimensional array would be $O(N * M * K)$ because you would need to allocate space for N rows, M columns, and K depth. Each element takes up a fixed amount of space, so the total space required is proportional to the product of N, M, and K.

However, if the size of each element in the array is not fixed, and it can vary depending on the data type or other factors, then you need to consider the space complexity on a per-element basis. For example, if each element in the array is an object that requires L

units of space, then the space complexity would be $O(N * M * K * L)$.

It's important to note that the space complexity only accounts for the storage required by the array itself and does not include any auxiliary data structures or variables used in algorithms or functions that operate on the array.

*When row is not equal to column and depth is not equal to row and column then space created by array is $depth * row * column$, hence it is $O(depth * row * column)$ here.*

*if $depth = p$, $row = m$ and $column = n$, then space complexity = $O(p * m * n)$.*

*When row is equal to column and depth is equal to row and column then space created by array is $depth * row * column = n^3$, hence it is $O(n^3)$.*