Linear Regression

$y = mx + c$

Study	Hours (x)	Score (y)
	2	50
	4	60
	6	70
	8	80
	10	95
	11	?

$y \propto x$

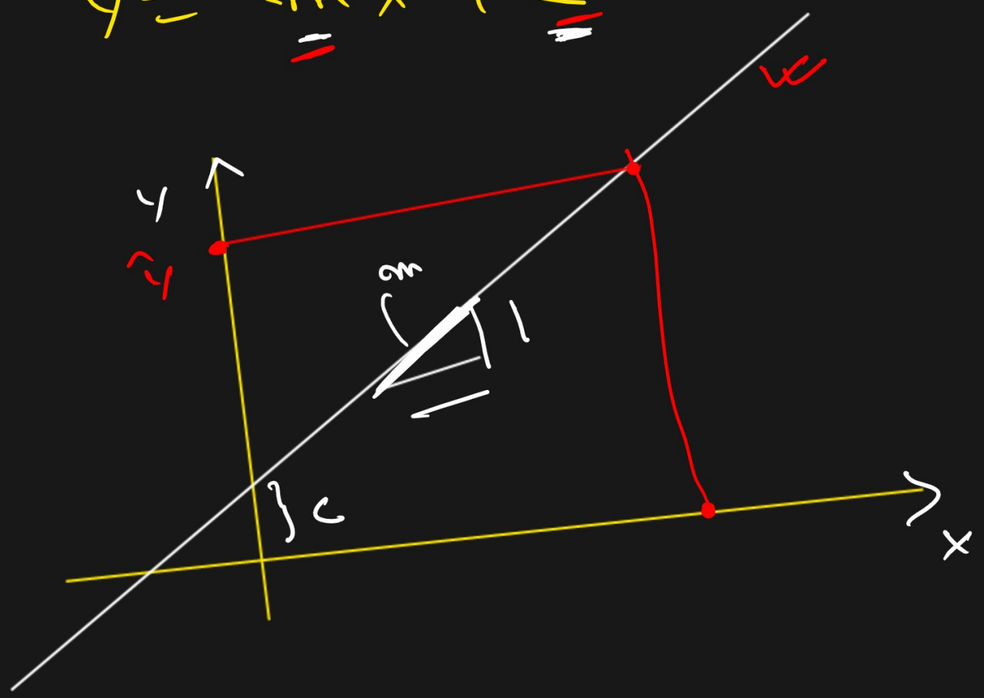
$\rightarrow y = wx + b$ — ①

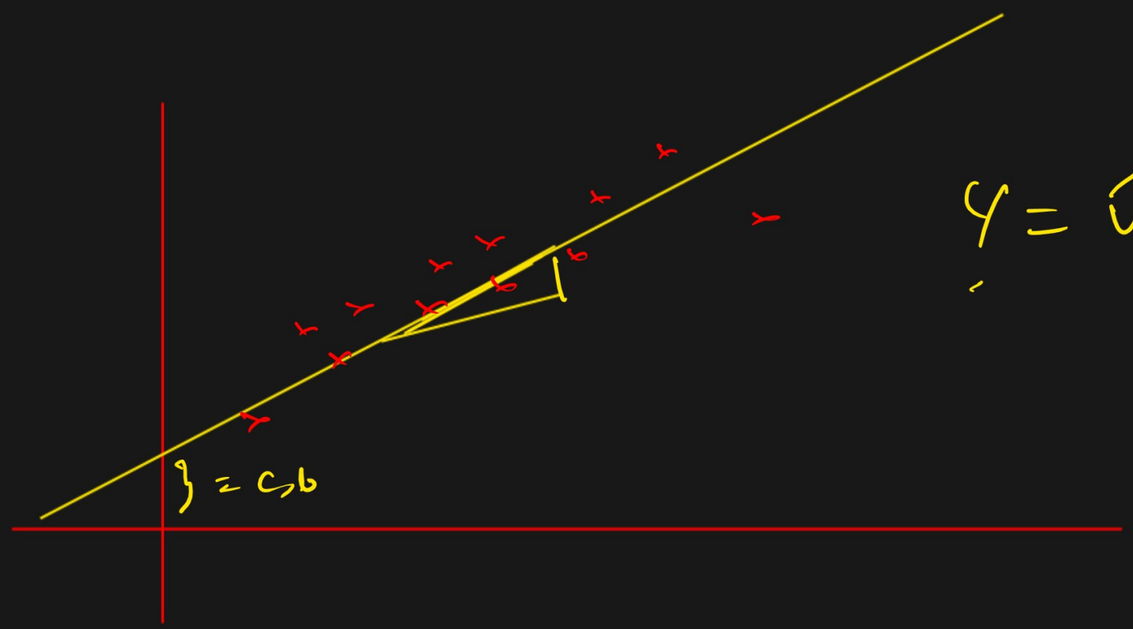
$y = wx^2 + b$

$y = mx + c$

$m = \left(\frac{y}{x}\right)$ $\quad c = 0$

$y = 2x + 5$





$$\hat{y} = wx + b \quad \text{— (1)}$$

$$w = 0$$

$$b = 0$$

$$\hat{y} = 0 \cdot x + 0 = 0$$

$$\hat{y} = 0 \cdot 2 + 0 = 0$$

$$y = 50$$ ← min

learning Rate → 0.05, 0.01, 0.001,

$$w_n = w_0 - \eta \left(\frac{\partial L}{\partial w}\right)_{x}$$

$$b_n = b_0 - \eta \frac{\partial L}{\partial b}$$

MSE (mean squared error) → $\frac{1}{n} \sum (\hat{y} - y)^2 = L$

$$L = \frac{1}{n}\sum_{i=1}^{n} (\hat{y} - y)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n} (\underbrace{wx+b}_{z} - y)^2$$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w}$$

$$= \frac{\partial L}{\partial w} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w}$$

$$= \frac{\partial L}{\partial w}$$

$$z = wx + b - y$$

$$L = \frac{1}{n}\sum(z)^2$$

$$\frac{\partial L}{\partial z} = \frac{2}{n}\sum(z)$$

$$\frac{\partial z}{\partial w} = \frac{\partial (wx + b - y)}{\partial w}$$

$$= x$$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial w}$$

$$= \frac{2}{n}(z) \cdot x$$

$$\frac{\partial L}{\partial w} = \frac{2}{n}(wx + b - y) \cdot x \quad \text{--- (2)}$$

$$\frac{\partial z}{\partial b} = \frac{\partial}{\partial b}(wx + b - y)$$

$$= 1$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial b}$$

$$= \frac{2}{n} z \cdot 1$$

$$= \frac{2}{n} z$$

$$= \frac{2}{n}(wx+b-y)$$

$$\frac{\partial L}{\partial b} = \frac{2}{n}(wx+b-y) \quad \text{— (3)}$$

$$w_n = w_0 - \eta\left(\frac{\partial L}{\partial w}\right)$$

$$b_n = b_0 - \eta\frac{\partial L}{\partial b}$$

$$\frac{\partial L}{\partial w} = \frac{2}{n}\sum_{i=1}^{n}\begin{pmatrix}(\hat{w}x^i + \hat{b} - \hat{y}^i)\,x^i \\ 0\end{pmatrix}$$

$$\frac{\partial L}{\partial b} = \frac{2}{n}\sum_{i=1}^{n}\begin{pmatrix}(\hat{y}^i - (wx^i + b)) \\ 0\end{pmatrix}$$

$$w = 0$$

$$b = 0$$

$$\eta = 0.01$$

$$\frac{\partial L}{\partial w} = \frac{2}{5}\left[(-50).2 + (-60).4 + (-70).6 + (-80).8 + (-95).10\right]$$

$$= -940$$

$$\frac{\partial L}{\partial b} = \frac{2}{5}\left[-50 - 60 - 70 - 80 - 95\right]$$

$$= -142$$

$$w_{n} = w_{0} - \eta \frac{\partial L}{\partial w}$$

$$= 0 - 0.01 \cdot (-940)$$

$$= 9.4$$

$$b_{n} = b_{0} - \eta \frac{\partial L}{\partial b}$$

$$= 0 - 0.01 \cdot (-142)$$

$= 1.42$

$w = 9.4$

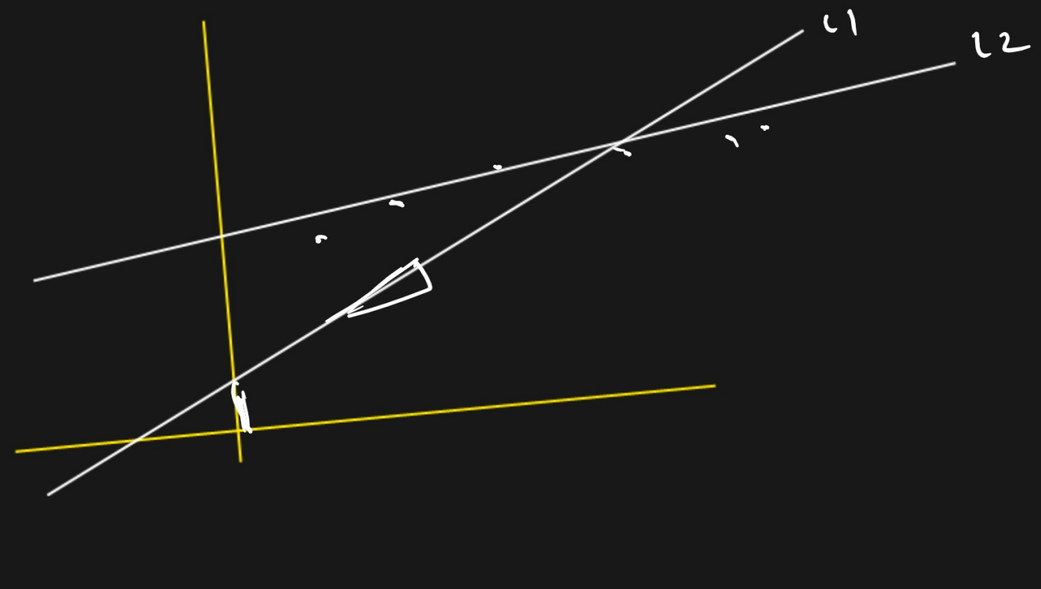
$b = 1.42$

$\hat{y} = 9.4x + 1.42$

$\hat{y} = 9.4 \cdot 2 + 1.42$

$= 20.22$

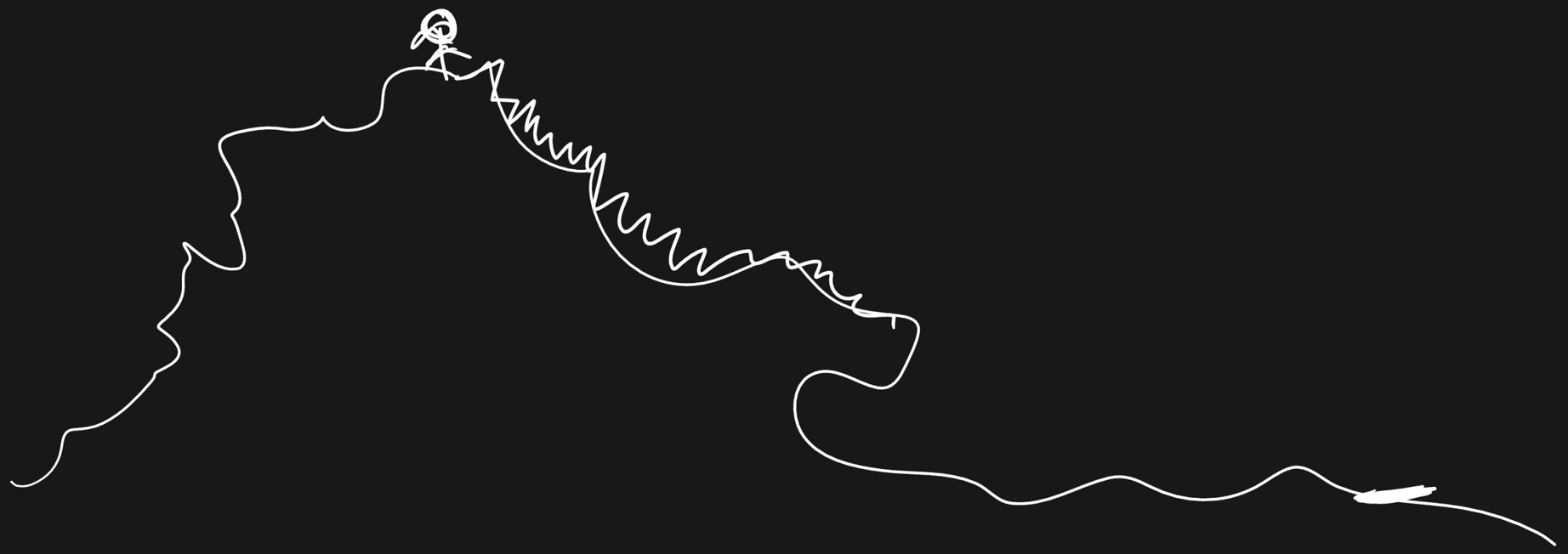
$y = 50$



$$\underline{MSE=0} = \frac{1}{n}\sum(\hat{y}-y)^2 \approx 0$$

$w = 0$

$b = 0$



$$y = \underline{\omega} x + \underline{b}$$

$$\hat{y} = \bar{w}\bar{x} + \bar{b}$$

Cost function = loss function = Error function

$$L = \underline{\underline{MSE}} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y} - y)^2$$

$$w_n = w_o - \eta \frac{\partial L}{\partial w} \quad \text{Gradients}$$

$$b_n = b_o - \eta \frac{\partial L}{\partial b}$$