Assignment - Probability

Q1.: What is Probability and Event with its type? Write rules of probability. Also, explain Mutually Exclusive \and Non-Exclusive Events in Detail with help of Example.

Answer: Probability is a measure of how likely an event is to occur in a random experiment. An event is a set of outcomes from an experiment. There are different types of events in probability, such as:

- **Simple event:** an event that consists of exactly one outcome. For example, getting a 6 when rolling a die is a simple event.
- **Compound event:** an event that consists of two or more outcomes. For example, getting an even number when rolling a die is a compound event.
- **Certain event:** an event that is sure to occur in any experiment. For example, getting a number less than 7 when rolling a die is a certain event.
- **Impossible event:** an event that cannot occur in any experiment. For example, getting a number greater than 6 when rolling a die is an impossible event.
- **Equally likely events:** events that have the same probability of occurring. For example, getting a 1, 2, 3, 4, 5, or 6 when rolling a die are equally likely events.
- **Complementary events:** two events that are mutually exclusive and exhaustive. That is, they cannot occur together and their union is the sample space. For example, getting a head or a tail when tossing a coin are complementary events.
- **Mutually exclusive events:** two or more events that cannot occur together. That is, their intersection is the empty set. For example, getting a 1 or a 2 when rolling a die are mutually exclusive events.
- **Exhaustive events:** a set of events that covers all the possible outcomes of an experiment. That is, their union is the sample space. For example, getting a head, a tail, or a side when tossing a coin are exhaustive events.
- **Independent events:** two or more events that do not affect each other's occurrence. That is, the probability of one event does not change the probability of another event. For example, getting a head when tossing a coin and getting a 6 when rolling a die are independent events.
- **Dependent events:** two or more events that affect each other's occurrence. That is, the probability of one event changes the probability of another event. For example, drawing a card from a deck and drawing another card without replacing the first one are dependent events.

Rules of Probability:

Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. Using probability, one can predict only the chance of an event to occur, i.e., how likely they are going to happen. There are probability rules that you can follow to find out the probability of those events. Here are some of the basic probability rules:

- **Addition Rule:** Whenever an event is the union of two other events, say A and B, then P (A or B) = P (A)+P (B)−P (A∩B).

- **Complementary Rule:** Whenever an event is the complement of another event, specifically, if A is an event, then P (not A)=1−P (A) or P (A') = 1 - P (A').

- **Conditional Rule:** When event A is already known to have occurred and probability of event B is desired, then P (B, given A)=P (A and B)P (A, given B). It can be vice versa in case of event B.

- **Multiplication Rule**: Whenever an event is the intersection of two other events, that is, events A and B need to occur simultaneously, P (A and B) = P (A) • P (B|A) or P (B)*P (A|B).

Mutually Exclusive \and Non-Exclusive Events:

Mutually exclusive events are events that cannot occur at the same time. For example, when you toss a coin, you can get either heads or tails, but not both. The probability of getting heads and tails at the same time is zero. Therefore, heads and tails are mutually exclusive events.

Non-exclusive events are events that can occur at the same time. For example, when you roll a die, you can get an even number and a number less than 4 at the same time. The probability of getting an even number and a number less than 4 at the same time is not zero. Therefore, even numbers and numbers less than 4 are non-exclusive events.

To calculate the probability of mutually exclusive events, you can use the formula:

P(A or B) = P(A) + P(B)

This means that the probability of either event A or event B occurring is the sum of the probabilities of each event occurring.

To calculate the probability of non-exclusive events, you can use the formula:

P(A or B) = P(A) + P(B) - P(A and B)

This means that the probability of either event A or event B occurring is the sum of the probabilities of each event occurring minus the probability of both events occurring at the same time.

Examples to illustrate these formulas:

- Suppose you toss a coin and roll a die. What is the probability of getting heads or a 6?

a. Heads and 6 are mutually exclusive events, so you can use the formula P(A or B) = P(A) + P(B).
b. The probability of getting heads is 1/2 and the probability of getting a 6 is 1/6.
c. Therefore, the probability of getting heads or a 6 is 1/2 + 1/6 = 2/3.

- Suppose you draw a card from a standard deck of 52 cards. What is the probability of getting a king or a red card?

a. King and red card are non-exclusive events, so you can use the formula P(A or B) = P(A) + P(B) - P(A and B).
b. The probability of getting a king is 4/52 and the probability of getting a red card is 26/52.
c. The probability of getting a king and a red card is 2/52, since there are two red kings in the deck.
d. Therefore, the probability of getting a king or a red card is 4/52 + 26/52 - 2/52 = 28/52.

Q.2: State Bayes Theorem and write it's applications. A consulting firm rents car from three agencies such that 50% from agency L, 30% from agency M and 20% from agency N. If 90% of the cars from L, 70% of cars from M and 60% of the cars from N are in good conditions (i) what is the probability that the firm will get a car in good condition? (ii) if a car is in good condition, what is probability that it has come from agency N?

Answer: Bayes' Theorem is named after Reverend Thomas Bayes. It is a very important theorem in mathematics that is used to find the probability of an event, based on prior knowledge of conditions that might be related to that event. Bayes theorem is also known as the Bayes Rule or Bayes Law. It is used to determine the conditional probability of event A when event B has already happened. The general statement of Bayes' theorem is "The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A and the probability of A divided by the probability of event B." i.e.

$$P(A|B) = P(B|A)P(A) / P(B)$$

where,

$P(A)$ and $P(B)$ are the probabilities of events A and B

$P(A|B)$ is the probability of event A when event B happens

$P(B|A)$ is the probability of event B when A happens

Applications of Bayes Theorem:

1. **Naive Bayes' Classifiers:** These are machine learning models that use Bayes' Theorem to classify data into different categories based on the features and the prior probabilities of each category. They are called naive because they assume that the features are independent of each other given the category. Naive Bayes' classifiers are widely used in text analysis, spam filtering, sentiment analysis, etc.

2. **Discriminant Functions and Surfaces:** These are mathematical functions that use Bayes' Theorem to assign a data point to one of several possible classes based on the posterior probabilities of each class. They can also define decision boundaries or surfaces that separate the classes in the feature space. Discriminant functions and surfaces are used in pattern recognition, image analysis, medical diagnosis, etc.

3. **Bayesian Parameter Estimation:** This is a method of estimating the unknown parameters of a statistical model using Bayes' Theorem and prior distributions. It provides a way of incorporating prior knowledge or beliefs into the estimation process and obtaining a posterior distribution of the parameters that reflects the uncertainty and the evidence. Bayesian parameter estimation is used in various fields such as finance, economics, physics, biology, etc.

Now,

Solution:

Let A be the event that the firm gets a car in good condition, and E1, E2, E3 be the events that the car is from agency L, M, N respectively. Then, using Bayes' theorem, we have:

(i) $P(A) = P(A|E1)P(E1) + P(A|E2)P(E2) + P(A|E3)P(E3)$

$= 0.9(0.5) + 0.7(0.3) + 0.6(0.2)$

$= 0.78$

So the probability that the firm will get a car in good condition is 0.78.

(ii) $P(E3|A) = P(A|E3)P(E3) / P(A)$

$= 0.6(0.2) / 0.78$

$= 0.1538$

So the probability that the car is from agency N, given that it is in good condition, is 0.1538.

Q.3: Suppose you have tested positive for a disease; what is the probability that you actually have the disease if: • P(T=1|D=1) = .95 (true positive) • P(T=1|D=0) = .10 (false positive) • P(D=1) = .01 (prior) Where T= Test and D=Disease.

Solution:

Bayes' theorem

$P(D=1|T=1) = P(T=1|D=1)P(D=1) / P(T=1)$

where

$P(D=1|T=1)$ is the probability of having the disease given a positive test result,

$P(T=1|D=1)$ is the probability of a positive test result given the disease,

$P(D=1)$ is the prior probability of having the disease

$P(T=1)$ is the probability of a positive test result.

Using the given values:

$P(D=1|T=1) = 0.95 \times 0.01 / P(T=1)$

To find

$P(T=1)$, we can use the law of total probability, which states that:

$P(T=1) = P(T=1|D=1)P(D=1) + P(T=1|D=0)P(D=0)$

where

$P(T=1|D=0)$ is the probability of a false positive and

$P(D=0)$ is the probability of not having the disease.

Using the given values:

$P(T=1) = 0.95 \times 0.01 + 0.10 \times 0.99 = 0.1085$

Therefore, the final answer is:

$P(D=1|T=1) = 0.95 \times 0.01 / 0.1085 = 0.0876$ (approx)

This means that the probability of actually having the disease after testing positive is about 8.76%.

Q.4: Give a Brief on Probability Distribution Functions and it's Type.

Answer: A probability distribution function is a mathematical function that describes the probability of different possible values of a random variable. It can be used to model various phenomena, such as coin tosses, dice rolls, heights, weights, test scores, etc.

There are two main types of probability distribution functions: discrete and continuous.

Discrete probability distribution functions assign probabilities to a finite or countable set of values, such as the number of heads in a coin toss.

Continuous probability distribution functions assign probabilities to an infinite or uncountable set of values, such as the height of a person.

Some examples of discrete probability distribution functions are:

- Binomial distribution: It models the number of successes in a fixed number of independent trials, each with a constant probability of success. For example, the number of heads in 10 coin tosses follows a binomial distribution with 10 trials and 0.5 probability of success.
- Poisson distribution: It models the number of events that occur in a fixed interval of time or space, given a constant average rate of occurrence. For example, the number of phone calls received by a call center in an hour follows a Poisson distribution with a certain average rate.
- Geometric distribution: It models the number of trials needed to get the first success in a sequence of independent trials, each with a constant probability of success. For example, the number of coin tosses needed to get the first head follows a geometric distribution with 0.5 probability of success.

Some examples of continuous probability distribution functions are:

- Normal distribution: It models the distribution of many natural and social phenomena that tend to cluster around a central value, with symmetric variation on either side. For example, the height of adult humans follows a normal distribution with a certain mean and standard deviation.
- Uniform distribution: It models the distribution of phenomena that are equally likely to occur in any interval of a given range. For example, the angle of a spinner follows a uniform distribution between 0 and 360 degrees.
- Exponential distribution: It models the distribution of the time between events that occur in a Poisson process, i.e., a process that has a constant average rate of occurrence. For example, the time between phone calls received by a call center follows an exponential distribution with a certain average rate.