Robotics Research Center | **Summer School 2021**

Assignment-1: Basic Math

Released: May 20^{th} | *Deadline: May 27^{th}*

Instructions

- This assignment is designed to get you familiar with the basic mathematical foundations required in machine learning and multi-view geometry, and brush up your concepts.
- For handwritten answers, use a scanning application on your phone to scan the handwritten answers and convert them to a pdf. Use ilovepdf to merge various pdf files. As the name suggests, we love pdf's, so please submit a single pdf file as **rollnumber.pdf**.
- This assignment is NOT graded. This will not help you increase your GPA. So please, for the love of God, do not copy. First try doing it yourself, and if you don't get it, ask us or your fellow RRC friends for help. It's okay if you submit wrong answers. What's more important is that you understand the concepts.
- The deadline is May 27, 23:59. We'll let you know the submission instructions soon - we'll try opening up a portal somewhere.

Question 1: MLE

1. Estimate the parameters (mean and variance) of a Gaussian distribution using MLE.
2. Find the optimal classification threshold for two equally probable classes represented by the univariate distributions $\mathbb{N}(23, \sigma^2)$ and $\mathbb{N}(33, \sigma^2)$.
3. For the classes in Q2, how does the optimal classification threshold change if the variance of the first class becomes double of the current value? Try out the same when the variance of the second class becomes double, keeping that of the first class unchanged.

Question 2: Function of a continuous random variable

Let X be a continuous random variable with PDF:

$$f_X(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} \quad \forall x \in \mathbb{R}$$

Let $Y = X^2$. Find:

1. $f_Y(y)$
2. $\mathbb{E}[X]$
3. $\sigma^2[X]$

Question 3: Computing covariance matrix

Given a set of 2D points X ,

$$X = \{[1, 1]^T, [2, 2]^T, [3, 3]^T, [4, 4]^T, [5, 5]^T\}$$

1. Compute covariance matrix (consider the two coordinates as random variables) for the data points.

2. How does the covariance matrix change when a new data point $[4, 3]^T$ is added to the set X?

Question 4: Gradients and Optimization

1. (Gradient and Hessian) Compute the gradient and the hessian of the three-variable function $f(x, y, z) = 5xy^2z$.

2. (Taylor Series) Find a Maclaurin Series (which is a Taylor Series whose derivatives are computed at zero) approximation for $f(x) = cos(6x)$ upto the fifth derivative.

3. (Lagrange Multipliers) Find the maximum and minimum values of $f(x, y) = 3x - 6y$ subject to the constraint $4x^2 + 2y^2 = 25$. You should try plotting the function using an online graph calculator like desmos and incude the graph in your pdf. You can check out similar problems here.

4. (Gradient Descent) The Gradient Descent algorithm aims to minimise the objective function by taking steps in the direction of the negative of the gradient. We start at a point x_0 and move a positive distance α in the direction of the negative gradient. We keep doing this until we reach the optimum value. You can answer the following questions in 2-3 sentences:

   - Write the gradient descent equation which expresses the update in the value using the gradient. Use the following notation: x_{n+1} is the updated value, x_n is the current value, α is the constant multiplied with the gradient ∇ of the function $f(x_n)$.
   - We aim to minimise our objective function. In gradient descent, why do we follow the **negative** of the gradient, and not the positive? What is the direction the gradient of the function is pointing us in?
   - The α (the term being multiplied with the gradient ∇) in the gradient descent equation is called the *step size*. Very simply, by looking at the words *step* and *size*, can you explain what it may mean in the context of the blindfolded mountain climber we discussed in the lecture?
   - Do you think gradient descent might be able to tell whether the minimum value it has found is local or global?

Question 5: Quadratic Function and Definiteness

Consider the equation

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - 2x_2 + x_3)^2$$

1. Find the 3×3 matrix S that satisfies the above equation.

2. Find its rank, eigen values and determinant.

3. Is S, a positive-definite matrix ?