

Assignment 1(ICSE Class 10 2017)

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Question 11.

Solve the following system of linear equations using matrix method:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9$$

$$\frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52$$

$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

Solution:

Let $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$

$$a + b + c = 9 \quad (1)$$

$$2a + 5b + 7c = 52 \quad (2)$$

$$2a + b - c = 0 \quad (3)$$

This System of equations can be written as

$$AX = B, \text{ where} \quad (4)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (5)$$

and

$$B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \quad (6)$$

Now,

$\therefore A^{-1}$ exists and it has a unique

solution.

$$A_{11} = -12 \quad A_{12} = 16 \quad A_{13} = -8$$

$$A_{21} = 2 \quad A_{22} = -3 \quad A_{23} = 1$$

$$A_{31} = 2 \quad A_{32} = -5 \quad A_{33} = 3$$

$$\text{adj}.A = \begin{bmatrix} -12 & 16 & -8 \\ 2 & -3 & 1 \\ 2 & -5 & 3 \end{bmatrix}^T \quad (7)$$

$$A^{-1} = \frac{\text{adj}.A}{|A|} \quad (8)$$

$$A^{-1} = -\frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \quad (9)$$

$$X = A^{-1}B \quad (10)$$

$$X = -\frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -108 + 104 + 0 \\ 144 - 156 - 0 \\ -72 + 52 + 0 \end{bmatrix} \quad (12)$$

$$= -\frac{1}{4} \begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \quad (14)$$

$$a = 1, b = 3, c = 5$$

$$\text{Thus, } \frac{1}{x} = 1 \implies x = 1,$$

$$\frac{1}{y} = 3 \implies y = \frac{1}{3}$$

$$\frac{1}{z} = 5 \implies z = \frac{1}{5}$$

$$\therefore x = 1, y = \frac{1}{3}, z = \frac{1}{5} \quad (15)$$