Assignment 1(ICSE Class 10 2017)

NIMMALA AVINASH(CS21BTECH11039)

May 16, 2022

Question

Question 11.

Solve the following system of linear equations using matrix method:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 9$$
$$\frac{2}{x} + \frac{5}{y} + \frac{7}{z} = 52$$
$$\frac{2}{x} + \frac{1}{y} - \frac{1}{z} = 0$$

Solution

Solution:

Let $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$

$$a + b + c = 9 \tag{1}$$

$$2a + 5b + 7c = 52 \tag{2}$$

$$2a + b - c = 0 \tag{3}$$

This System of equations can be written as

$$AX = B, where \tag{4}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \tag{5}$$

and

$$B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \tag{6}$$

Now,

$\therefore A^{-1}$ exists and it has a unique solution.

$$A_{11} = -12 \quad A_{12} = 16 \quad A_{13} = -8$$
$$A_{21} = 2 \quad A_{22} = -3 \quad A_{23} = 1$$
$$A_{31} = 2 \quad A_{32} = -5 \quad A_{33} = 3$$

$$adj.A = \begin{bmatrix} -12 & 16 & -8 \\ 2 & -3 & 1 \\ 2 & -5 & 3 \end{bmatrix}^T \tag{7}$$

$$A^{-1} = \frac{adj.A}{|A|} \tag{8}$$

$$A^{-1} = -\frac{1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \tag{9}$$

$$X = A^{-1}B \tag{10}$$

$$X = -\frac{1}{4}\begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -108+104+0 \\ 144-156-0 \\ -72+52+0 \end{bmatrix} \tag{12}$$

$$= -\frac{1}{4}\begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} \tag{13}$$

$$= \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \tag{14}$$

$$a = 1, b = 3, c = 5$$

Thus,

$$\frac{1}{x} = 1 \implies x = 1 \tag{15}$$

$$\frac{1}{y} = 3 \implies y = \frac{1}{3} \tag{16}$$

$$\frac{1}{z} = 5 \implies z = \frac{1}{5} \tag{17}$$

$$\therefore x = 1, y = \frac{1}{3}, z = \frac{1}{5} \tag{18}$$

LaTeX