Assignment1(CambridgeUS)

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Question:

Show that $\binom{n}{k} p^k q^{n-k} \simeq \frac{1}{\sqrt{2\pi npq}} e^{\frac{-(k-np)^2}{2npq}}$ with p+q = 1 is a special case of $\frac{n!}{k_1!\cdots k_r!}\; p_1^{k_1} \cdots p_r^{k_r} \simeq \frac{e^{\{-\frac{1}{2}[\frac{(k_1-np_1)^2}{np_1}+\dots\dots+\frac{(k_r-np_r)}{np_r}]\}}}{\sqrt{(2\pi n)^{r-1} p_1 \cdots p_r}}$ obtained with
$r = 2, k_1 = k, k_2 = n - k, p_1 = p, p_2 = 1 - p.$

Solution

With $r = 2, k_1 = k, k_2 = n - k, p_1 = p, p_2 = 1 - p = q$,we obtain

$$k_1 - np_1 = k - np \tag{1}$$

$$k_2 - np_2 = n - k - nq = np - k \tag{2}$$

hence,the bracket in general equation equals:-

$$\frac{(k_1 - np_1)^2}{np_1} + \frac{(k_2 - np_2)^2}{np_2} = \frac{(k - np)^2}{n}(\frac{1}{p} + \frac{1}{q}) \tag{3}$$

$$\frac{(k - np)^2}{n}(\frac{1}{p} + \frac{1}{q}) = \frac{(k - np)^2}{npq} \tag{4}$$

Solution

From L.H.S of general equation,

$$L.H.S = \frac{n!}{k_1! \cdots k_r!} p_1^{k_1} \cdots p_r^{k_r} \tag{6}$$

$$\implies = \frac{n!}{k!.(n-k)!} p^k (1-p)^{n-k} \tag{7}$$

$$\implies L.H.S = \binom{n}{k} p^k (1-p)^{n-k} \tag{8}$$

Solution

From R.H.S of general equation

$$R.H.S = \frac{e^{\left\{\frac{-1}{2}\left[\frac{(k_1-np_1)^2}{np_1}+\cdots+\frac{(k_r-np_r)^2}{np_r}\right]\right\}}}{\sqrt{(2\pi n)^{r-1}p_1\cdots p_r}} \tag{9}$$

$$\tag{10}$$

From equation-(3)

$$\implies = \frac{e^{\left\{\frac{-1}{2}\left[\frac{(k-np)^2}{np(1-p)}\right]\right\}}}{\sqrt{(2\pi n)^1 p(1-p)}} \tag{11}$$

$$\tag{12}$$

$$\therefore R.H.S = \frac{e^{\frac{-(k-np)^2}{2np(1-p)}}}{\sqrt{2\pi np(1-p)}} \tag{13}$$

Join L.H.S and R.H.S of modified general equation i.e., (eq-(7)and eq-(11))

$$\binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{e^{\frac{-(k-np)^2}{2np(1-p)}}}{\sqrt{2\pi np(1-p)}} \tag{14}$$

$$(q = 1 - p) \tag{15}$$

$$\binom{n}{k} p^k q^{n-k} \simeq \frac{1}{\sqrt{2\pi npq}} e^{\frac{-(k-np)^2}{2npq}} \tag{16}$$