Assignment6

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Question:

Suppose x and y have joint density

$$f_{xy}(x,y) = \begin{cases} 1 & 0 \leq x \leq 2, 0 \leq y \leq 1, 2y \leq x \\ 0 & \textit{otherwise} \end{cases} \tag{1}$$

show that $z = x + y$ has density

$$f_{xy}(x,y) = \begin{cases} (1/3)z & 0 \leq z \leq 2 \\ 2 - (2/3)z & 2 \leq z \leq 3 \\ 0 & \textit{elsewhere} \end{cases} \tag{2}$$

Solution:

$$F_Z(z) = P(X + Y \leq z) \tag{3}$$

$$= \begin{cases} P(X + Y \leq z), & 0 < z < 2 \\ 1 - P(X + Y > z), & 2 < z < 3 \end{cases} \tag{4}$$

where, (5)

$$2y \leq x, \implies 3y \leq x + y \implies y \leq \frac{z}{3} \tag{6}$$

$$2y \leq x \leq z - y \tag{7}$$

$$0 \leq y \leq \frac{z}{3} \tag{8}$$

$$= \begin{cases} \int_0^{z/3} \int_{2y}^{z-y} f_{XY}(x, y) dx dy, & 0 < z < 2 \\ 1 - \int_{2z/3}^{2} \int_{z-x}^{x/2} f_{XY}(x, y) dy dx, & 2 < z < 3 \end{cases} \tag{9}$$

(10)

solution

$$= \begin{cases} \int_0^{z/3}(z-3y)dy, & 0<z<2 \\ 1-\int_{2z/3}^{2}(\frac{3x}{2}-z)dx & 2<z<3 \end{cases} \tag{11}$$

$$= \begin{cases} \frac{z^2}{6}, & 0<z<2 \\ 2z-\frac{z^2}{3}-2, & 2<z<3 \end{cases} \tag{12}$$

Thus

$$f_Z(z) = \frac{d}{dz}(F_Z(z)) \tag{13}$$

$$f_Z(z) = \begin{cases} \frac{1}{3}z, & 0<z<2 \\ 2-\frac{2z}{3}, & 2<z<3 \\ 0, & otherwise \end{cases} \tag{14}$$