Assignment 6

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Question:

Suppose x and y have joint density

$$f_{xy}(x,y) = \begin{cases} 1 & 0 \le x \le 2, 0 \le y \le 1, 2y \le x \\ 0 & otherwise \end{cases} \tag{1}$$

show that $z = x + y$ has density

$$f_{xy}(x,y) = \begin{cases} (1/3)z & 0 \le z \le 2 \\ 2 - (2/3)z & 2 \le z \le 3 \\ 0 & elsewhere \end{cases} \tag{2}$$

Solution:

$$F_Z(z) = P(X + Y \le z) \tag{3}$$

$$= \begin{cases} P(X+Y \le z), & 0 < z < 2 \\ 1 - P(X+Y > z), & 2 < z < 3 \end{cases} \tag{4}$$

$$where, \tag{5}$$

$$2y \le x, \implies 3y \le x + y \implies y \le \frac{z}{3} \tag{6}$$

$$2y \le x \le z - y \tag{7}$$

$$0 \le y \le \frac{z}{3} \tag{8}$$

$$= \begin{cases} \int_0^{z/3} \int_{2y}^{z-y} f_{XY}(x,y)dxdy, & 0 < z < 2 \\ 1 - \int_{2z/3}^{2} \int_{z-x}^{x/2} f_{XY}(x,y)dydx, & 2 < z < 3 \end{cases} \tag{9}$$

$$= \begin{cases} \int_0^{z/3} (z - 3y)dy, & 0 < z < 2 \\ 1 - \int_{2z/3}^{2} (\frac{3x}{2} - z)dx & 2 < z < 3 \end{cases} \tag{10}$$

$$= \begin{cases} \frac{z^2}{6}, & 0 < z < 2 \\ 2z - \frac{z^2}{3} - 2, & 2 < z < 3 \end{cases} \tag{11}$$

Thus

$$f_Z(z) = \frac{d}{dz}(F_Z(z)) \tag{12}$$

$$f_Z(z) = \begin{cases} \frac{1}{3}z, & 0 < z < 2 \\ 2 - \frac{2z}{3}, & 2 < z < 3 \\ 0, & otherwise \end{cases} \tag{13}$$