Assignment 7

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Question:

The events A,B and C are such that

$$P(A) = P(B) = P(C) = 0.5 \tag{1}$$

$$P(AB) = P(AC) = P(BC) = P(ABC) = 0.25 \tag{2}$$

show that the zero-one random variables associated with these events are not independent;they are,however,independent in pairs.

Solution:

$$P\{x_A = 1, x_B = 1, x_C = 1\} = P(ABC) = \frac{1}{4} \tag{3}$$

$$P\{x_A = 1\} = P(A) = \frac{1}{2} \tag{4}$$

$$P\{x_B = 1\} = P(B) = \frac{1}{2} \tag{5}$$

$$P\{x_C = 1\} = P(c) = \frac{1}{2} \tag{6}$$

$$P\{x_A = 1, x_B = 1, x_C = 1\} \neq P\{x_A = 1\}P\{x_B = 1\}P\{x_C = 1\} \tag{7}$$

hence x_A, x_B, x_C are not independent. But

$$P\{x_A = 1, x_B = 1\} = P(AB) = \frac{1}{4} = p\{x_A = 1\}p\{x_C = 1\} \tag{8}$$

similarly for any other combination,e.g.,
since $P(A) = P(AB) + P(A\overline{B})$,we conclude that
$P(A\overline{B}) = P(A) - P(AB)$

$$P(A\overline{B}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \tag{9}$$

$$P(\overline{B}) = 1 - P(B) = \frac{1}{2} \tag{10}$$

$$p\{x_A = 1, x_B = 0\} = P(A\overline{B}) = \frac{1}{4} \tag{11}$$

$$p\{x_B = 0\} = P(\overline{B}) = \frac{1}{2} \tag{12}$$

$$\implies p\{x_A = 1, x_B = 0\} = p\{x_A = 1\}p\{x_B = 0\} \tag{13}$$

So,these are independent in pairs.