

# Assignment7

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## Question:

The events  $A, B$  and  $C$  are such that

$$P(A) = P(B) = P(C) = 0.5 \quad (1)$$

$$P(AB) = P(AC) = P(BC) = P(ABC) = 0.25 \quad (2)$$

show that the zero-one random variables associated with these events are not independent; they are, however, independent in pairs.

## Solution:

$$P\{x_A = 1, x_B = 1, x_C = 1\} = P(ABC) = \frac{1}{4} \quad (3)$$

$$P\{x_A = 1\} = P(A) = \frac{1}{2} \quad (4)$$

$$P\{x_B = 1\} = P(B) = \frac{1}{2} \quad (5)$$

$$P\{x_C = 1\} = P(c) = \frac{1}{2} \quad (6)$$

$$P\{x_A = 1, x_B = 1, x_C = 1\} \neq P\{x_A = 1\}P\{x_B = 1\}P\{x_C = 1\} \quad (7)$$

hence  $x_A, x_B, x_C$  are not independent.

# Solution

But,

$$P\{x_A = 1, x_B = 1\} = P(AB) = \frac{1}{4} = p\{x_A = 1\}p\{x_C = 1\} \quad (8)$$

similarly for any other combination, e.g.,

since  $P(A) = P(AB) + P(A\bar{B})$ , we conclude that

$$P(A\bar{B}) = P(A) - P(AB)$$

$$P(A\overline{B}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \quad (9)$$

$$P(\overline{B}) = 1 - P(B) = \frac{1}{2} \quad (10)$$

$$p\{x_A = 1, x_B = 0\} = P(A\overline{B}) = \frac{1}{4} \quad (11)$$

$$p\{x_B = 0\} = P(\overline{B}) = \frac{1}{2} \quad (12)$$

$$\implies p\{x_A = 1, x_B = 0\} = p\{x_A = 1\}p\{x_B = 0\} \quad (13)$$

So, these are independent in pairs.