Assignment9

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Question:

Show that if X(t) is BL as in and Δ = $2\pi/\sigma$,then

$$X(t) = 4\sin^2\frac{\sigma t}{2}\sum_{n=-\infty}^{\infty}[\frac{X(n\Delta)}{(\sigma t-2n\pi)^2}+\frac{X'(n\Delta)}{\sigma(\sigma t-2n\pi)}] \tag{1}$$

Solution:

From Papoulis sampling expansion ,if we have N functions such as $P_1(\omega$,t),$\cdots$ $P_N(\omega$,t) such that//

$$H_1(\omega)P_1(\omega,\tau)+\cdots+H_N(\omega)P_N(\omega,\tau)=1 \tag{2}$$

$$H_1(\omega+c)P_1(\omega,\tau)+\cdots+H_N(\omega+c)P_N(\omega,\tau)=e^{jc\tau} \tag{3}$$

W.K.T

N = 2,$H_1(\omega)=1, H_2(\omega)=j\omega, c=\sigma$

$$H_1(\omega)P_1(\omega,\tau)+H_2(\omega)P_2(\omega,\tau)=1 \tag{4}$$

$$H_1(\omega+\sigma)P_1(\omega,\tau)+H_2(\omega+\sigma)P_2(\omega,\tau)=e^{j\sigma\tau} \tag{5}$$

$$\implies P_1(\omega,\tau)+j\omega P_2(\omega+\sigma)=1 \tag{6}$$

$$P_1(\omega,\tau)+j(\omega+\sigma)P_2(\omega,\tau)=e^{j\sigma\tau} \tag{7}$$

$$\implies P_1(\omega,\tau)=1-\frac{\omega}{\sigma}(e^{j\sigma\tau}-1) \tag{8}$$

$$P_2(\omega,\tau)\frac{1}{j\sigma}(e^{j\sigma\tau}-1) \tag{9}$$

$$\tag{10}$$

$$p_k(\tau)=\frac{1}{\sigma}\int_{-\sigma}^{0}P_k(\omega,\tau)e^{j\omega\tau}d\omega\cdots wherek=1 \tag{11}$$

$$\implies p_1(\tau)=\frac{1}{\sigma}\int_{-\sigma}^{0}p_1(\omega,\tau)e^{j\omega\tau}d\omega \tag{12}$$

$$\implies p_1(\tau)=\frac{1}{\sigma}\int_{-\sigma}^{0}(1-\frac{\omega}{\sigma}(e^{j\sigma\tau}-1))e^{j\omega\tau}d\omega \tag{13}$$

$$\implies p_1(\tau)=\frac{1}{\sigma}(\frac{e^{j\omega\tau}}{j\tau})_{-\sigma}^{0}-(\frac{e^{j\sigma\tau}-1}{\sigma^2})\int_{-\sigma}^{0}e^{j\omega\tau}\omega d\omega \tag{14}$$

$$\tag{15}$$

after integrating we get

$$p_1(\tau)=\frac{2-2\cos(\sigma\tau)}{\sigma^2\tau^2} \tag{16}$$

$$p_1(\tau)=\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2} \tag{17}$$

$$similarly, p_2(\tau)=\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau} \tag{18}$$

$$x(t+\tau)=\sum_{n=-\infty}^{\infty}[y_1(t+nNT)P_1(\tau-nNT)+\cdots \tag{19}$$

$$\cdots+y_N(t+nNT)p_N(\tau-nNT)] \tag{20}$$

$$witht=0, N=2, P_1andP_2, \tag{21}$$

$$\tau=t-nT, T=2\pi/\sigma, \Delta=2\pi/\sigma \tag{22}$$

$$x(\tau)=\sum_{n=-\infty}^{\infty}[x(n\Delta)P_1(\tau)+x'(n\Delta)P_2(\tau)] \tag{23}$$

$$W.K.T, p_1(\tau)=\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2} \tag{24}$$

$$p_2(\tau)=\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau} \tag{25}$$

$$x(\tau)=\sum_{n=-\infty}^{\infty}[x(n\Delta)\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2}+x'(n\Delta)\frac{4\sin^2(\sigma}{\sigma^2\tau} \tag{26}$$

$$\tau=t-nT=t-2n\pi/\sigma \implies \tau\sigma=\sigma t-2n\pi \tag{27}$$

$$\tag{28}$$

$$x(\tau)=4\sin^2(\frac{\sigma\tau}{2})\sum_{n=-\infty}^{\infty}[\frac{x(n\Delta)}{(\sigma t-2n\pi)^2}+\frac{x'(n\Delta)}{\sigma(\sigma t-2n\pi)}] \tag{29}$$

$$HenceProved$$