Assignment9

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Subsections

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Question:

Show that if X(t) is BL as in and $\Delta = 2\pi/\sigma$,then

$$X(t) = 4\sin^2\frac{\sigma t}{2}\sum_{n=-\infty}^{\infty}[\frac{X(n\Delta)}{(\sigma t - 2n\pi)^2} + \frac{X'(n\Delta)}{\sigma(\sigma t - 2n\pi)}] \tag{1}$$

Solution

From Papoulis sampling expansion ,if we have N functions such as $P_1(\omega$,t),$\cdots$ $P_N(\omega$,t) such that//

$$H_1(\omega)P_1(\omega,\tau)+\cdots+H_N(\omega)P_N(\omega,\tau)=1 \tag{2}$$

$$H_1(\omega+c)P_1(\omega,\tau)+\cdots+H_N(\omega+c)P_N(\omega,\tau)=e^{jc\tau} \tag{3}$$

W.K.T
N $= 2, H_1(\omega) = 1, H_2(\omega) = j\omega, c = \sigma$

$$H_1(\omega)P_1(\omega,\tau) + H_2(\omega)P_2(\omega,\tau) = 1 \tag{4}$$

$$H_1(\omega+\sigma)P_1(\omega,\tau) + H_2(\omega+\sigma)P_2(\omega,\tau) = e^{j\sigma\tau} \tag{5}$$

$$\implies P_1(\omega,\tau) + j\omega P_2(\omega+\sigma) = 1 \tag{6}$$

$$P_1(\omega,\tau) + j(\omega+\sigma)P_2(\omega,\tau) = e^{j\sigma\tau} \tag{7}$$

$$\implies P_1(\omega,\tau) = 1 - \frac{\omega}{\sigma}(e^{j\sigma\tau} - 1) \tag{8}$$

$$P_2(\omega,\tau)\frac{1}{j\sigma}(e^{j\sigma\tau} - 1) \tag{9}$$

$$\tag{10}$$

Solution

$$p_k(\tau) = \frac{1}{\sigma}\int_{-\sigma}^{0} P_k(\omega,\tau)e^{j\omega\tau}d\omega \cdots wherek = 1 \tag{11}$$

$$\implies p_1(\tau) = \frac{1}{\sigma}\int_{-\sigma}^{0} p_1(\omega,\tau)e^{j\omega\tau}d\omega \tag{12}$$

$$\implies p_1(\tau) = \frac{1}{\sigma}\int_{-\sigma}^{0} (1 - \frac{\omega}{\sigma}(e^{j\sigma\tau} - 1))e^{j\omega\tau}d\omega \tag{13}$$

$$\implies p_1(\tau) = \frac{1}{\sigma}(\frac{e^{j\omega\tau}}{j\tau})_{-\sigma}^{0} - (\frac{e^{j\sigma\tau} - 1}{\sigma^2})\int_{-\sigma}^{0} e^{j\omega\tau}\omega d\omega \tag{14}$$

$$\tag{15}$$

after integrating we get

$$p_1(\tau) = \frac{2 - 2\cos(\sigma\tau)}{\sigma^2\tau^2} \tag{16}$$

$$p_1(\tau) = \frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2} \tag{17}$$

$$similarly, p_2(\tau) = \frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau} \tag{18}$$

$$x(t+\tau) = \sum_{n=-\infty}^{\infty} [y_1(t+nNT)P_1(\tau - nNT) + \cdots \tag{19}$$

$$\cdots + y_N(t+nNT)p_N(\tau - nNT)] \tag{20}$$

$$witht = 0, N = 2, P_1 andP_2, \tag{21}$$

$$\tau = t - nT, T = 2\pi/\sigma, \Delta = 2\pi/\sigma \tag{22}$$

Solution

$$x(\tau) = \sum_{n=-\infty}^{\infty} [x(n\Delta)P_1(\tau) + x'(n\Delta)P_2(\tau)] \tag{23}$$

$$W.K.T, p_1(\tau) = \frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2} \tag{24}$$

$$p_2(\tau) = \frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau} \tag{25}$$

$$x(\tau) = \sum_{n=-\infty}^{\infty} [x(n\Delta)\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau^2} + x'(n\Delta)\frac{4\sin^2(\sigma\tau/2)}{\sigma^2\tau}] \tag{26}$$

$$\tau = t - nT = t - 2n\pi/\sigma \implies \tau\sigma = \sigma t - 2n\pi \tag{27}$$

$$\tag{28}$$

Solution:

$$x(\tau) = 4\sin^2(\frac{\sigma\tau}{2})\sum_{n=-\infty}^{\infty}[\frac{x(n\Delta)}{(\sigma t - 2n\pi)^2} + \frac{x'(n\Delta)}{\sigma(\sigma t - 2n\pi)}] \tag{29}$$

$$HenceProved$$