Assignment 4

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Example 4-15

Determine the conditional density $f(x||X-\eta| \le k\sigma)$ of an $N(\eta;\sigma)$ random variable.

Solution:

$$P|X-\eta| \le K\sigma = P\eta - k\sigma \le X \le \eta + k\sigma = 2\int_0^k \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \tag{1}$$

Let M = $|X-\eta| \le k\sigma$

$$f(x|M) = \frac{d}{dx}(F(x|M)) \tag{2}$$

$$= \frac{d}{dx}(F(x||X-\eta| \le k\sigma)) \tag{3}$$

$$= \frac{d}{dx}(F(x|\eta - k\sigma \le X \le \eta + k\sigma)) \tag{4}$$

W.K.T

$$F(x|b \le X \le a) = \frac{P\{X \le x, b \le X \le a\}}{P\{b \le X \le a\}} \tag{5}$$

$$If x \ge a, then \{X \le x, b \le X \le a\} = \{b \le X \le a\}.Hence \tag{6}$$

$$= F(x|b \le X \le a) = \frac{F(a)-F(b)}{F(a)-F(b)} = 1 \tag{7}$$

$$If b \le x \le a, then \{X \le x, b \le X \le a\} = \{b \le X \le x\}.Hence \tag{8}$$

$$= F(x|b \le X \le a) = \frac{F(x)-F(b)}{F(a)-F(b)} \tag{9}$$

$$if(x \le b)F(x|b \le x \le a) = 0 \tag{10}$$

From eq - (9) and (4)

$$f(x|M) = \frac{d}{dx}(\frac{F(x)-F(b)}{F(a)-F(b)}) \tag{11}$$

$$= \frac{f(x)}{F(a)-F(b)} \tag{12}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{13}$$

$$x -> (x-\eta)^2/2(\sigma)^2 \tag{14}$$

$$a = \eta + k\sigma \tag{15}$$

$$b = \eta - k\sigma \tag{16}$$

$$F(a) - f(b) = F(\eta + k\sigma) - F(\eta - k\sigma) \tag{17}$$

$$\implies F(\eta + k\sigma) - F(\eta - k\sigma) = P(|X-\eta| \le k\sigma) \tag{18}$$

$$\implies f(x|M) = \frac{1}{P(|X-\eta| \le k\sigma)} \frac{e^{\frac{-(x-\eta)^2}{2(\sigma)^2}}}{\sigma\sqrt{2\pi}} M \quad = |X - \tag{19}$$

$$f(x||X-\eta| \le k\sigma) = \frac{1}{P(|X-\eta| \le k\sigma)} \frac{e^{\frac{-(x-\eta)^2}{2(\sigma)^2}}}{\sigma\sqrt{2\pi}} \tag{20}$$