Assignment 4

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Question

Example 4-15

Determine the conditional density $f(x||X-\eta| \leq k\sigma)$ **of an** $N(\eta;\sigma)$ **random variable.**

Solution

$$P|X-\eta| \leq K\sigma = P\eta - k\sigma \leq X \leq \eta + k\sigma = 2\int_0^k \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \qquad (1)$$

Let M $= |X-\eta| \leq k\sigma$

$$f(x|M) = \frac{d}{dx}(F(x|M)) \qquad (2)$$

$$= \frac{d}{dx}(F(x||X-\eta| \leq k\sigma)) \qquad (3)$$

$$= \frac{d}{dx}(F(x|\eta - k\sigma \leq X \leq \eta + k\sigma)) \qquad (4)$$

solution

W.K.T

$$F(x|b \leq X \leq a) = \frac{P\{X \leq x, b \leq X \leq a\}}{P\{b \leq X \leq a\}} \tag{5}$$

$$If x \geq a, then\{X \leq x, b \leq X \leq a\} = \{b \leq X \leq a\}.Hence \tag{6}$$

$$= F(x|b \leq X \leq a) = \frac{F(a) - F(b)}{F(a) - F(b)} = 1 \tag{7}$$

$$If b \leq x \leq a, then\{X \leq x, b \leq X \leq a\} = \{b \leq X \leq x\}.Hence \tag{8}$$

$$= F(x|b \leq X \leq a) = \frac{F(x) - F(b)}{F(a) - F(b)} \tag{9}$$

$$if(x \leq b)F(x|b \leq x \leq a) = 0 \tag{10}$$

Solution

From eq - (9) and (4)

$$f(x|M) = \frac{d}{dx}(\frac{F(x) - F(b)}{F(a) - F(b)}) \tag{11}$$

$$= \frac{f(x)}{F(a) - F(b)} \tag{12}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \tag{13}$$

$$x- > (x - \eta)^2 / 2(\sigma)^2 \tag{14}$$

$$a = \eta + k\sigma \tag{15}$$

$$b = \eta - k\sigma \tag{16}$$

$$F(a) - f(b) = F(\eta + k\sigma) - F(\eta - k\sigma) \tag{17}$$

$$\implies F(\eta + k\sigma) - F(\eta - k\sigma) = P(|X - \eta| \leq k\sigma) \tag{18}$$

Solution

$$\implies f(x|M) = \frac{1}{P(|X-\eta| \leq k\sigma)} \frac{e^{\frac{-(x-\eta)^2}{2(\sigma)^2}}}{\sigma\sqrt{2\pi}} \tag{19}$$

$$M = |X-\eta| \leq k\sigma \tag{20}$$

$$f(x||X-\eta| \leq k\sigma) = \frac{1}{P(|X-\eta| \leq k\sigma)} \frac{e^{\frac{-(x-\eta)^2}{2(\sigma)^2}}}{\sigma\sqrt{2\pi}} \tag{21}$$