

# Random Numbers

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```
wget https://github.com/gadepall/
probability/raw/master/manual/codes/
coeffs.h
```

1.2 Load the uni.dat file into python and plot the empirical CDF of  $U$  using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \quad (1.1)$$

## Abstract

This manual provides a simple introduction to the generation of random numbers

**Solution:** The following code plots Fig. 1.2

```
wget https://github.com/gadepall/
probability/raw/master/manual/codes/
cdf_plot.py
```

## 1 Uniform Random Numbers

Let  $U$  be a uniform random variable between 0 and 1.

1.1 Generate  $10^6$  samples of  $U$  using a C program and save into a file called uni.dat .

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/gadepall/
probability/raw/master/manual/codes/
exrand.c
```

1.3 Find a theoretical expression for  $F_U(x)$ .

1.4 The mean of  $U$  is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^N U_i \quad (1.2)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \quad (1.3)$$

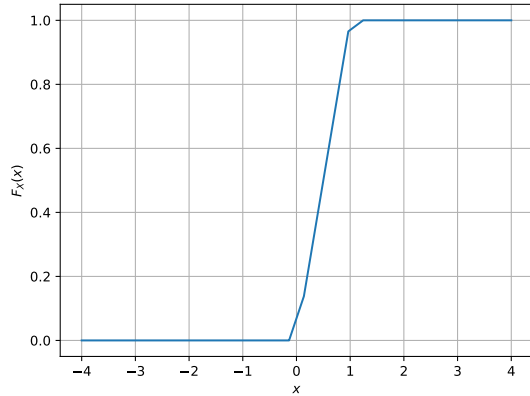


Figure 1.2: The CDF of  $U$

Write a C program to find the mean and variance of  $U$ .

1.5 Verify your result theoretically given that

$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \quad (1.4)$$

## 2 Central Limit Theorem

2.1 Generate  $10^6$  samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \quad (2.1)$$

using a C program, where  $U_i, i = 1, 2, \dots, 12$  are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat

2.2 Load gau.dat in python and plot the empirical CDF of  $X$  using the samples in gau.dat. What properties does a CDF have?

Figure 2.2: The CDF of  $X$

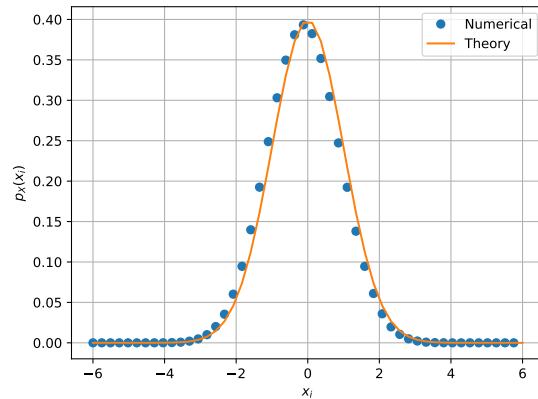


Figure 2.3: The PDF of  $X$

**Solution:** The CDF of  $X$  is plotted in Fig. 2.2

2.3 Load gau.dat in python and plot the empirical PDF of  $X$  using the samples in gau.dat. The PDF of  $X$  is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \quad (2.2)$$

What properties does the PDF have?

**Solution:** The PDF of  $X$  is plotted in Fig. 2.3 using the code below

```
wget https://github.com/gadepall/
probability/raw/master/manual/codes/
pdf_plot.py
```

2.4 Find the mean and variance of  $X$  by writing a C program.

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

### **3 From Uniform to Other**

3.1 Generate samples of

$$V = -2 \ln(1 - U) \quad (3.1)$$

and plot its CDF.

3.2 Find a theoretical expression for  $F_V(x)$ .