





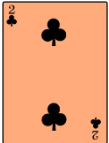

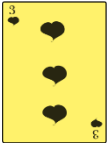


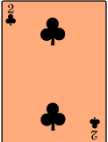
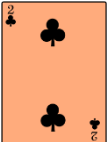


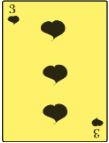




**WHY COMBINATORIAL
OPTIMIZATION PROBLEMS
ARE DIFFICULT TO SOLVE?**

CARD GAME EXAMPLE



x_1	x_2	x_3
		
		
		
		
		
		

Size of search space = 6

DRONE NAVIGATION EXAMPLE



Size of search space = 24

x_1	x_2	x_3	x_4

SIZE OF SEARCH SPACE

$$n! = n(n - 1)(n - 2) \dots 3 \times 2 \times 1$$

Where n is the number of objects (variables)

COST OF SOLVING COMBINATORIAL PROBLEMS

n

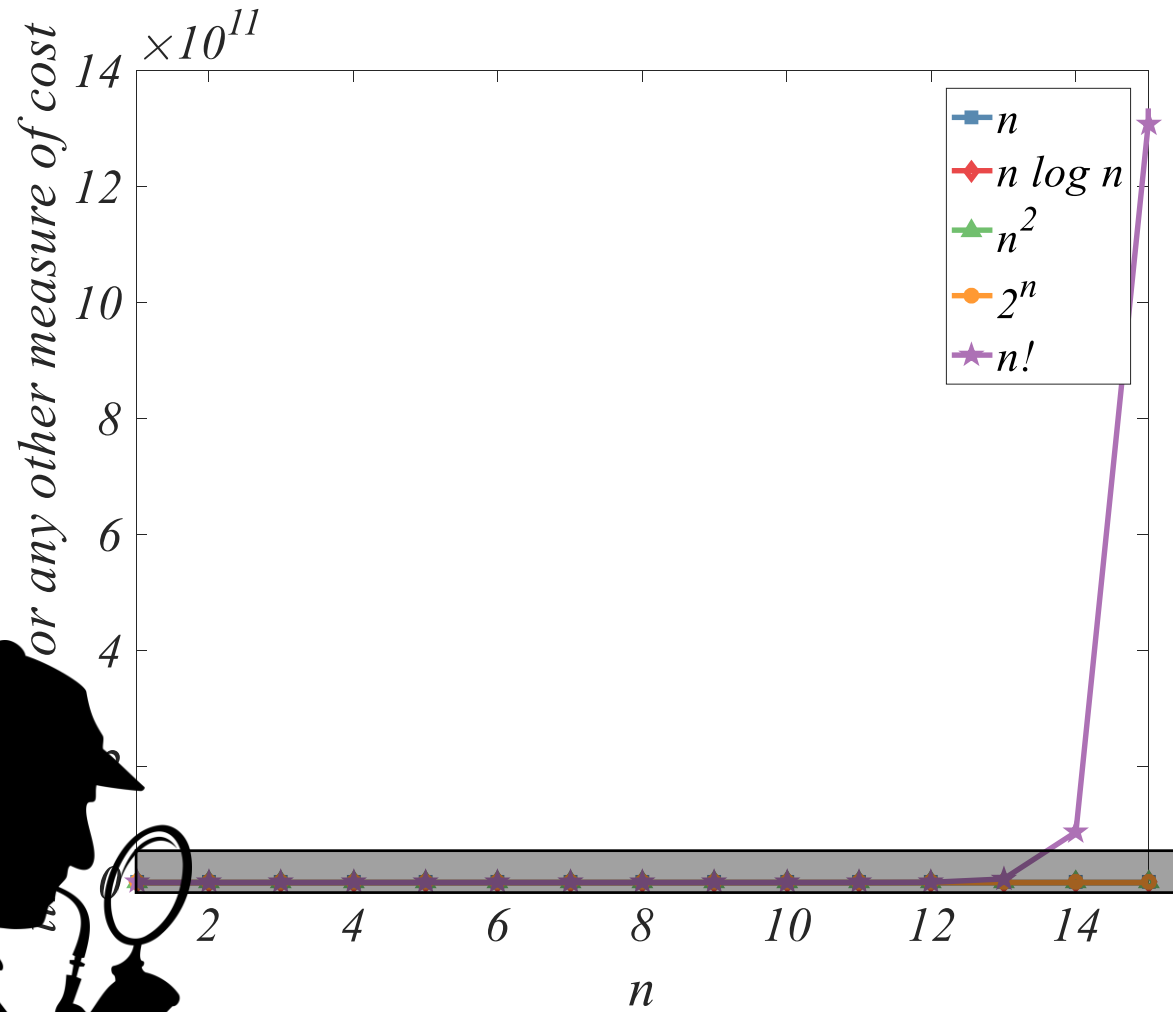
$n \log n$

n^2

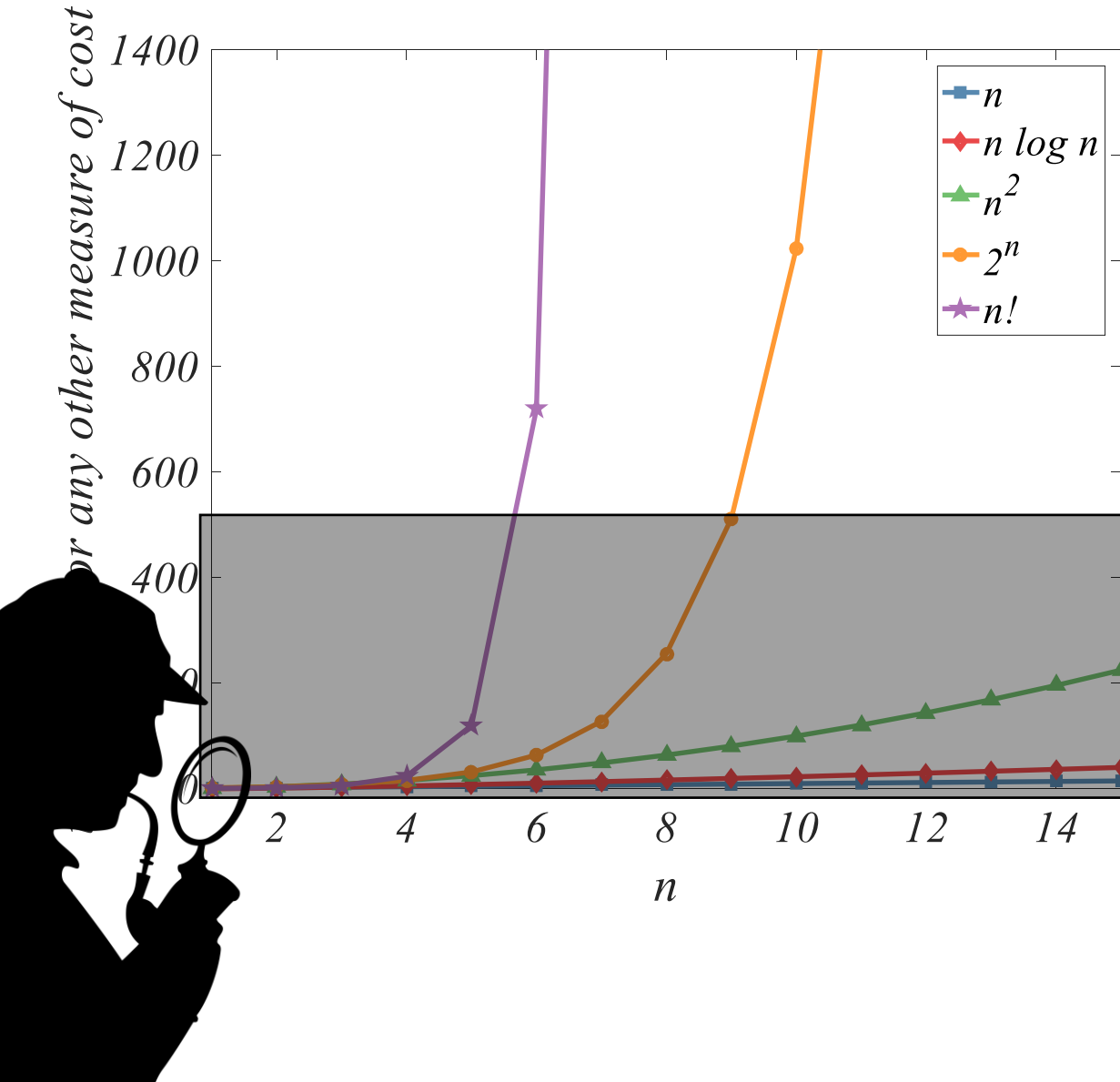
2^n

$n!$

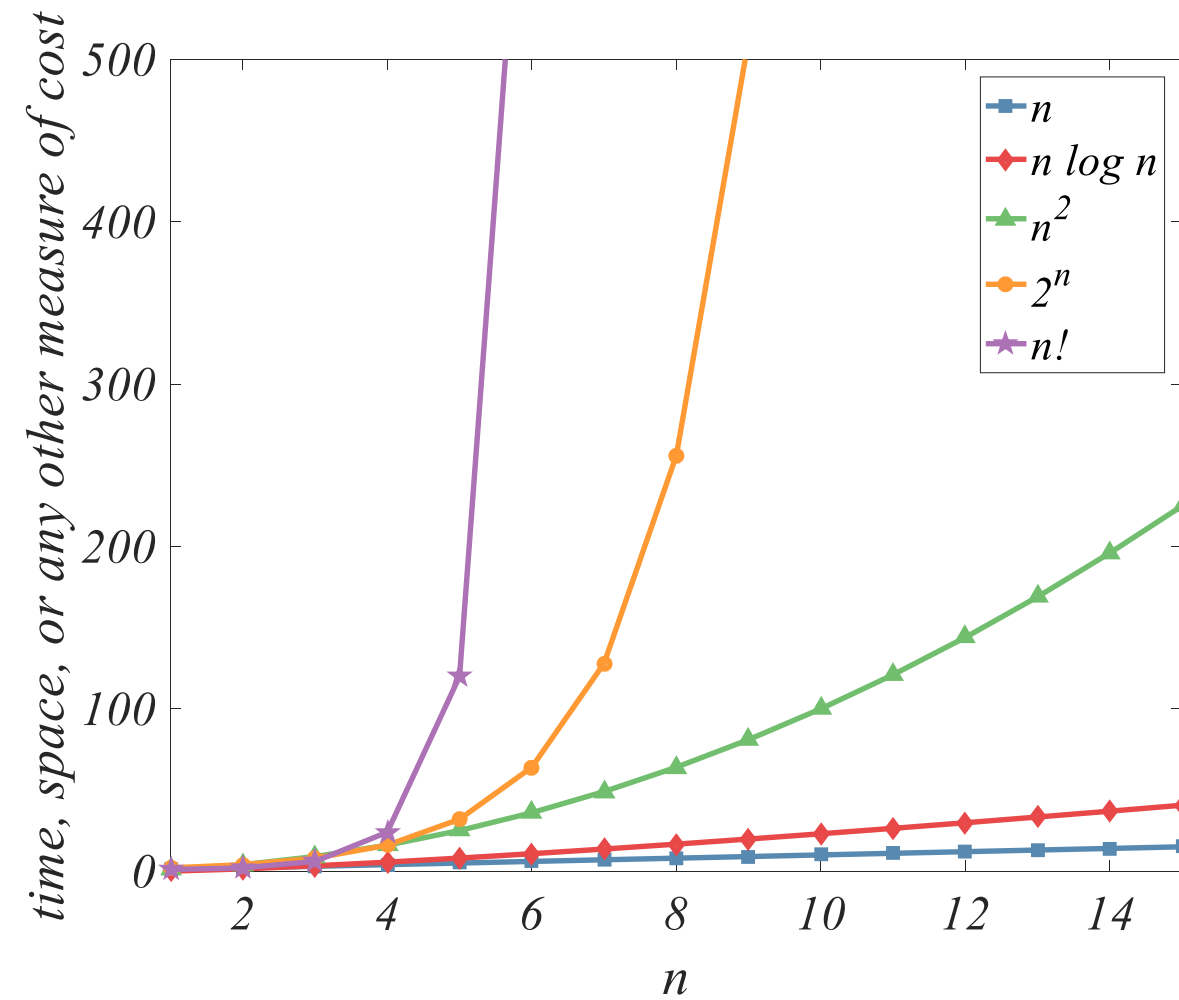
COST OF SOLVING COMBINATORIAL PROBLEMS



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COST OF SOLVING COMBINATORIAL PROBLEMS

$$n! > 2^n > n^2 > n \log n > n$$

COST OF SOLVING COMBINATORIAL PROBLEMS

As $n \rightarrow \infty$

Factorial > Exponential > polynomial > Logarithmic > Linear

$n!$ > 2^n > n^2 > $n \log n$ > n

$3^{\sqrt{n}}$

\sqrt{n}

$\ln n$

$2n$

$\cosh n$

$n^2 + 1$

$(\ln n)^3$

$5n$

e^x

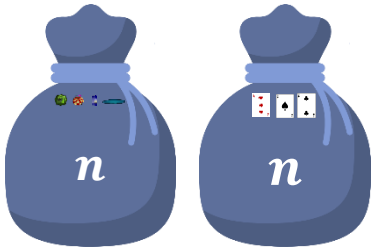
$n\sqrt{1+n^2}$

$\ln(\ln n)$

$10n$

LET'S BLOW YOUR MIND 😊

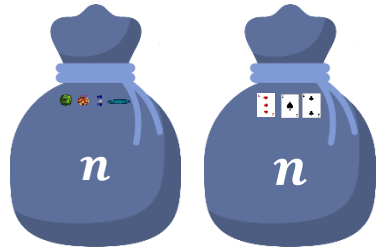
n	$n!$	Time (s,d,y)
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1 calculation
per second

LET'S BLOW YOUR MIND 😊

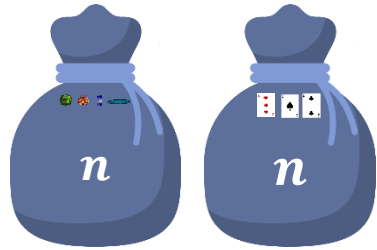
n	$n!$	Time (s,d,y)
3	$3! = 6$	3 seconds



1 calculation
per second

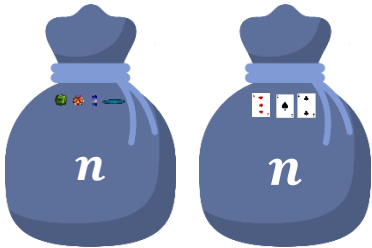
LET'S BLOW YOUR MIND 😊

n	$n!$	Time (s,d,y)
3	$3! = 6$	3 seconds
5	$5! = 120$	120 seconds



1 calculation
per second

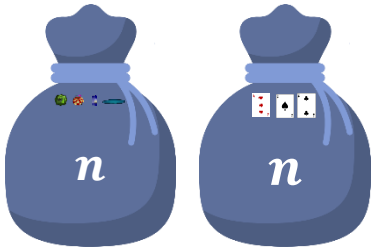
LET'S BLOW YOUR MIND 😊



1 calculation
per second

<i>n</i>	<i>n</i> !	Time (s,d,y)	
3	3! = 6	3	seconds
5	5! = 120	120	seconds
10	10! = 3628800	42	days

LET'S BLOW YOUR MIND 😊

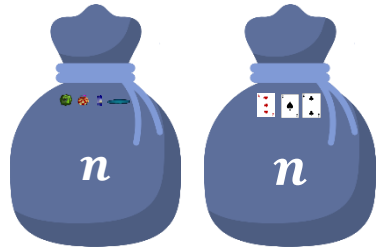


1 calculation
per second

n	$n!$	Time (s,d,y)	
3	$3! = 6$	3	seconds
5	$5! = 120$	120	seconds
10	$10! = 3628800$	42	days
11	$11! = 39916800$	462	days ≈ 1 year

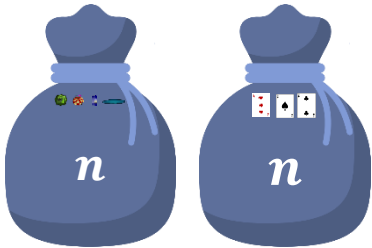
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1 calculation
per second

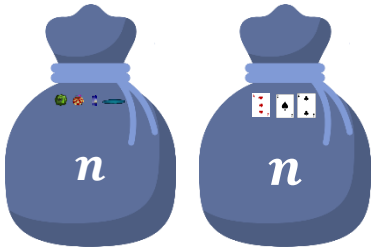
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12	$12! = 479001600$	5544	days ≈ 15 years
13	$13! = 6227020800$	72072	days ≈ 197 years

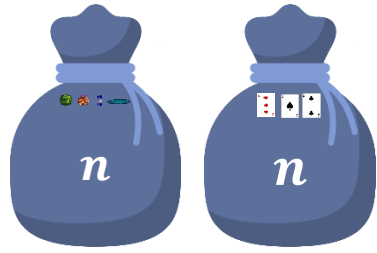
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13	13! = 6227020800	72072	days ≈ 197 years
14	14! = 87178291200	2764	years

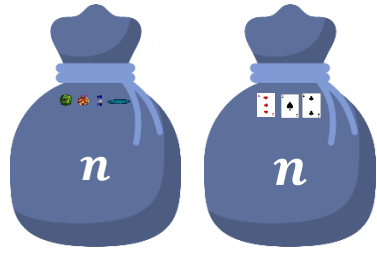
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15	15! = 1.3076744e+12	41000	years

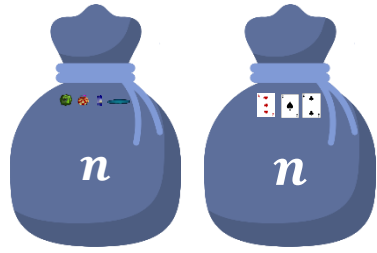
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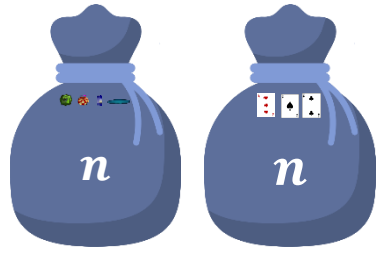
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16	16! = 2.092279e+13	663457	years
17	17! = 3.5568743e+14	≈ 11M	years

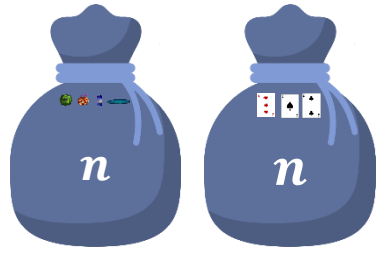
LET'S BLOW YOUR MIND 😊



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16	$16! = 2.092279\text{e}+13$	663457 years
17	$17! = 3.5568743\text{e}+14$	\approx 11M years
18	$18! = 6.4023737\text{e}+15$	\approx 203M years

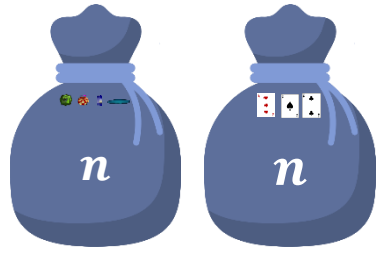
LET'S BLOW YOUR MIND 😊



1 calculation
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19	19! = 1.216451e+17	≈ 4 B years

LET'S BLOW YOUR MIND 😊



1 calculation
per second

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18	$18! = 6.4023737\text{e}+15$	\approx 203M years
19	$19! = 1.216451\text{e}+17$	\approx 4 B years
20	$20! = 2.432902\text{e}+18$	\approx 77B years

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$n = 19 \approx 4 \text{ B years}$

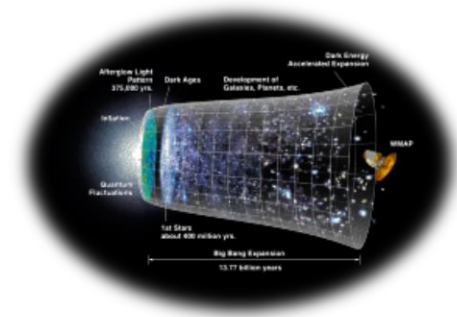
Earth



4.543 billion years old

$n = 20 \approx 77 \text{ B years}$

Big Bang



14 billion years old

LET'S BLOW YOUR MIND 😊

n	$n!$	Time (s,d,y)
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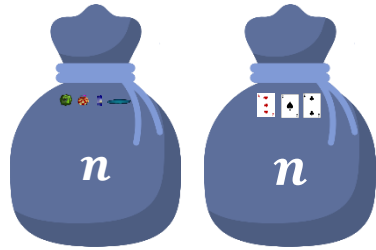


2.0GHz CPU

2,000,000,000
calculation per second

LET'S BLOW YOUR MIND 😊

<i>n</i>	<i>n!</i>	Time (s,d,y)	
3	3! = 6	1.5e-9	seconds
5	5! = 120	6e-8	seconds
10	10! = 3628800	0.0018144	seconds



2.0GHz CPU

2,000,000,000
calculation per second

LET'S BLOW YOUR MIND 😊

n	$n!$	Time (s,d,y)	
3	$3! = 6$	1.5e-9	seconds
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10	$10! = 3628800$	0.0018144	seconds
11	$11! = 39916800$	0.0199584	seconds
12	$12! = 479001600$	0.2395008	seconds
13	$13! = 6227020800$	3.1135104	seconds
14	$14! = 87178291200$	43.5891456	seconds
15	$15! = 1.3076744\text{e}+12$	653 s \approx 10	minutes

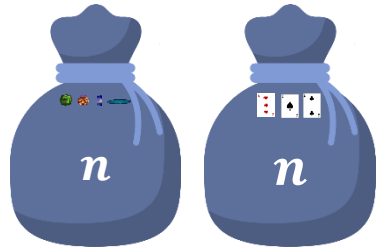


2.0GHz CPU

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15	$15! = 1.3076744\text{e}+12$	653 s \approx 10	minutes
16	$16! = 2.092279\text{e}+13$	10461 \approx 2.9	hours
17	$17! = 3.5568743\text{e}+14$	177843 \approx 2	days
18	$18! = 6.4023737\text{e}+15$	3201186 \approx 37	days
19	$19! = 1.216451\text{e}+17$	\approx 1.2	years
20	$20! = 2.432902\text{e}+18$	\approx 38	years

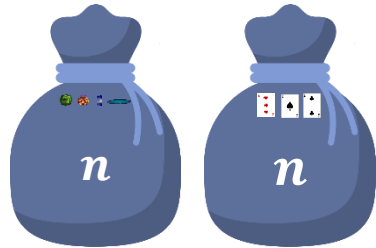


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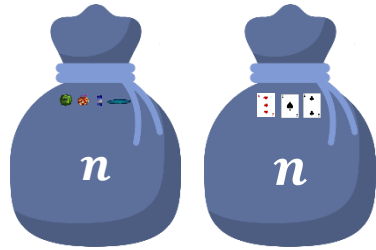
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20	20! = 2.432902e+18	≈ 38	years
21	21! = 5.1090942e+19	≈ 810	years



2.0GHz CPU
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21	21! = 5.1090942e+19	≈ 810	years
22	22! = 1.1240007e+21	≈ 18000	years

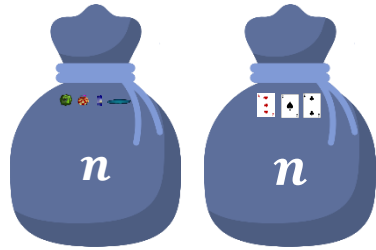


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22	22! = 1.1240007e+21	≈ 18000	years
23	23! = 2.5852017e+22	≈ 410000	years

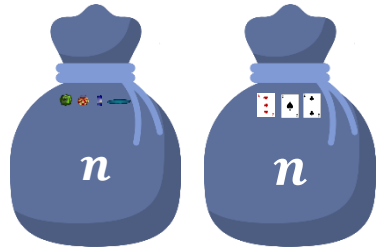


2.0GHz CPU

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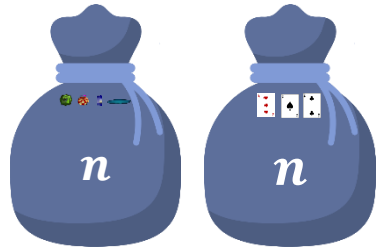
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23	23! = 2.5852017e+22	≈ 410000	years
24	24! = 6.204484e+23	≈ 9.9 M	years



2.0GHz CPU
2,000,000,000
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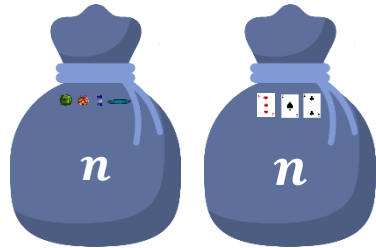
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21	21! = 5.1090942e+19	≈ 810	years
22	22! = 1.1240007e+21	≈ 18000	years
23	23! = 2.5852017e+22	≈ 410000	years
24	24! = 6.204484e+23	≈ 9.9 M	years
25	25! = 1.551121e+25	≈ 245M	years



2.0GHz CPU
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13	13! = 6227020800	3.1135104	seconds
14	14! = 87178291200	43.5891456	seconds
15	15! = 1.3076744e+12	653 s ≈ 10	minutes
16	16! = 2.092279e+13	10461 ≈ 2.9	hours
17	17! = 3.5568743e+14	177843 ≈ 2	days
18	18! = 6.4023737e+15	3201186 ≈ 37	days
19	19! = 1.216451e+17	≈ 1.2	years
20	20! = 2.432902e+18	≈ 38	years
21	21! = 5.1090942e+19	≈ 810	years
22	22! = 1.1240007e+21	≈ 18000	years
23	23! = 2.5852017e+22	≈ 410000	years
24	24! = 6.204484e+23	≈ 9.9 M	years
25	25! = 1.551121e+25	≈ 245M	years
26	26! = 4.0329146e+26	≈ 6B	years



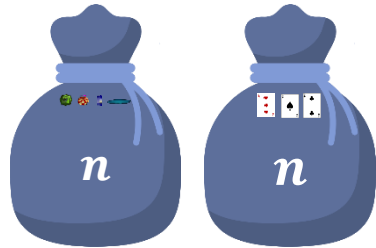
2.0GHz CPU

2,000,000,000
calculation per second



LET'S BLOW YOUR MIND 😊

<i>n</i>	<i>n</i> !	Time (s,d,y)	
3	3! = 6	1.5e-9	seconds
5	5! = 120	6e-8	seconds
10	10! = 3628800	0.0018144	seconds
11	11! = 39916800	0.0199584	seconds
12	12! = 479001600	0.2395008	seconds
13	13! = 6227020800	3.1135104	seconds
14	14! = 87178291200	43.5891456	seconds
15	15! = 1.3076744e+12	653 s ≈ 10	minutes
16	16! = 2.092279e+13	10461 ≈ 2.9	hours
17	17! = 3.5568743e+14	177843 ≈ 2	days
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21	21! = 5.1090942e+19	≈ 810	years
22	22! = 1.1240007e+21	≈ 18000	years
23	23! = 2.5852017e+22	≈ 410000	years
24	24! = 6.204484e+23	≈ 9.9 M	years
25	25! = 1.551121e+25	≈ 245M	years
26	26! = 4.0329146e+26	≈ 6B	years
27	27! = 1.0888869e+28	≈ 172B	years



2.0GHz CPU

2,000,000,000
calculation per second



COMBINATORIAL EXPLOSION



COMBINATORIAL EXPLOSION

