ANT COLONY OPTIMIZAION: MATHEMATICAL MODELS

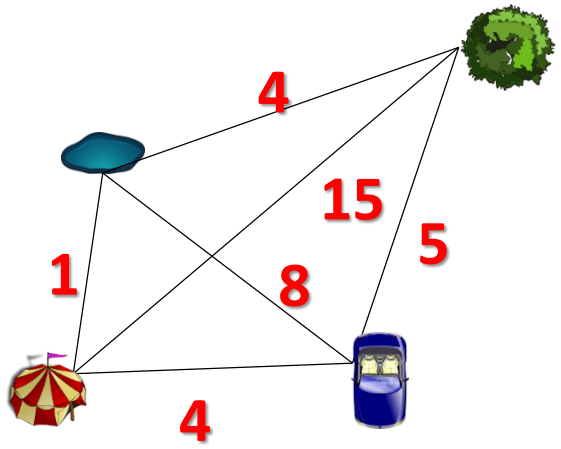
MATHEMATICAL MODEL OF ACO

- **Pheromone (model and vaporization)**
- **Decision making**

PHEROMONE MATRIX

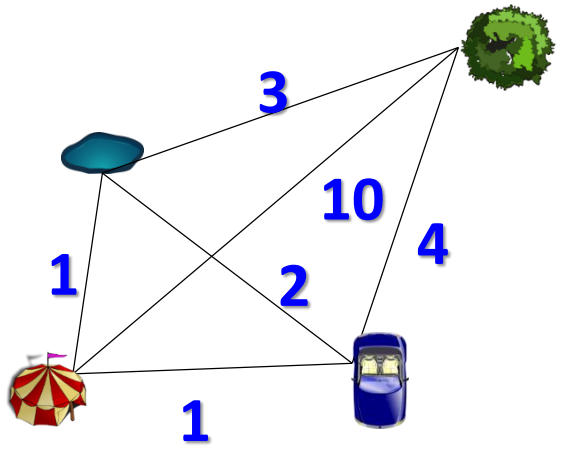
	0	5	15	4
	5	0	4	8
	15	4	0	1
	4	8	1	0

Cost matrix (distance)



	0	4	10	3
	4	0	1	2
	10	1	0	1
	3	2	1	0

Pheromone matrix



INSPIRATION

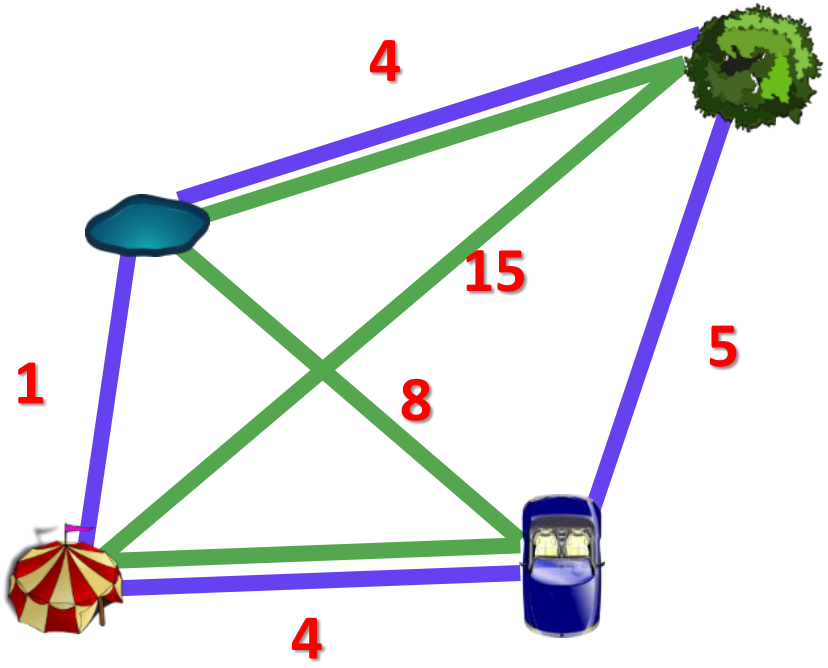
$$\Delta\tau_{i,j}^{k} = \begin{cases} \frac{1}{L_k} & k^{th} ant\ travels\ on\ the\ edge\ i,j \\ 0 & othereise \end{cases}$$

$$\tau_{i,j}^{k} = \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$$ **Without vaporization**

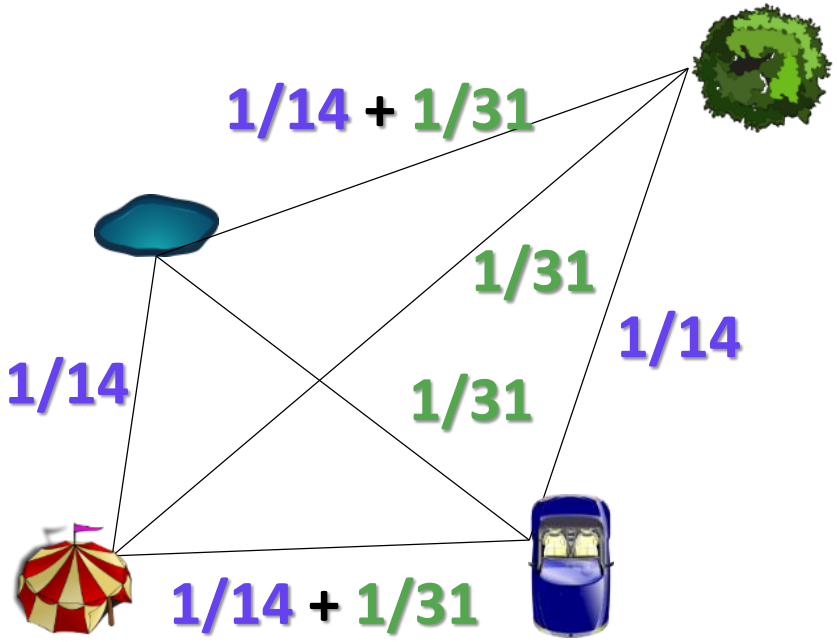
$$\tau_{i,j}^{k} = (1-\rho)\,\tau_{i,j} + \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$$ **With vaporization**

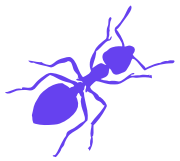
PHEROMONE MATRIX

Cost graph

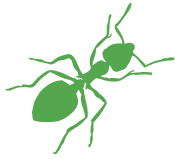


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$$

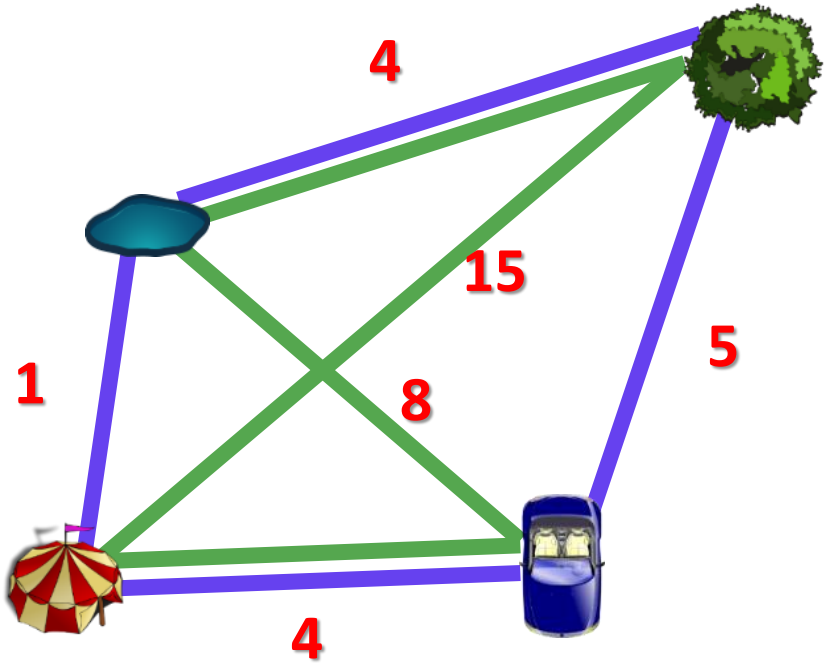


$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{31}$$

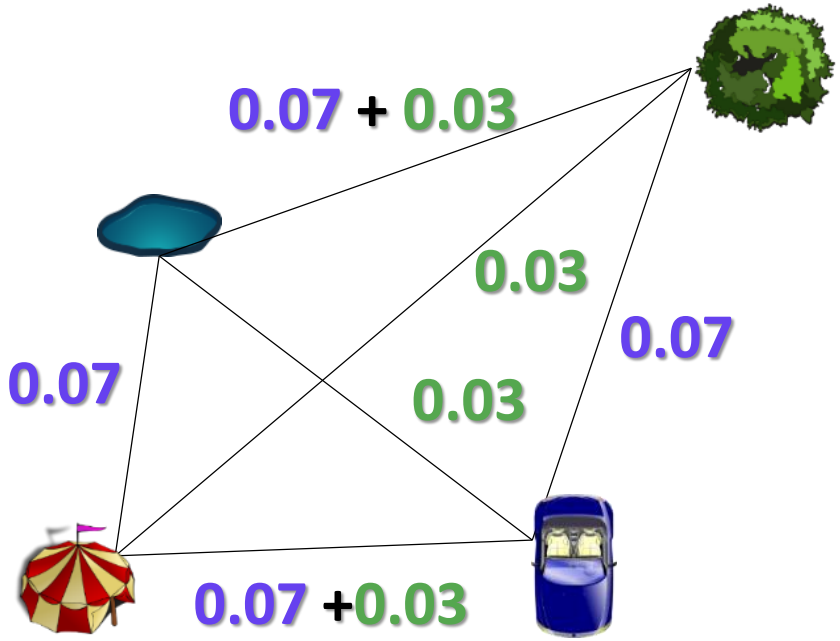
$$\tau_{i,j} = \sum_{k=1}^{m} \Delta\tau_{i,j}^k$$

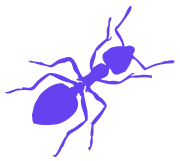
PHEROMONE MATRIX

Cost graph

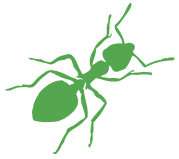


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$$

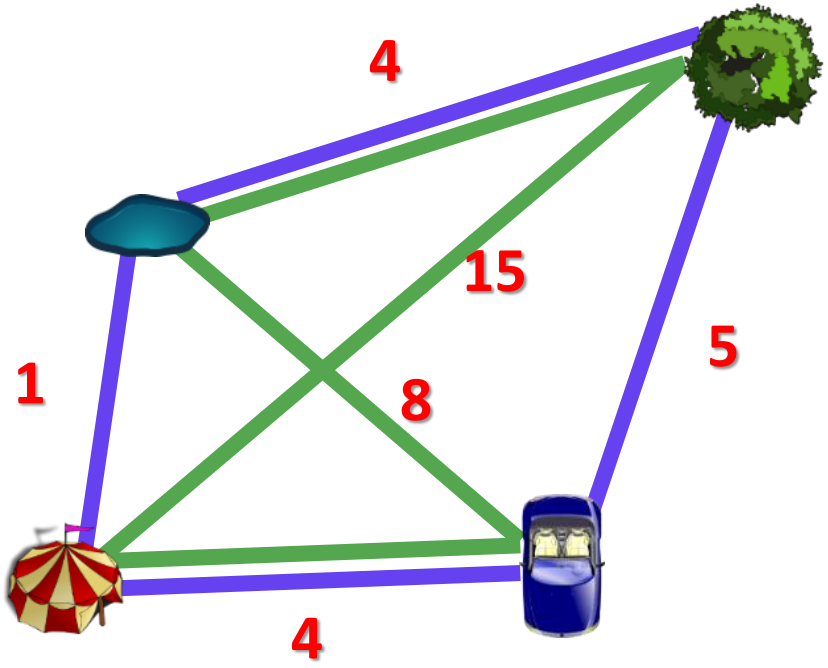


$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{31}$$

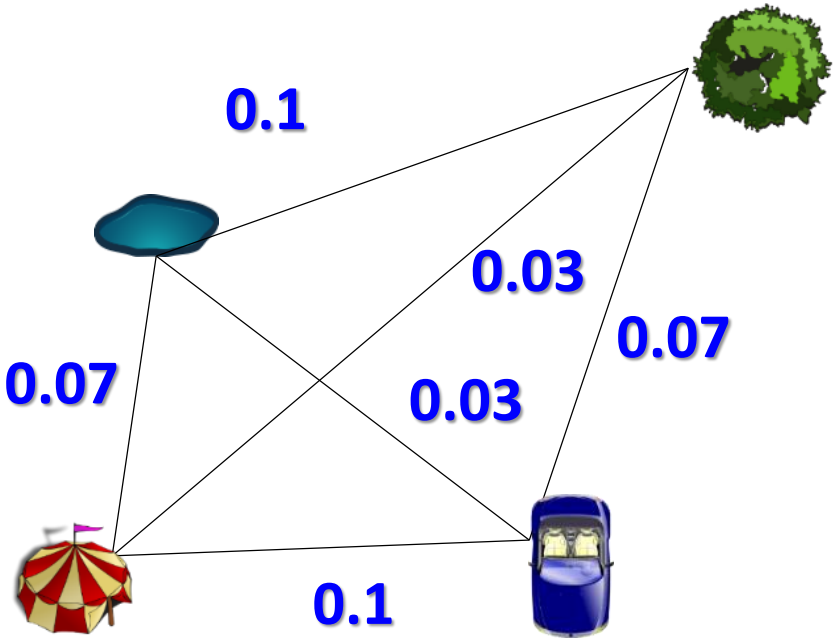
$$\tau_{i,j} = \sum_{k=1}^{m} \Delta\tau_{i,j}^k$$

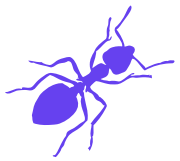
PHEROMONE MATRIX

Cost graph

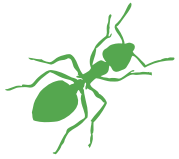


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$$

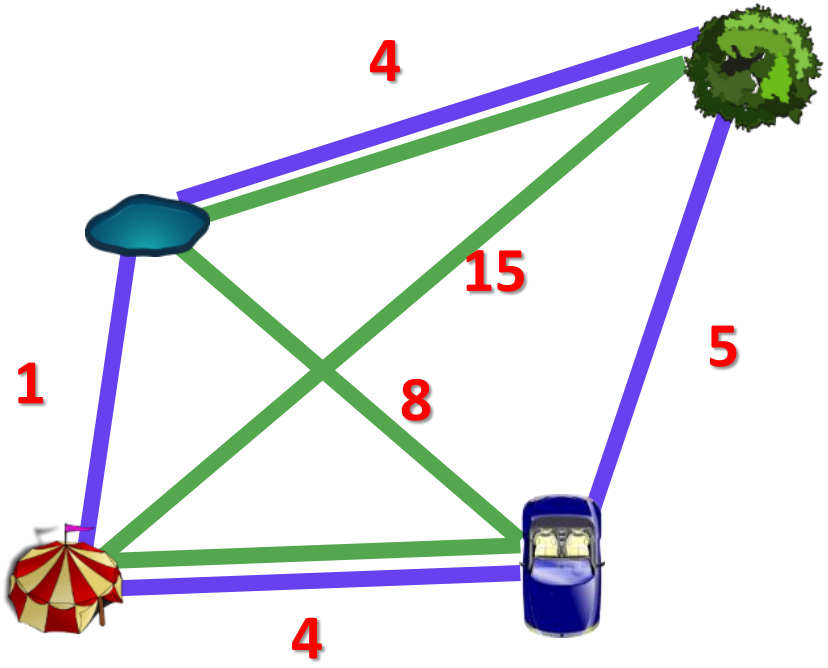


$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{31}$$

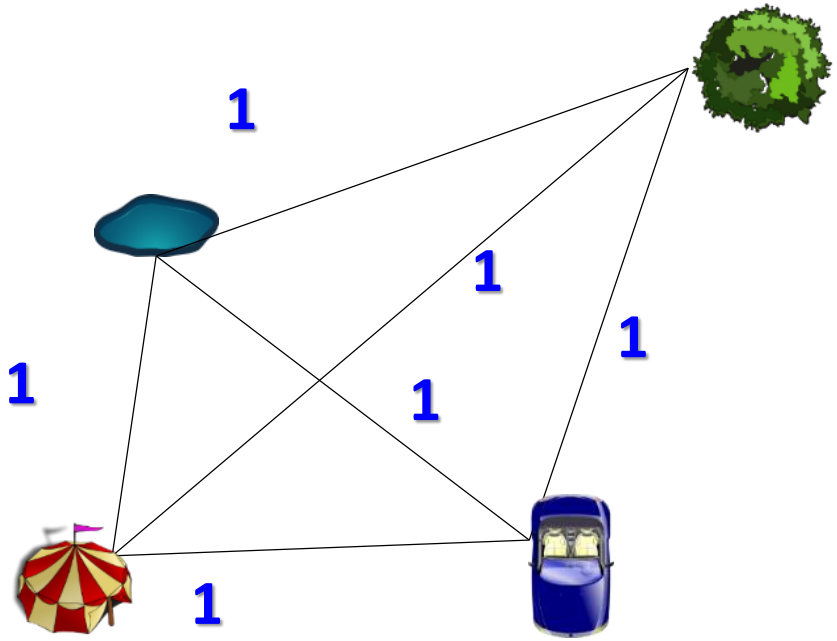
$$\tau_{i,j} = \sum_{k=1}^{m} \Delta\tau_{i,j}^k$$

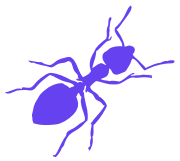
PHEROMONE MATRIX

Cost graph

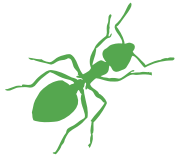


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^{1} = \frac{1}{14}$$



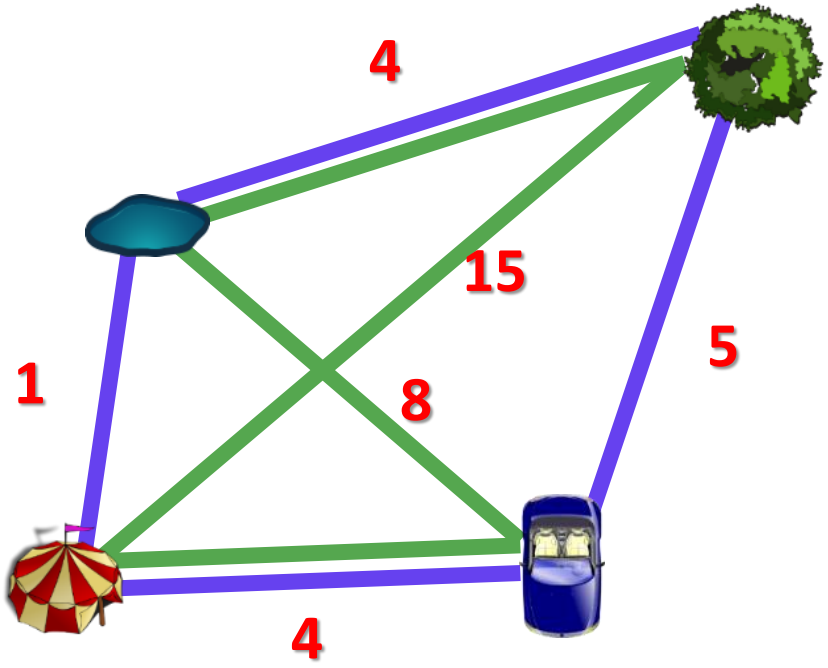
$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^{2} = \frac{1}{31}$$

$$\tau_{i,j} = (1-\rho)\,\tau_{i,j} + \sum_{k=1}^{m} \Delta\tau_{i,j}^{k}$$

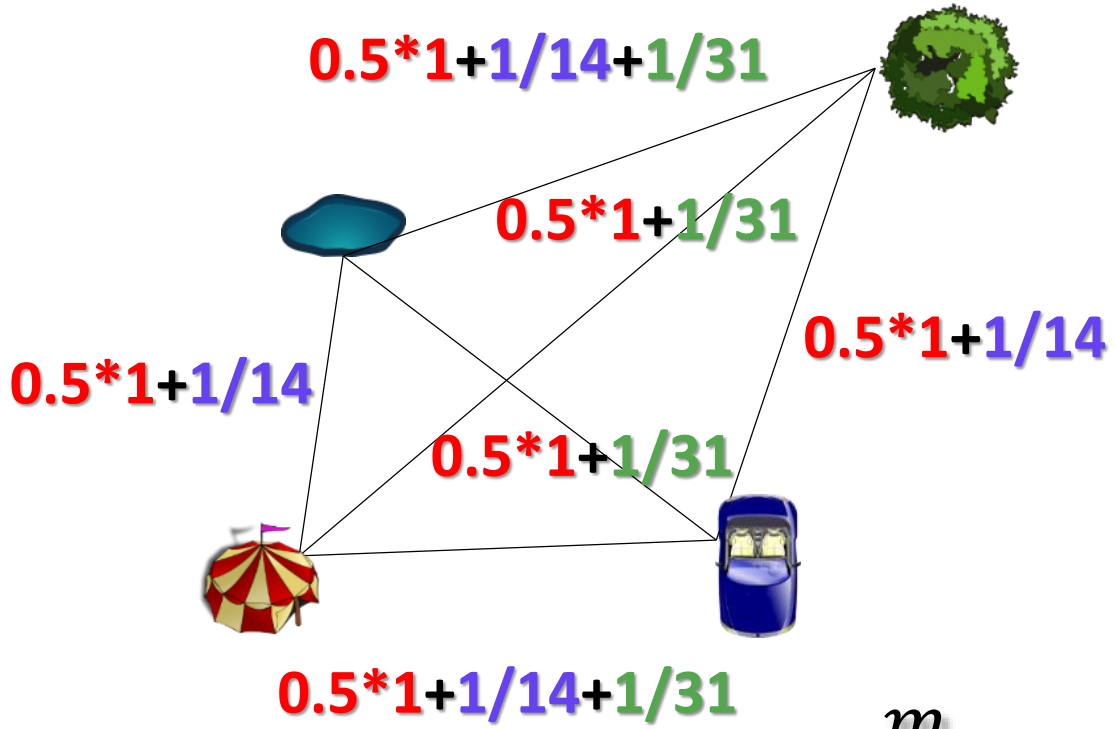
PHEROMONE MATRIX

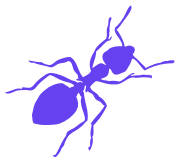
$$\rho = 0.5$$

Cost graph

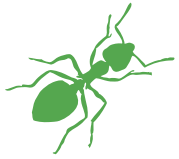


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$$



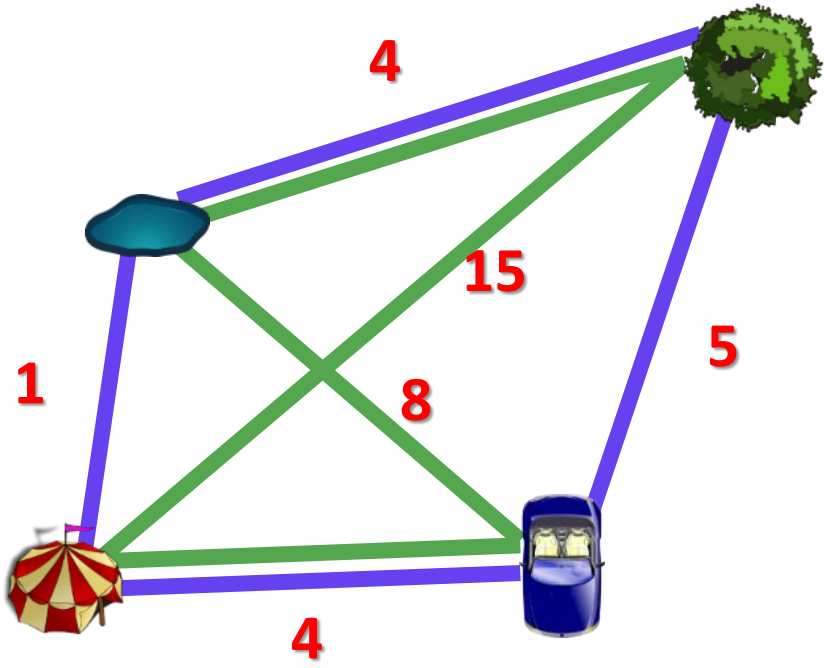
$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{31}$$

$$\tau_{i,j} = (1-\rho)\,\tau_{i,j} + \sum_{k=1}^{m} \Delta\tau_{i,j}^k$$

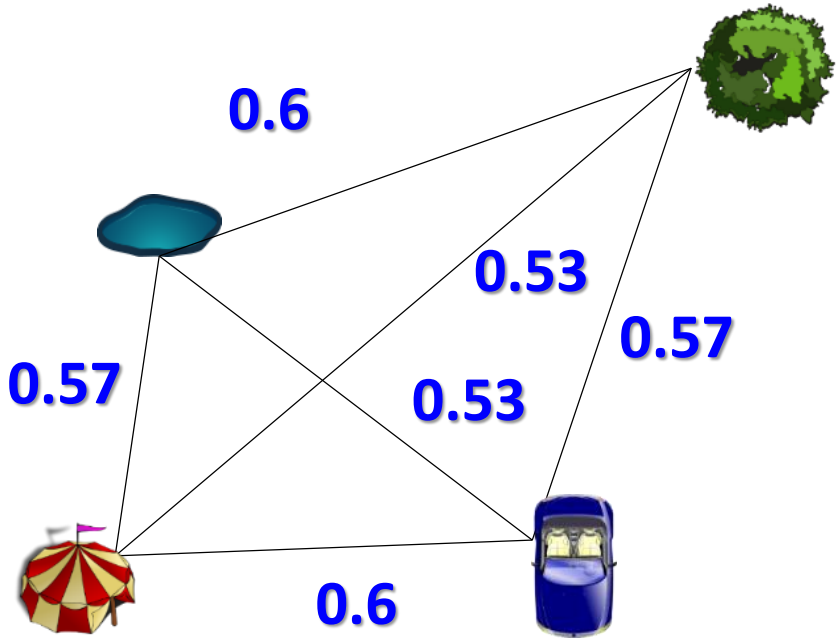
PHEROMONE MATRIX

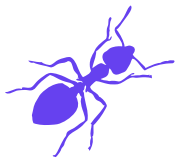
$$\rho = 0.5$$

Cost graph

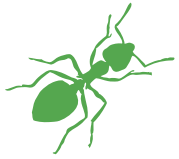


Pheromone graph





$$L_1 = 14 \rightarrow \Delta\tau_{i,j}^1 = \frac{1}{14}$$



$$L_2 = 31 \rightarrow \Delta\tau_{i,j}^2 = \frac{1}{31}$$

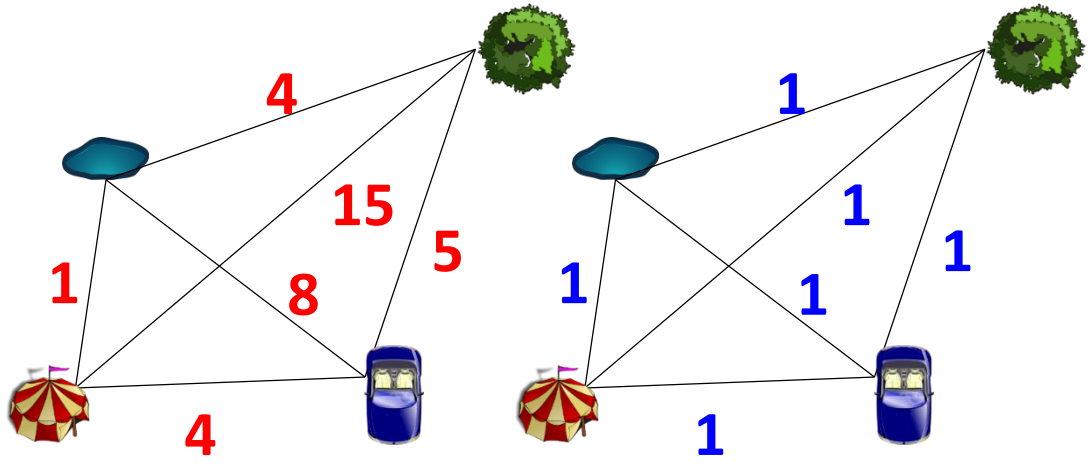
$$\tau_{i,j} = (1-\rho)\,\tau_{i,j} + \sum_{k=1}^{m} \Delta\tau_{i,j}^k$$

CALCULATING THE PROBABILITIES

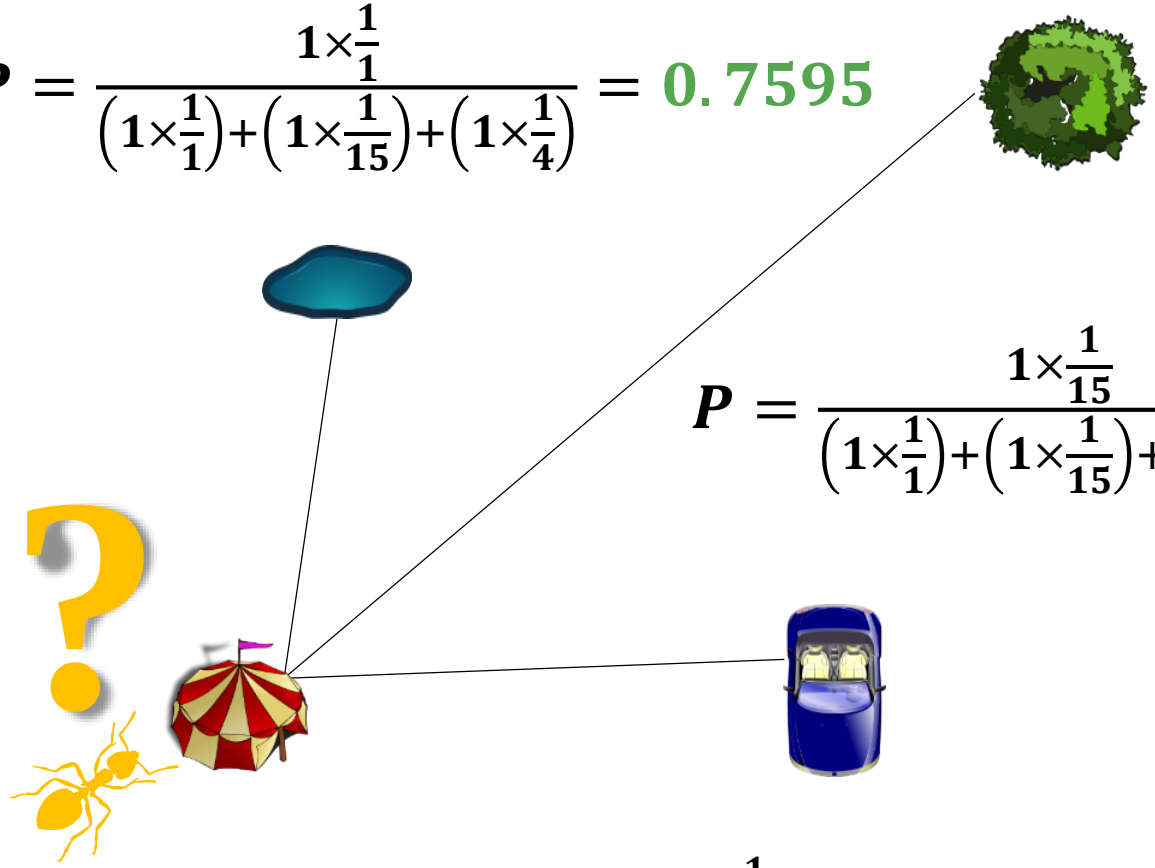
$$P_{i,j} = \frac{(\tau_{i,j})^{\alpha}(\eta_{i,j})^{\beta}}{\sum\left((\tau_{i,j})^{\alpha}(\eta_{i,j})^{\beta}\right)}$$

$$where: \eta_{i,j} = \frac{1}{\mathrm{L}_{i,j}}$$

NUMERICAL EXAMPLE



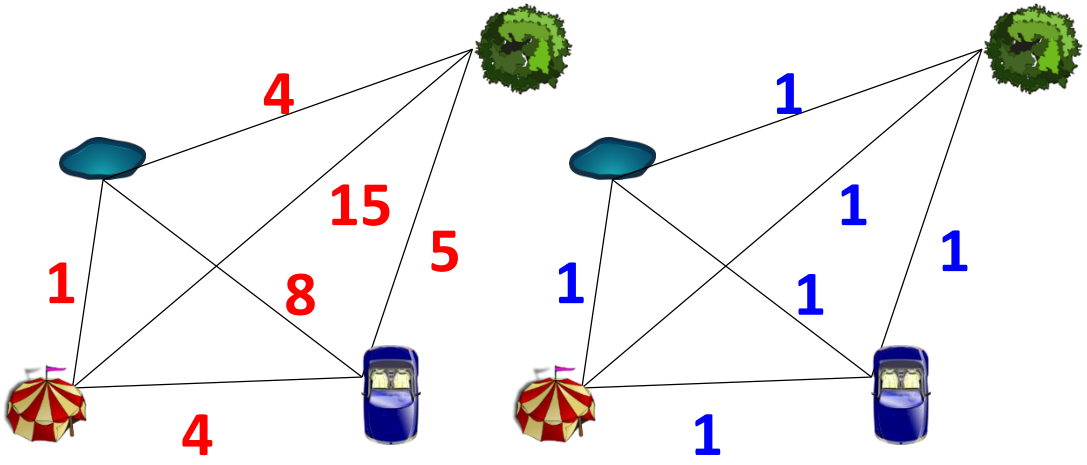
$$P = \frac{1\times\frac{1}{1}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}\right)+\left(1\times\frac{1}{4}\right)} = 0.7595$$

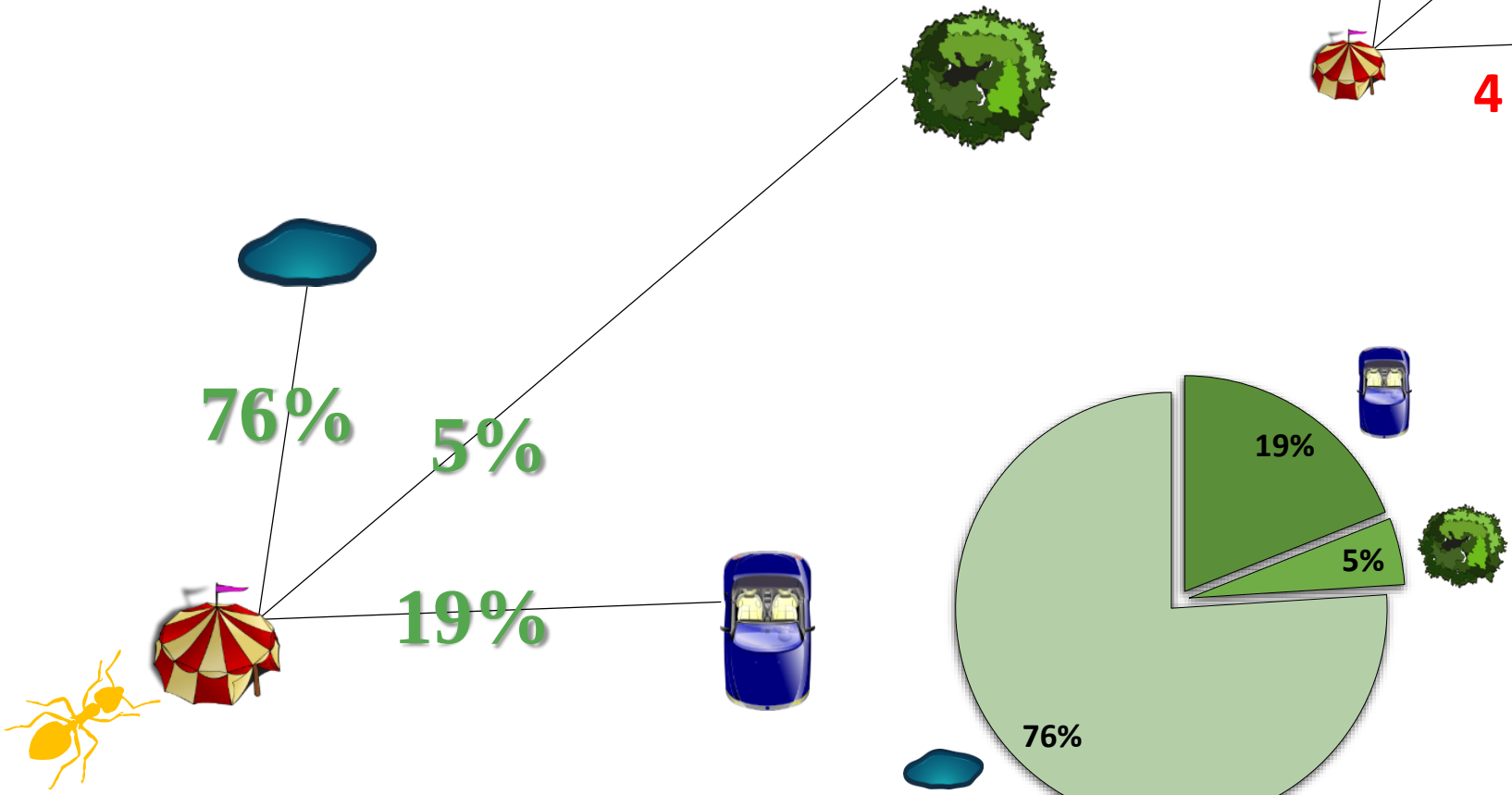


$$P = \frac{1\times\frac{1}{15}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}\right)+\left(1\times\frac{1}{4}\right)} = 0.0506$$

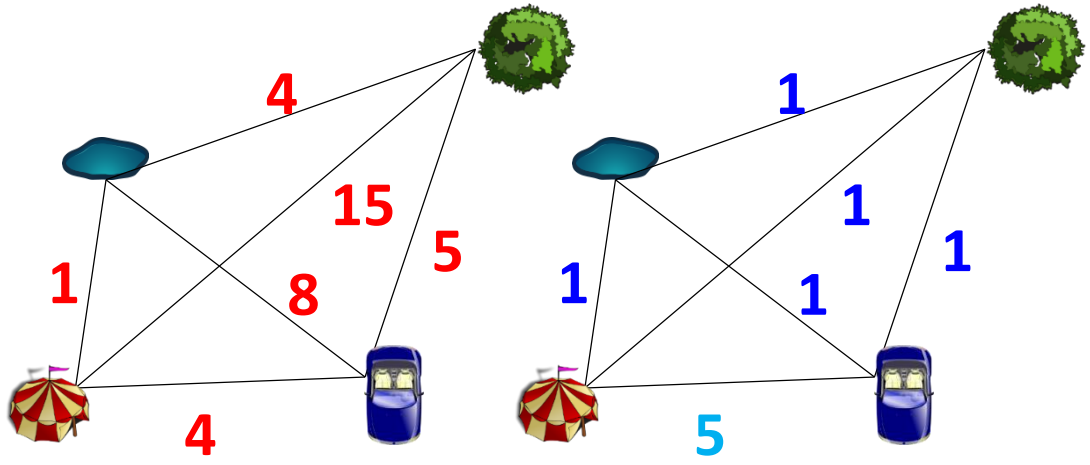
$$P = \frac{1\times\frac{1}{4}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}\right)+\left(1\times\frac{1}{4}\right)} = 0.1899$$

NUMERICAL EXAMPLE

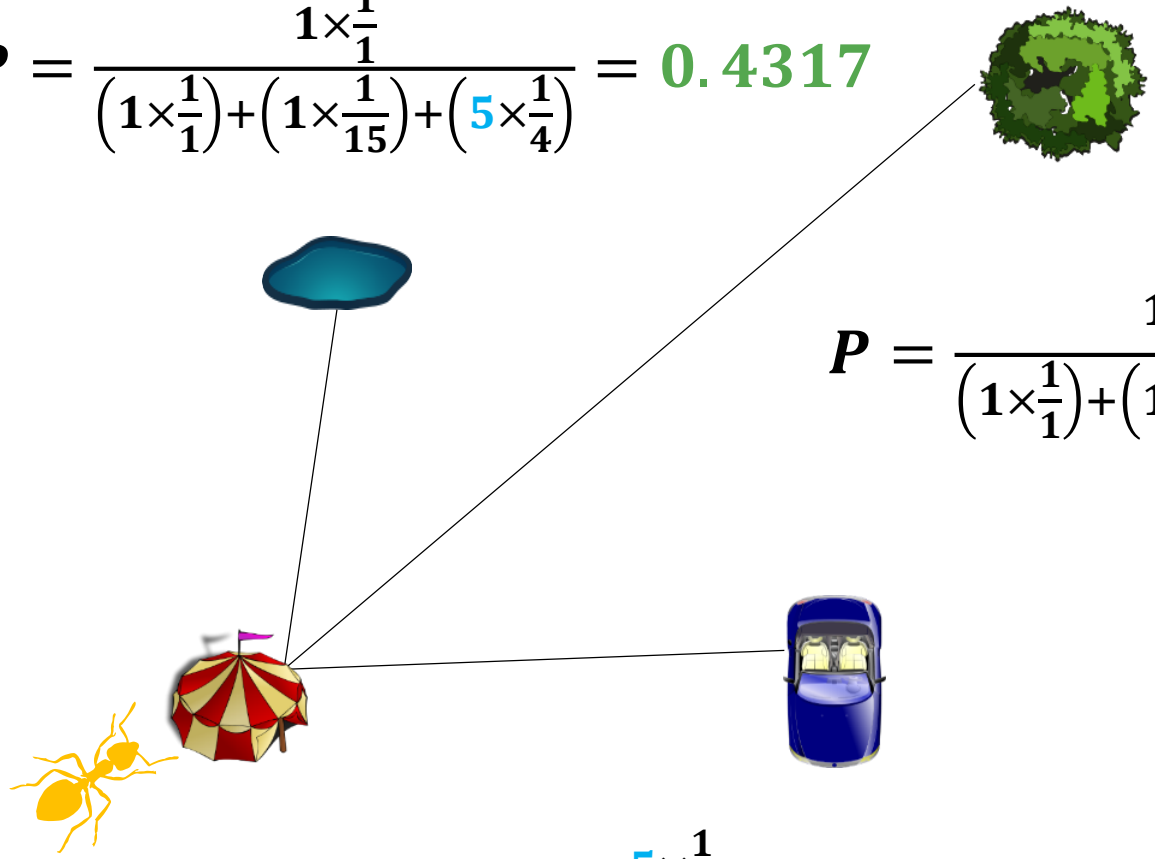




NUMERICAL EXAMPLE



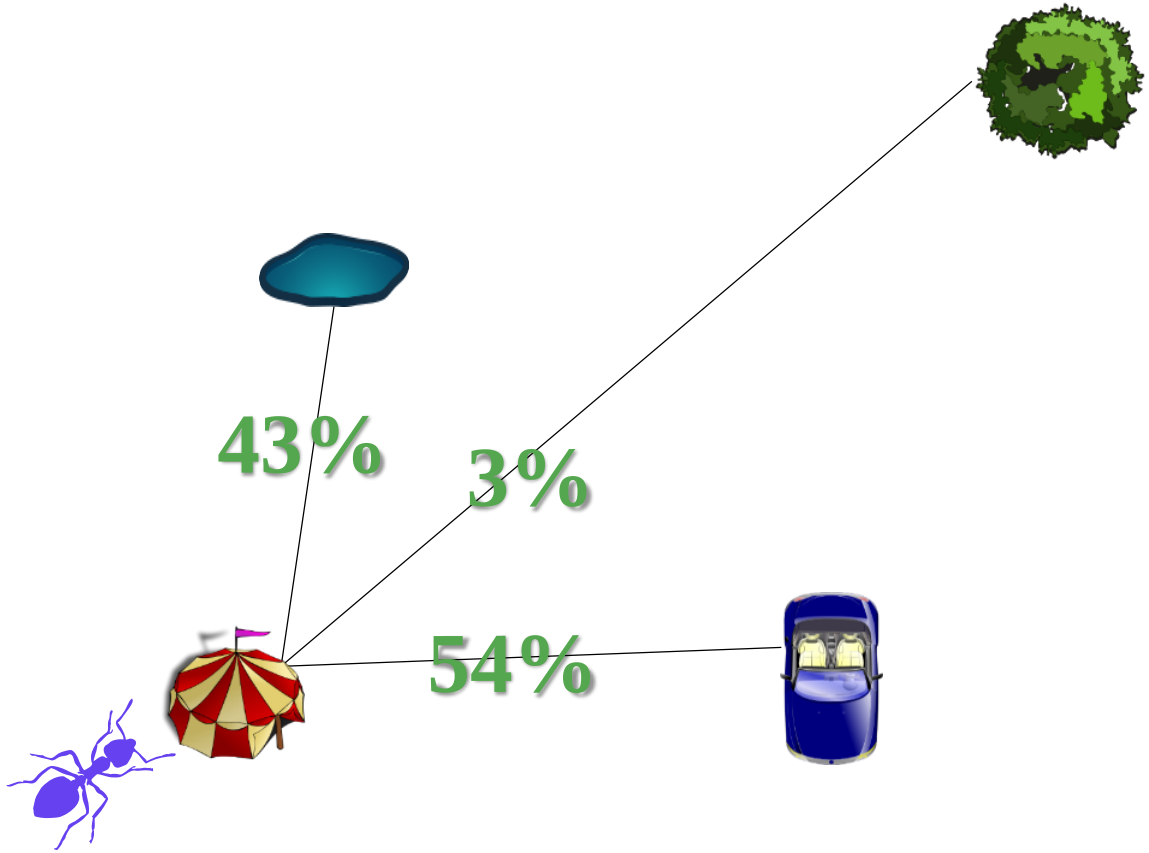
$$P = \frac{1\times\frac{1}{1}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}\right)+\left(5\times\frac{1}{4}\right)} = 0.4317$$

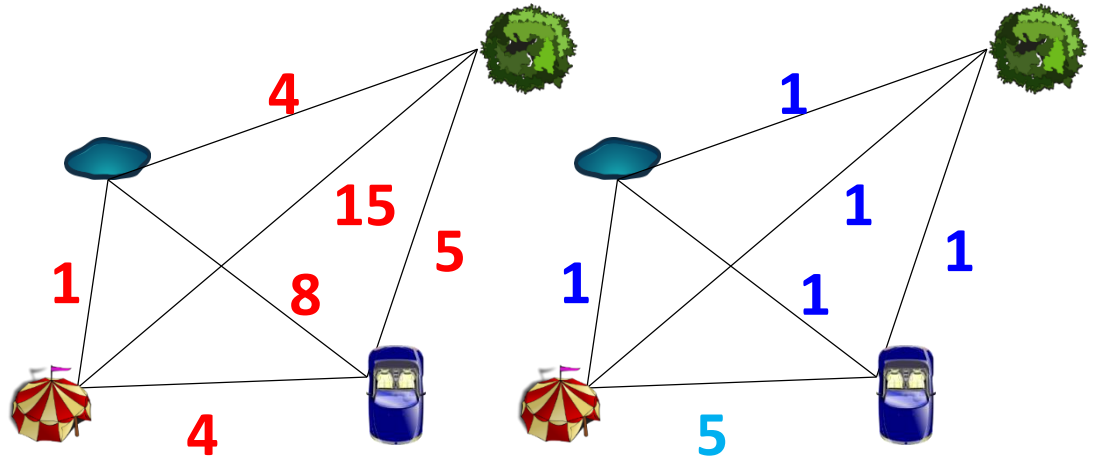


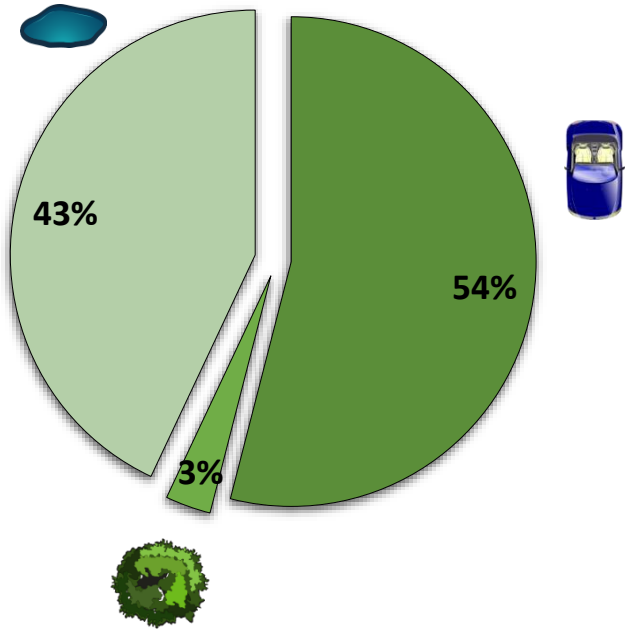
$$P = \frac{1\times\frac{1}{15}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}+\right)\left(5\times\frac{1}{4}\right)} = 0.0288$$

$$P = \frac{5\times\frac{1}{4}}{\left(1\times\frac{1}{1}\right)+\left(1\times\frac{1}{15}\right)+\left(5\times\frac{1}{4}\right)} = 0.5396$$

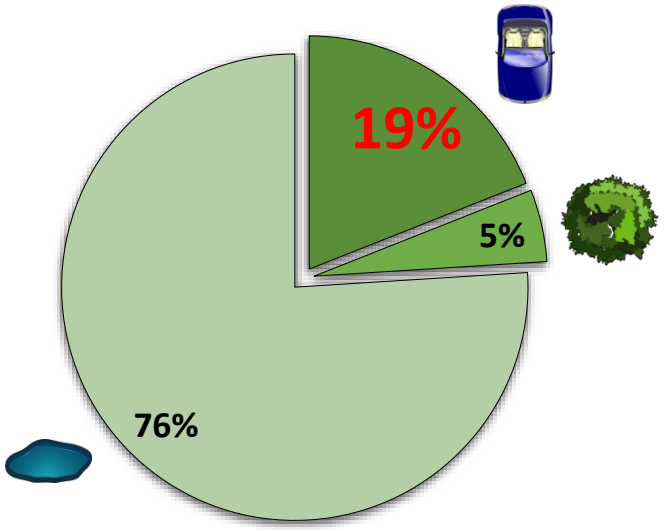
NUMERICAL EXAMPLE

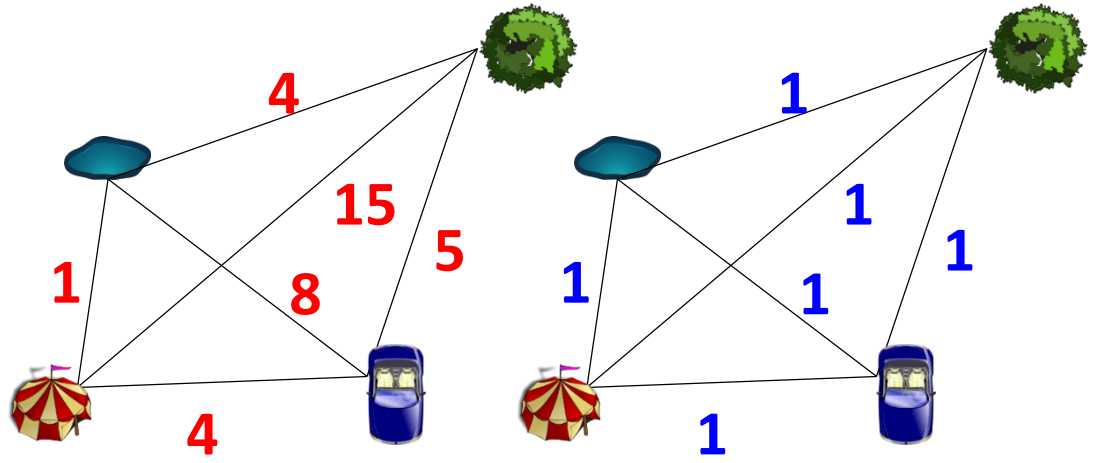


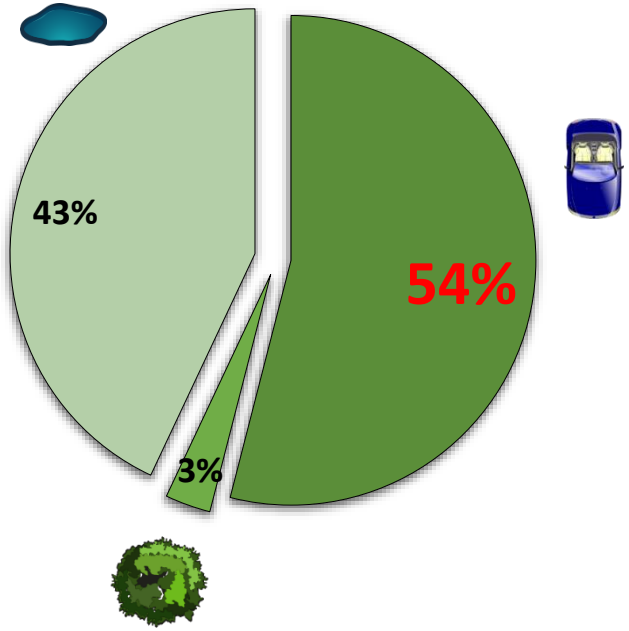


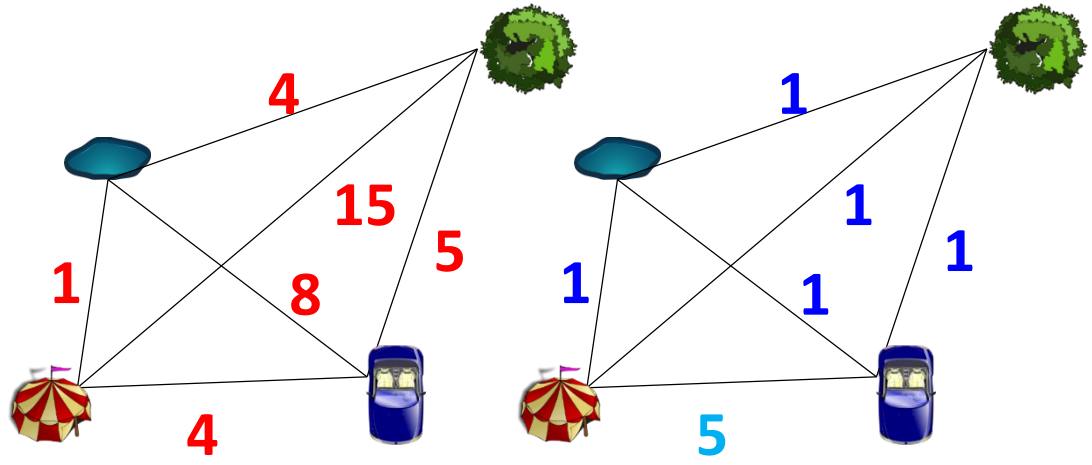


NUMERICAL EXAMPLE

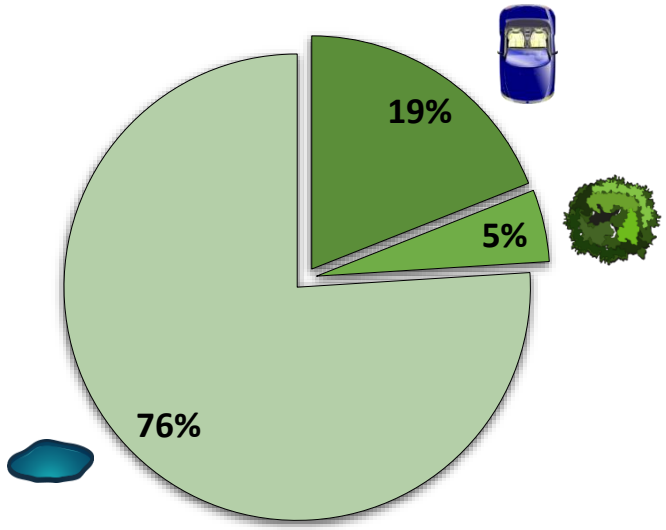




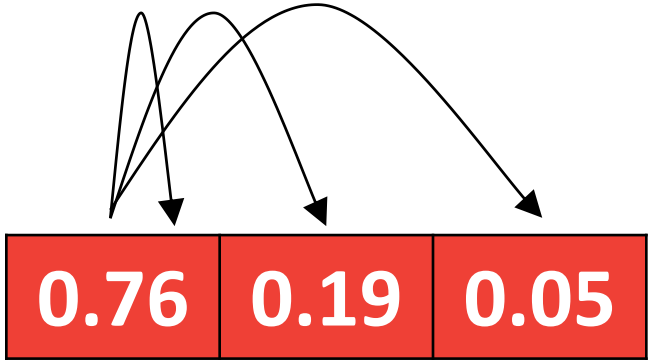




ROULETTE WHEEL

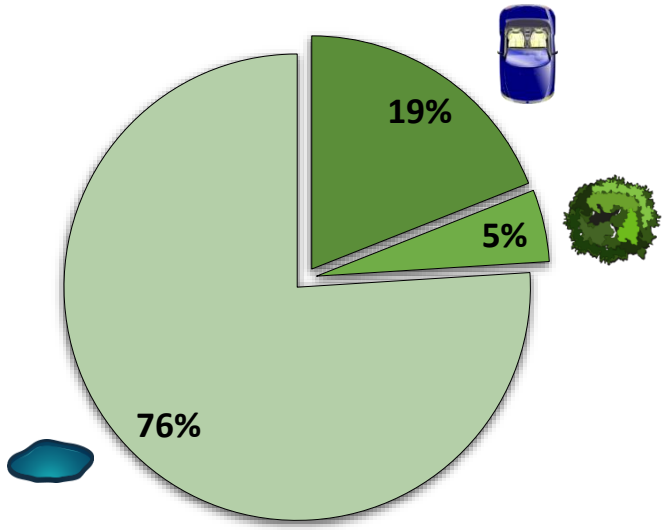


Probabilistic

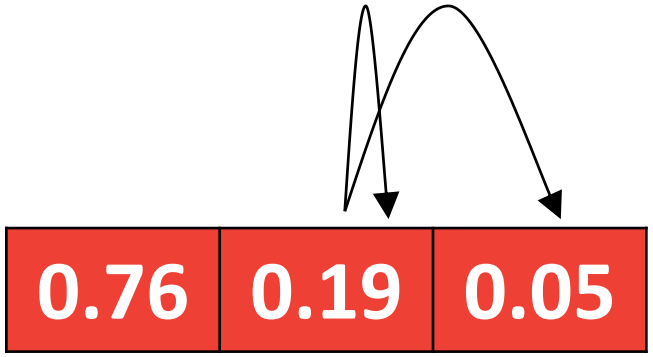


Cumulative sum

ROULETTE WHEEL



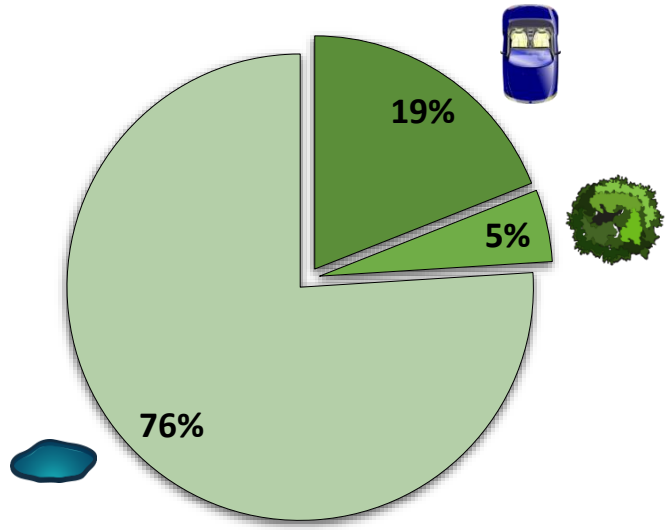
Probabilistic



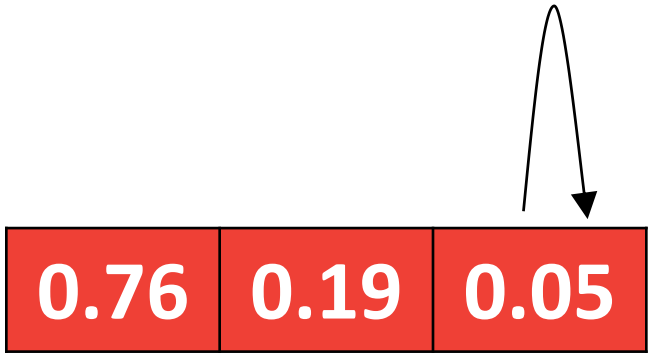
Cumulative sum

1	0.24	

ROULETTE WHEEL

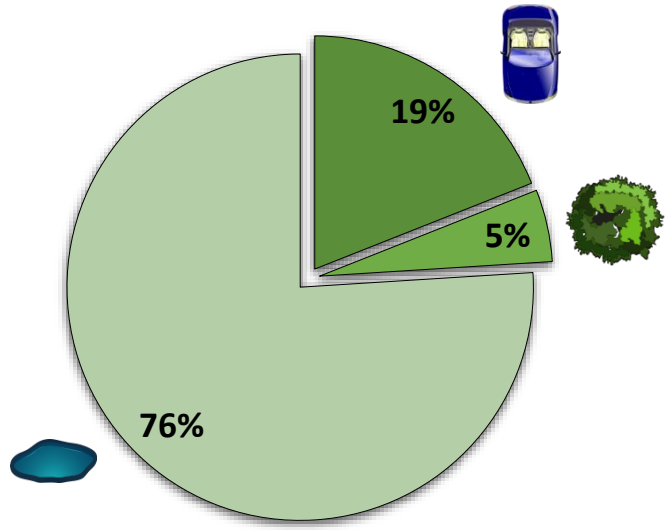


Probabilistic



Cumulative sum

ROULETTE WHEEL



Probabilistic

0.76	0.19	0.05

Cumulative sum

1	0.24	0.05

A random number (r) in [0,1]

$$\begin{cases} 0.24 < r \leq 1.00 \\ 0.05 < r \leq 0.24 \\ 0.00 \leq r \leq 0.05 \end{cases}$$